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ABSTRACT

The importance of tail risk for asset pricing has been widely recognized in theory and extensively studied in equity markets empirically. We construct one of the first model-free measures of tail risk for fixed-income markets using proprietary datasets of swaptions, denoted as `TAIL`, which captures tail risk across different horizons and recessionary states with high or low interest rates. `TAIL` has strong predictive power for returns on Treasuries, corporate bonds, mortgage-backed securities, fixed-income hedge funds, and even equities, suggesting that interest rate tail risk is priced in all major financial markets. We document strong links between `TAIL` and economic fundamentals.

Keywords: Fixed-income, Swaption, Tail risk

JEL classification: G12, G13

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1 Introduction

The importance of tail risk or rare disaster risk for asset pricing has been widely recognized in the literature (Tsai and Wachter, 2015). Rietz (1988) is probably the first study to show that low-probability economic disasters, such as the Great Depression, can explain the “equity premium puzzle” of Mehra and Prescott (1985). Global economic and financial crises in the past several decades have renewed the interest in this topic. For example, recent theoretical works, such as Barro (2006), Gabaix (2012), Gourio (2011), Julliard and Ghosh (2012), Martin (2013), and Wachter (2013), have shown that models with macroeconomic (consumption and GDP) tail risk can account for the high equity premium, low interest rate, excess market return volatility, aggregate market return predictability, and cross-sectional stock return variations.¹ While a range of innovative empirical measures of tail risk have been developed for stock markets, such as in Bollerslev and Todorov (2011, 2013), Kelly and Jiang (2014), and Gao, Gao, and Song (2017), little work has been done on tail risk of interest rate in fixed-income markets.

Dramatic movements in interest rates could have profound effects on all sectors of fixed-income markets, and even the entire financial system, given the systematic role of interest rates in discounting all cash flows. In particular, prices of fixed-income securities could be severely affected by dramatic changes in interest rates, which we refer to as tail risk in contrast to the risk caused by moderate fluctuations of interest rates. Several distinguishing features of interest rate tail risk, relative to equity tail risk, make it particularly informative about economic conditions and monetary policies. First, interest rate tail risk can differentiate the recessional economic states with high and low interest rates, e.g., due to hyperinflation and serious deflation, respectively. In contrast, equity tail risk is about market downturns and does not differentiate these two recessional states. Second, interest rate tail risk has a term structure inherited from the term structure of interest rates, unlike equity tail risk associated an infinite maturity. Such a term structure of tail risk can deliver valuable information about the permanent and transitory nature of extreme shocks to the economy. Third, interest rate tail risk is closely related to monetary policies of central banks in stimulating or slowing down the economy.

In this paper, we provide one of the first comprehensive studies of tail risk in fixed-income markets using a proprietary dataset of interest rate swaptions for about 20 years. We construct a model-free

¹Two recent works, Weitzman (2007) and Malmendier and Nagel (2011), study the formation of macro tail risk belief, and the risk-taking behavior of economic agents who experienced tail risk events, respectively.

measure of interest rate tail risk based on swaptions, denoted as T\AA ILL , by generalizing the concept of variance swaps for equities to interest rate swaps. Interest rate swaps and swaptions, one of the largest and most liquid segments of fixed-income markets and also one of the most important tools for interest rate risk management, are closely related to most other fixed-income securities.² LIBOR and swap rates represent the funding costs of most financial institutions, whereas swaptions have been widely used to hedge risks of fixed-income securities with embedded options, such as mortgage-backed securities and callable agency securities (Duarte, 2008). Moreover, “though the institutional structures of dollar swap and U.S. Treasury markets are different, some of the basic distributional characteristics of the associated yields are similar” (Dai and Singleton, 2000). Therefore, tail risk shocks to Treasury yields are likely to manifest in swap rates and swaption prices.

T\AA ILL is based on a *variance contract on swap rates* we develop that generalizes variance swaps for equities (Bakshi and Madan, 2000; Britten-Jones and Neuberger, 2000; Carr and Wu, 2009) to interest rate swaps. A variance swap on stock returns allows one to bet on the realized variance of stock returns and can be replicated by a portfolio of equity options. Similarly, a *variance contract on swap rates* allows one to hedge the risk of realized variance of interest rate swap rates and can be replicated using a portfolio of swaptions. However, a swaption is more complicated than an equity option because its payoff depends on both the underlying forward swap rate and a stochastic annuity discount factor; the payoff of an equity option depends only on the underlying stock price (Carr and Wu, 2009). To purely focus on the tail risk of swap rates, we define the payoff of the *variance contract on swap rates* as the realized variance of swap rates multiplied by the annuity discount factor. This adjustment makes it possible to replicate the *variance contract on swap rates* using a portfolio of swaptions.³

Our measure of tail risk equals the price difference between two swaption-based replication portfolios of the *variance contract on swap rates*. The first portfolio accounts for mild variations of swap rates, whereas the second incorporates extreme variations induced by dramatic changes in swap rates associated

²According to the Bank for International Settlements, the notional value outstanding of interest rate swaps (swaptions) exceeds \$379 (\$50) trillion on a net basis as of June 2012. In contrast, the notional value of all exchange-traded interest rate futures and options is only \$52 trillion. A 2009 survey by the International Swaps and Derivatives Association reports that 88.3% of Fortune Global 500 companies use swaps and swaptions to manage interest rate risk. Conversations with interest rate swap and swaption traders confirm that products with the sum of swaption maturity and swap tenor less than 30 years are fairly liquid.

³As a by-product, our design of the *variance contract on swap rates* can be used to compute model-free implied volatility for swaptions, which is similar to the VIX of the Chicago Board of Exchange (CBOE).

with tail events. We show that T\AA ILL captures all the high-order moments (≥ 3) of the jump measure associated with infrequent but extreme moves in swap rates. T\AA ILL has several appealing features. First, a term structure of T\AA ILL can be constructed using swaptions with various swap tenors, which captures the tail risk of swap rates over a range of investment horizons.⁴ Second, although T\AA ILL s of different investment horizons are computed under different annuity measures, they represent prices for protection against extreme swings in swap rates at these horizons and hence can be compared directly.⁵ Third, T\AA ILL is applicable to any finite swaption maturity (rather than shrinking maturity, which is required by measures based on high-frequency data), which is particularly important because many fixed-income portfolios involve long-term interest rate risk exposures.

We empirically estimate T\AA ILL using a proprietary database of interest rate swaptions provided by two of the largest inter-dealer brokers in interest rate derivatives markets, Barclays Capital and J.P. Morgan. The swaption prices are specified along three dimensions: swap tenor (ranging from 6 months to 30 years), swaption maturity (1 month to 10 years), and strike (200 basis points below and above at-the-money strike). The number of strikes for each swap tenor and swaption maturity is as many as 13, including deeply out-of-the-money strikes that are important for quantifying tail risk. The data cover a long sample period from June 1993 through January 2013, spanning two NBER-defined recessions, including the recent financial crisis, and periods of great inflation and deflation concern.

We first document a pronounced skew in the average implied volatility of swaptions, with OTM receiver swaptions having higher implied volatility than ATM swaptions. The volatility skew reflects potential tail risk in swap rates given the fat-tailed distribution of forward swap rates. Our estimates of one-month T\AA ILL (i.e., tail risk implied by swaptions with one-month to maturity) with swap tenors of 1, 2, 5, 10, 20, and 30 years show that the tail risk of swaps rates is significantly time-varying and captures episodes of high tail risk, such as the collapse of LTCM, the 2002 stock market downturn, and especially the 2007-2008 financial crisis. Moreover, swap rates exhibit both upside and downside risks conditional on different economic scenarios. The magnitude of tail risk is larger at the short-horizon than long-horizon,

⁴It is important to note that the term structure here refers to the tenor of swaps rather than the maturity of swaptions. Although the equity tail measure can be computed over various option horizons, there is no term structure associated with the underlying equity index (Bollerslev and Todorov, 2011).

⁵In contrast, implied moments of swap rates from swaptions are tied to annuity measures at specific swaption maturities and swap tenors, and a direct comparison does not deliver a clear economic meaning due to the impact of the stochastic discount rate (Trolle and Schwartz, 2014).

suggesting that tail risk has a mean-reverting pattern.

We document strong predictive power of TAILL for returns of important fixed-income securities including US Treasury bonds, corporate bonds, commercial mortgage-backed securities (CMBS), and fixed-income hedge funds. For example, in univariate regressions, a one-standard-deviation-change in TAILL of 5-year tenor induces a 0.48, 0.43, 0.40, and 0.31 standard deviation change in the excess returns of Treasury bonds, corporate bonds, CMBS, and hedge fund return indices, respectively. Multivariate regressions show that, collectively TAILL s at all swap tenors explain up to 30%, 60%, and 20% of the variations in excess returns of Treasury and corporate bonds, CMBS, and fixed-income hedge funds, respectively. The predictive power of TAILL remains significant after controlling for the equity tail measures of Bollerslev and Todorov (2011) and Kelly and Jiang (2014), as well as the powerful bond return predictors of Cochrane and Piazzesi (2005) and Ludvigson and Ng (2009). Moreover, the return predictability of TAILL prevails in-sample and out-of-sample and extends to equity returns. In addition, although our analysis focuses mainly on tail risk at the one-month horizon, TAILL s constructed using one-year swaptions have even stronger predictive power for one-year holding period returns. Finally, the return predictability of TAILL is robust to excluding the recent financial crisis and multiple ways of constructing returns on Treasury securities.

Similar to measures of equity tail risk based on financial asset prices, our TAILL measure also contains the tail risk premium and does not quantify the tail risk of economic fundamentals directly. To better understand the economic nature of tail risk, we study contemporaneous relations between TAILL and economic variables that measure macroeconomic risk, credit risk, mortgage hedging activity, and funding liquidity. We find that the macro and credit risk factors tend to be strongly correlated with TAILL at long swap tenors (10 to 20 years). The mortgage hedging activity has strong correlations with TAILL at intermediate tenors (5 to 10 years), consistent with the fact that most MBS have durations of 5-10 years. Liquidity factors are largely correlated with TAILL at short tenors. Given that TAILL captures the market perception of tail risk over a future period of time, we examine the predictive power of TAILL for future values of these economic variables. We find that short-term TAILL has significant predictive power for future funding liquidity and mortgage factors, whereas long-term TAILL has significant predictive power for market liquidity, credit, mortgage hedging, and macro factors, with R^2 s between 15% and 40%. Such

predictive power remains strong up to a year. Overall, we find close connections between TALL and fundamental economic factors.

Our study contributes to the empirical literature of disaster risk, which focuses on equities mostly (e.g., Bali, Cakici, and Whitelaw, (2014), Bollerslev and Todorov (2011, 2013); Chapman and Gallmeyer (2014); Kelly and Jiang (2014)).⁶ Differently, we propose an innovative measure of interest rate tail risk and document the importance of time-varying tail risk premium for all major fixed-income securities. Our analysis provides novel findings on tail risk – the differential pricing of tail risk in different economic states (of high and low interest rates) and across different investment horizons (of long-term and short-term). Such characteristics cannot be uncovered using measures of equity tail risk that does not differentiate deflation and hyperinflation scenarios and has one single maturity for the underlying security (aggregate stock market). Combined with the documented connection between TALL and fundamental economic factors, these findings suggest that an extension of macro rare disaster risk models with liquidity and mortgage hedging factors can potentially explain tail risk dynamics at different maturities and their strong predictive power for both bond and equity returns.⁷ We leave this extension for future exploration.

Our paper also contributes to the large literature on bond risk premia, which has identified a set of factors with predictive power for bond excess returns. These include a “tent-shape” factor by Cochrane and Piazzesi (2005), a cycles factor by Cieslak and Povala (2011), macro factors by Cooper and Priestley (2009), Ludvigson and Ng (2009), and Joslin, Priebsch, and Singleton (2012), funding liquidity by Fontaine and Garcia (2011), a hidden factor by Duffee (2011), skewness risk by Trolle and Schwartz (2014), and mortgage convexity by Hanson (2012) and Malkhozov et al. (2013). Our paper shows that the time-varying tail risk premium is an important component of bond excess returns. We also show that the predictive power of TALL for asset returns (across the fixed-income market) is robust to volatility factors.⁸

Our paper is also related to the literature on interest rate volatility and derivatives pricing. Many studies investigate the “unspanned stochastic volatility” of interest rates and parametric models for pricing interest rate caps/floors and swaptions, such as Collin-Dufresne and Goldstein (2002), Collin-Dufresne,

⁶A related literature studies the explanatory power of tail risk for equity option prices, such as Backus, Chernov, and Martin (2011), Bates (2008), Drechsler (2013), Liu, Pan, and Wang (2005), and Seo and Wachter (2015) among others.

⁷A recent study, Tsai (2013), calibrates standard macro rare disaster risk models to match certain average characteristics of Treasury yield curves.

⁸Mueller, Vedolin, and Yen (2013) shows that model-free implied volatility measures constructed from Treasury futures options have strong predictive power for bond returns. To control for volatility factors, we include both at-the-money Black-implied volatility of swaptions and a model-free implied volatility based on swaptions.

Goldstein, and Jones (2009), Heidari and Wu (2003), Jacobs and Karoui (2009), Li and Zhao (2006), Longstaff, Santa-Clara, and Schwartz (2001), Fan, Gupta, and Ritchken (2003), Han (2007), Jarrow, Li, and Zhao (2007), and Trolle and Schwartz (2014) among others. Our study differs from these in two important ways. First, we propose a model-free tail risk measure without relying on any parametric model. Second, we focus on the risk premia of fixed-income asset returns driven by tail risk rather than fitting derivative prices.

The rest of the paper is organized as follows. In section 2, we introduce the *variance contract on swap rates* and its two swaption-based replication portfolios, through which we construct a model-free measure of tail risk. Section 3 documents a volatility skew in swaptions and provides empirical estimates of TAIL . Section 4 studies return predictability of TAIL for a wide range of fixed-income securities as well as equities. Section 5 investigates the link between TAIL and fundamental economic variables. Section 6 concludes. The appendix provides technical details. A separate Internet Appendix provides details on the data and more empirical results.

2 Tail Risk Measure Based on Swaptions

In this section, we develop a model-free measure of tail risk for fixed-income markets based on interest rate swaptions. After a brief discussion of swaptions, we develop the *variance contract on swap rates* and construct a tail risk measure based on the price difference between two replication portfolios of the *variance contract on swap rates* using swaptions.

2.1 Interest Rate Swaption

Consider a forward start fixed versus floating interest rate swap with a start date T_m and maturity date T_n . The fixed annuity payments are made on a pre-specified set of dates, $T_{m+1} < T_{m+2} < \dots < T_n$, with the intervals equally spaced by δ , which equals six months in U.S. swaption markets. The floating payments tied to the three-month LIBOR are made quarterly at $T_{m+1} - \delta/2$, T_{m+1} , $T_{m+2} - \delta/2$, T_{m+2} , \dots , $T_n - \delta/2$, and T_n .⁹

At time T_m , the value of the floating leg equals par, and the time- t value of the floating leg is $D(t, T_m)$,

⁹We assume that both the fixed and floating legs pay \$1 principal at T_n .

where $D(t, T)$ is the time- t price of a zero-coupon bond maturing at time T . The time- t value of the fixed leg is equal to $D(t, T_n) + A_{m,n}(t)$, where $A_{m,n}(t) \equiv \sum_{j=m+1}^n D(t, T_j)$ is the present value of an annuity associated with the fixed leg of the forward swap contract, also known as the “price value of the basis point” (PVBPP) of a swap. The time- t forward swap rate, $S_{m,n}(t)$, is the rate on the fixed leg that makes the present value of the swap contract equal to zero at t :

$$S_{m,n}(t) = \frac{D(t, T_m) - D(t, T_n)}{A_{m,n}(t)}. \quad (1)$$

This forward swap rate becomes the spot swap rate $S_{m,n}(T_m)$ at time T_m .

A swaption gives its holder the right but not the obligation to enter into an interest rate swap either as a fixed leg (payer swaption) or as a floating leg (receiver swaption) with a pre-specified fixed coupon rate. The underlying security of a swaption is a forward start interest rate swap contract. For example, let T_m be the expiration date of the swaption, K be the coupon rate on the swap, and T_n be the final maturity date of the swap. The payer swaption allows the holder to enter into a swap at time T_m with a remaining term of $T_n - T_m$ and to pay the fixed annuity of K . At time t , this swaption is usually called a $(T_m - t)$ into $(T_n - T_m)$ payer swaption, also known as a $(T_m - t)$ by $(T_n - T_m)$ payer swaption, where $(T_m - t)$ is the option maturity and $(T_n - T_m)$ is the tenor of the underlying swap. Because the value of the floating leg will be par at time T_m , the payer swaption is equivalent to a put option on a bond with a coupon rate K and a remaining maturity of $T_n - T_m$, where the strike of this put option is \$1. Similarly, the receiver swaption is equivalent to a call option on the same coupon bond with a strike price of \$1.

Let $\mathcal{P}_{m,n}(t; K)$ and $\mathcal{R}_{m,n}(t; K)$ denote the time- t value of a European payer and receiver swaption, respectively, expiring at T_m with strike K on a forward start swap for the time period between T_m and T_n . At the option expiration date T_m , the payer swaption has a payoff of

$$[1 - D(T_m, T_n) - KA_{m,n}(T_m)]^+ = A_{m,n}(T_m) [S_{m,n}(T_m) - K]^+,$$

where equation (1) evaluated at T_m is used. Therefore, the time- t ($< T_m$) price of this payer swaption is

given by

$$\mathcal{P}_{m,n}(t; K) = \mathbb{E}_t^{\mathbb{Q}} \left\{ e^{-\int_t^{T_m} r(s) ds} A_{m,n}(T_m) [S_{m,n}(T_m) - K]^+ \right\} = A_{m,n}(t) \mathbb{E}_t^{\mathbb{A}^{m,n}} \{ [S_{m,n}(T_m) - K]^+ \}, \quad (2)$$

where \mathbb{Q} is the risk-neutral measure and $\mathbb{A}^{m,n}$ is the annuity measure with $A_{m,n}(t)$ as the numeraire. That is, the Radon-Nikodym derivative of the annuity measure with respect to the risk-neutral measure is $\frac{d\mathbb{A}^{m,n}}{d\mathbb{Q}} = e^{-\int_t^{T_m} r(s) ds} \frac{A_{m,n}(T_m)}{A_{m,n}(t)}$. Similarly, the time- t price of the receiver swaption is given by

$$\mathcal{R}_{m,n}(t; K) = A_{m,n}(t) \mathbb{E}_t^{\mathbb{A}^{m,n}} \{ [K - S_{m,n}(T_m)]^+ \}. \quad (3)$$

We note from (2) and (3) that a swaption is tied to two sources of uncertainty: (i) the forward swap rate $S_{m,n}(t)$, and (ii) the swap's PVBP realized at time T_m , $A_{m,n}(T_m)$. The change of measure from \mathbb{Q} to $\mathbb{A}^{m,n}$ allows us to focus on the risk of $S_{m,n}(t)$ and facilitates the pricing of swaptions.

2.2 Variance Contract on Swap Rates

In this section, we introduce the *variance contract on swap rates* and its two swaption-based replication portfolios, through which we construct our model-free measure of tail risk. Variance swaps on equities allow one to hedge the risk of the realized variance of stock returns. The *variance contract on swap rates* that we develop below allows us to hedge the risk of the realized variance of interest rate swap rates.

Definition 1: Variance Contract on Swap Rates At time t , the short leg promises to pay the long leg at T_m

$$A_{m,n}(T_m) \left[\left(\ln \frac{S_{m,n}(t + \Delta)}{S_{m,n}(t)} \right)^2 + \left(\ln \frac{S_{m,n}(t + 2\Delta)}{S_{m,n}(t + \Delta)} \right)^2 + \dots + \left(\ln \frac{S_{m,n}(T_m)}{S_{m,n}(T_m - \Delta)} \right)^2 \right], \quad (4)$$

the product of the realized variance of the log forward swap rate $\log S_{m,n}(t)$ over $[t, T_m]$ and the PVBP $A_{m,n}(T_m)$. In return, the long leg pays the short leg $A_{m,n}(T_m) \times \mathbb{V}\mathbb{P}_{m,n}(t)$ at T_m , where $\mathbb{V}\mathbb{P}_{m,n}(t)$ is determined at time t such that the value of the contract equals zero at initiation. We refer to $\mathbb{V}\mathbb{P}_{m,n}(t)$ as the variance price of the forward swap rate.

The *variance contract on swap rates* uses the sum of squared log changes to measure the realized variance of forward swap rates over $[t, T_m]$. Similar to the payoff of a swaption, the payoff of the *variance contract on swap rates* depends on the realized variance of forward swap rates as well as an annuity discount factor. This design makes it convenient to obtain the variance price $\mathbb{V}\mathbb{P}_{m,n}(t)$ by a change of the risk-neutral measure to the corresponding annuity measure. It also makes it easier to replicate the variance contract using swaptions given the similar payoff structures.

For convenience in pricing, a continuous-time setup is usually employed by letting $\Delta \rightarrow 0$. Then the present value of the payment made by the long leg equals

$$\mathbb{E}_t^{\mathbb{Q}} \left\{ e^{-\int_t^{T_m} r(s) ds} A_{m,n}(T_m) \times \mathbb{V}\mathbb{P}_{m,n}(t) \right\} = A_{m,n}(t) \times \mathbb{V}\mathbb{P}_{m,n}(t), \quad (5)$$

while that made by the short leg is

$$\begin{aligned} & \mathbb{E}_t^{\mathbb{Q}} \left\{ e^{-\int_t^{T_m} r(s) ds} A_{m,n}(T_m) \lim_{\Delta \rightarrow 0} \left[\left(\ln \frac{S_{m,n}(t+\Delta)}{S_{m,n}(t)} \right)^2 + \left(\ln \frac{S_{m,n}(t+2\Delta)}{S_{m,n}(t+\Delta)} \right)^2 + \dots + \left(\ln \frac{S_{m,n}(T_m)}{S_{m,n}(T_m-\Delta)} \right)^2 \right] \right\} \\ &= A_{m,n}(t) \mathbb{E}_t^{\mathbb{A}^{m,n}} \left\{ \lim_{\Delta \rightarrow 0} \left[\left(\ln \frac{S_{m,n}(t+\Delta)}{S_{m,n}(t)} \right)^2 + \left(\ln \frac{S_{m,n}(t+2\Delta)}{S_{m,n}(t+\Delta)} \right)^2 + \dots + \left(\ln \frac{S_{m,n}(T_m)}{S_{m,n}(T_m-\Delta)} \right)^2 \right] \right\}. \end{aligned} \quad (6)$$

The limit inside the expectation, denoted as $[\ln S_{m,n}, \ln S_{m,n}]_t^{T_m}$, is the quadratic variation of the log swap rate process and measures the continuous-time sum of squared log changes.

By choosing the variance price $\mathbb{V}\mathbb{P}_{m,n}(t)$ such that the time- t value of the variance contract equals zero, we have

$$\mathbb{V}\mathbb{P}_{m,n}(t) = \mathbb{E}_t^{\mathbb{A}^{m,n}} \left\{ \lim_{\Delta \rightarrow 0} \left[\left(\ln \frac{S_{m,n}(t+\Delta)}{S_{m,n}(t)} \right)^2 + \left(\ln \frac{S_{m,n}(t+2\Delta)}{S_{m,n}(t+\Delta)} \right)^2 + \dots + \left(\ln \frac{S_{m,n}(T_m)}{S_{m,n}(T_m-\Delta)} \right)^2 \right] \right\}. \quad (7)$$

That is, the variance price $\mathbb{V}\mathbb{P}_{m,n}(t)$ is the $\mathbb{A}^{m,n}$ -expectation of the quadratic variation of the forward swap rate $S_{m,n}(t)$ over $[t, T_m]$.

2.3 Tail Risk Measure via Two Swaption-Based Replication Portfolios

It has been shown that the payoff of a variance swap on equity can be replicated using a portfolio of out-of-the-money equity options. Similarly, the time-varying payoff of the *variance contract on swap rates* can be replicated using a portfolio of out-of-the-money swaptions written on $S_{m,n}(t)$. In this section, we consider two swaption-based replication portfolios for the variance contract, through which we construct our model-free measure of tail risk.

The first replication portfolio generalizes the algorithm used by CBOE in constructing VIX and focuses on the limit of the discrete sum of squared log changes:

$$\mathbb{IV}_{m,n}(t) \equiv \frac{2}{A_{m,n}(t)} \left\{ \int_{K > S_{m,n}(t)} \frac{1}{K^2} \mathcal{P}_{m,n}(t; K) dK + \int_{K < S_{m,n}(t)} \frac{1}{K^2} \mathcal{R}_{m,n}(t; K) dK \right\}, \quad (8)$$

where $T_m - t$ is the time-to-maturity. As observed from (8), this replication portfolio contains positions in out-of-the-money swaptions with a weight that is inversely proportional to their strikes. A similar replication portfolio based on equity options has been employed in the literature to construct model-free implied volatility measures (Bollerslev, Tauchen, and Zhou, 2009; Carr and Wu, 2009).

To see how $\mathbb{IV}_{m,n}(t)$ is related to $\mathbb{VP}_{m,n}(t)$, we consider a general process for the forward swap rate under the annuity measure $\mathbb{A}^{m,n}$:

$$\frac{dS_{m,n}(t)}{S_{m,n}(t-)} = \mu_t dt + \sigma(S_{m,n}(t-)) dW_t^{\mathbb{A}} + \int_{R_0} (e^x - 1) [\omega(dx, dt) - \nu_t(x) dx dt], \quad (9)$$

where $W_t^{\mathbb{A}}$ denotes an $\mathbb{A}^{m,n}$ -standard Brownian motion, R_0 denotes the real line excluding zero, $S_{m,n}(t-)$ denotes the forward swap rate just prior to any jump at time t , and the random counting measure $\omega(dx, dt)$ realizes a nonzero value for a given x if and only if the forward swap rate jumps from $S_{m,n}(t-)$ to $S_{m,n}(t) = S_{m,n}(t-)e^x$ at time t . The process $\nu_t(x)$ compensates the jump process so that the last term in Equation (9) is the increment of a pure jump martingale under the annuity measure $\mathbb{A}^{m,n}$. Equation (9) specifies the forward swap rate process as the summation of three terms: a process of finite variation, a purely continuous martingale, and a purely jump martingale. This decomposition is generic for a semi-martingale process, the most flexible process for asset prices. Moreover, we assume that the jump process is of finite variation: $\int_{R_0} (|x| \wedge 1) \nu_t(x) dx < \infty$.

The relation between the price of the replication portfolio, $\mathbb{IV}_{m,n}(t)$, and the variance price of the forward swap rate, $\mathbb{VP}_{m,n}(t)$, is given below.

Proposition 1. Under process (9), we have (i)

$$\begin{aligned}\mathbb{IV}_{m,n}(t) &= 2\mathbb{E}_t^{\mathbb{A}^{m,n}} \left(\int_t^{T_m} \frac{dS_{m,n}(t)}{S_{m,n}(t-)} - \ln \frac{S_{m,n}(T_m)}{S_{m,n}(t)} \right) \\ &= \mathbb{VP}_{m,n}(t) - 2\mathbb{E}_t^{\mathbb{A}^{m,n}} \int_t^{T_m} \int_{R_0} (1+x+x^2/2-e^x) \omega [dx, dt],\end{aligned}$$

and (ii) when the process of $S_{m,n}(t)$ is continuous,

$$\mathbb{IV}_{m,n}(t) = \mathbb{VP}_{m,n}(t).$$

Therefore, the replication portfolio in (8) perfectly replicates $\mathbb{VP}_{m,n}(t)$ when the forward swap rate does not jump and contains an error term when jumps are allowed.

The second replication portfolio relies on $Var_t^{\mathbb{A}^{m,n}} [\ln(S_{m,n}(T_m)/S_{m,n}(t))]$ and avoids discrete sum approximation:

$$\begin{aligned}\mathbb{V}_{m,n}(t) \equiv & \frac{2}{A_{m,n}(t)} \left\{ \int_{K>S_{m,n}(t)} \frac{1 - \ln(K/S_{m,n}(t))}{K^2} \mathcal{P}_{m,n}(t; K) dK \right. \\ & \left. + \int_{K<S_{m,n}(t)} \frac{1 - \ln(K/S_{m,n}(t))}{K^2} \mathcal{R}_{m,n}(t; K) dK \right\} - (\Psi_{m,n}(t))^2,\end{aligned}\quad (10)$$

where $\Psi_{m,n}(t) = \frac{1}{A_{m,n}(t)} \left\{ \int_{K>S_{m,n}(t)} \frac{1}{K^2} \mathcal{P}_{m,n}(t; K) dK + \int_{K<S_{m,n}(t)} \frac{1}{K^2} \mathcal{R}_{m,n}(t; K) dK \right\}$. The relation between $\mathbb{V}_{m,n}(t)$ and $\mathbb{VP}_{m,n}(t)$ is given below.

Proposition 2. Under process (9), we have (i) $\mathbb{V}_{m,n}(t) = Var_t^{\mathbb{A}^{m,n}} \left(\ln \frac{S_{m,n}(T_m)}{S_{m,n}(t)} \right)$ and (ii) $\mathbb{V}_{m,n}(t) = \mathbb{VP}_{m,n}(t)$.

Therefore, portfolio (10) can perfectly replicate $\mathbb{VP}_{m,n}(t)$ under process (9). In fact, this equivalence holds for general Lévy processes, which include most commonly used models for asset prices such as the geometric Brownian motion, jump diffusion, and infinite activity Lévy process.¹⁰

¹⁰The equivalence breaks down when σ_t follows a stochastic volatility process, which introduces a stochastic drift to the

Observe that the second replication portfolio in (10) differs from the first in (8) by assigning larger (smaller) weights to more deeply out-of-the-money receiver (payer) swaptions: As K declines (increases), i.e., receiver (payer) swaptions become more out-of-the-money, $1 - \ln(K/S_t)$ becomes larger (smaller). Since more deeply out-of-the-money swaptions protect investors against larger swap rate changes, it is conceivable that the difference between $\mathbb{V}_{m,n}(t)$ and $\mathbb{IV}_{m,n}(t)$ captures investor expectations about variations of swap rates induced by large swings in the swap rates.

Our tail risk measure for swap rates is defined as

$$\text{TALL}(a, b) \equiv \mathbb{V}_{m,n}(t) - \mathbb{IV}_{m,n}(t) = 2\mathbb{E}_t^{\mathbb{A}^{m,n}} \int_t^T \int_{R_0} (1 + x + x^2/2 - e^x) \omega[dx, dt]. \quad (11)$$

where $a = T_m - t$ and $b = T_n - T_m$ represent swaption maturity and swap tenor, respectively, and Propositions 1 and 2 are used in the second equality. Therefore, $\text{TALL}(a, b)$ captures all the high-order (≥ 3) moments of the random jump measure $\omega[dx, dt]$ associated with large jumps of the forward swap rate, given that $e^x - (1 + x + x^2/2) = x^3/3 + x^4/4 + \dots$.

Although our tail risk measure is computed under the annuity measure that changes with swaption maturity and swap tenor, they represent prices for protection against extreme swings in swap rates. Therefore, TALL s of different swap tenors and swaption maturities can be compared directly, since they measure the economic significance of jumps in interest rates over a range of investment horizons.

3 Empirical Estimates of Tail Risk

In this section, we first document a volatility skew for interest rate swaptions. Then we provide empirical estimates of TALL based on swaptions with one-month to maturity and various swap tenors.

3.1 Volatility Skew of Swaptions

We obtain daily LIBOR rates with maturities of 3, 6, 9, and 12 months, as well as daily 2-, 3-, 4-, 5-, 7-, 10-, 15-, 20-, 25-, 30-, and 35-year spot swap rates between June 1, 1993 and January 31, 2013 from J.P.

forward rate process. However, simulation studies show that the replication error is negligible, and hence the replication portfolio (10) is fairly accurate in tracking $\mathbb{VP}_{m,n}(t)$; see Du and Kapadia (2012) for further discussion of the accuracy of this replication portfolio for equity variance swaps.

Morgan. We bootstrap the swap rates to first obtain daily zero-curves. Then we construct the PVBP curve $A_{m,n}(t)$ and forward swap rate curve $S_{m,n}(t)$ up to 35 years according to (1).¹¹

Daily observations of (European) swaption prices are combined from J.P. Morgan and Barclays Capital, two of the largest inter-dealer brokers in interest rate derivatives markets. For the whole sample, swaption prices are quoted for six swap tenors (1, 2, 5, 10, 20, and 30 years) and eight swaption maturities (1, 3, 6, 9 months and 1, 2, 5, 10 years). The market convention is to quote swaption prices in terms of their log-normal implied volatility based on the Black (1976) formula.¹²

The swaption prices from J.P. Morgan are available between June 1, 1993 and January 31, 2013 with five strikes, including at-the-money-forward (ATMF), $\text{ATMF} \pm 100$, and $\text{ATMF} \pm 50$ basis points. The swaption prices from Barclays are between December 1, 2004 and January 31, 2013 with thirteen strikes, including ATMF, $\text{ATMF} \pm 200$, $\text{ATMF} \pm 150$, $\text{ATMF} \pm 100$, $\text{ATMF} \pm 75$, $\text{ATMF} \pm 50$, and $\text{ATMF} \pm 25$ basis points. In our empirical analysis, we use the swaption prices from J.P. Morgan from June 1, 1993 through December 1, 2004 and those from Barclays after December 1, 2004.¹³

As a standard procedure to obtain swaption prices on a continuum of strikes as requested in equations (8) and (10) (Carr and Wu, 2009), we interpolate implied volatilities across the range of the observed strikes and use the implied volatility of the lowest (highest) available strike to replace that of the strikes below (above).¹⁴ We further generate 200 implied volatility points equally spaced over a strike range with moneyness between $0.9 \times S_{m,n}(t)$ and $1.1 \times S_{m,n}(t)$, where $S_{m,n}(t)$ is the current forward swap rate on each day.

The top left panel of Figure 1 plots the average (over time series) implied volatility of 1-month swaptions against swap tenor and moneyness, whereas the top right panel plots the average at-the-money (1-month) swaption implied volatility across swap tenor. We see a monotonically declining at-the-money implied volatility over swap tenor, suggesting that short-term swap rates have higher volatility than long-term swap rates. We also see a volatility skew for swaptions, with out-of-the-money receiver swaptions

¹¹We first use a standard cubic spline algorithm to interpolate the swap rates at semiannual intervals from one year to 35 years. We then solve for the zero curve by bootstrapping the interpolated par curve with swap rates as par bond yields. The day count convention is 30/360 for the fixed leg, and Actual/360 for the floating leg.

¹²Many market participants think in terms of normal (or absolute or basis point) implied volatilities—the volatility parameter that, plugged into the normal pricing formula, matches a given price—as they are more uniform across the swaption grid and more stable over time than log-normal implied volatilities.

¹³All our empirical results remain unchanged if we use only the J.P. Morgan swaption data rather than a combination of the J.P. Morgan and Barclays Capital swaption data.

¹⁴We also use linear and spline-based methods of extrapolation and results are similar.

having higher implied volatility than at-the-money swaptions. The volatility skew suggests that forward swap rates follow a fat-tailed distribution, which reflects market concerns on potential extreme movements in swap rates, instead of the log-normal distribution of the Black (1976) model.¹⁵

The bottom left (right) panel plots the time series of implied volatilities, across multiple moneyness levels, of 1-month swaptions with 2-year (10-year) swap tenors. We see that the general level of volatility fluctuates dramatically over time. The level of skewness also becomes steeper when the implied volatility spikes, especially during the recent financial crisis. The time-varying level of skewness reflects varying levels of concern over tail risk in the market.

3.2 Empirical Estimates of TAIL

We provide empirical estimates of TAIL based on historical swaption prices. Given our focus on return prediction regression at the monthly frequency, we consider TAIL estimated from out-of-the-money swaptions with mainly one-month to maturity and 1-, 2-, 5-, 10-, 20-, and 30-year swap tenors in constructing the two replication portfolios for the *variance contract on swap rates*.¹⁶ For each day, we first compute swaption prices using the Black formula based on the generated implied volatility curve and then construct TAIL according to a discretization of (8) and (10). With daily estimates, we use end-of-month values to obtain a monthly time series of TAIL from June 1993 through January 2013.

Panel A of Table 1 reports the summary statistics on monthly estimates of one-month TAIL with swap tenors of 1, 2, 5, 10, 20, and 30 years between June 1993 and January 2013. The mean of TAIL is the lowest at the 1-year tenor and generally increases with swap tenors, although not monotonically, whereas the standard deviation is the highest at the 1-year tenor and declines monotonically with swap tenors. This suggests that the prices for insuring against extreme downside moves in short-term swap rates tend to be smaller but more volatile than those for long-term swap rates, given that positive (negative) TAIL reflects concern over rates going down (up), as shown in (11). Moreover, negative (positive) skewness of TAIL at short (long) tenors suggests that during our sample, concern over the tail risk for short- and long-term swap rates are different. Kurtosis exhibits a U-shape and is high at both short and long tenors

¹⁵Jarrow, Li, and Zhao (2007) made a similar observation on interest rate caps, which are written on LIBOR with very short maturities (3-month).

¹⁶Out-of-the-money swaptions are receiver swaptions with strikes less than the forward swap rate and payer swaptions with strikes higher than the forward swap rate.

and low in the middle, implying more severe concern on tail risk for short- and long-term swap rates. TAIL is also highly autocorrelated, across all tenors. Panel B of Table 1 shows that correlations between TAIL s at short-term (1- and 2-year) and long-term (20- and 30-year) tenors are as low as 0.31, suggesting that the tail risk at long and short horizons are far from being perfectly correlated.

To examine whether TAIL captures major tail events in interest rates, we provide, in Figure 2, time series plots of TAIL at 2- and 10-year tenors, denoted as $\text{TAIL}^{(2)}$ and $\text{TAIL}^{(10)}$ respectively, along with major economic and financial crises in the past two decades. During normal times, TAIL fluctuates around its time-series average of 0.01 with a standard deviation of 0.03 and is highly persistent with a monthly autocorrelation of over 90%. During crises, however, $\text{TAIL}^{(2)}$ and $\text{TAIL}^{(10)}$ exhibit extreme positive and negative spikes. For example, during the LTCM crisis, both $\text{TAIL}^{(2)}$ and $\text{TAIL}^{(10)}$ exhibit positive spikes, reflecting market concern over downward moves in interest rates. Both $\text{TAIL}^{(2)}$ and $\text{TAIL}^{(10)}$ spike after 9.11, the burst of the Internet bubble, and around the bankruptcies of Bear Sterns and Lehman Brothers, again reflecting market expectations of downward moves in interest rates. After 2010, however, $\text{TAIL}^{(2)}$ tends to take negative values and $\text{TAIL}^{(10)}$ tends to take positive values, meaning that the market is concerned about 2-year (10-year) swap rates going up (down). This pattern is especially salient around operation twist, where the Fed buys long-term bonds and sells short-term bonds. It is also the case around QE3, where $\text{TAIL}^{(2)}$ reaches its lowest value. The time series behaviors of TAIL at all tenors in Figure 3 closely resemble that of $\text{TAIL}^{(2)}$ and $\text{TAIL}^{(10)}$ in Figure 2. They tend to be highly correlated and all spike (up and down) around major financial crises, although TAIL s at short swap tenors tend to be more volatile. We also see divergence between TAIL s at short and long tenors after 2010, with the main risk that the short rate is going up and the long rate is going down. Overall, TAIL seems to be able to capture major tail events in many recent financial crises and characterize recessionary states with both upside and downside interest rate tail risk across various horizons.

Finally, we investigate the extent to which variations in TAIL are driven by variations in the term structure of swap rates. We extract the first three principal components (PCs) of monthly changes of forward swap rates, which capture virtually all term structure variations. Then we regress TAIL s at all swap tenors (1, 2, 5, 10, 20, and 30 years) on the three PCs, as well as the squared PCs to capture potential nonlinear effects. The regression R^2 s reported in Table 2 are fairly low: The largest is about

12% for TALL at long swap tenors when the squared PCs are included in the regression.¹⁷

We then obtain the PCs of the regression residuals of TALL at six swap tenors and report, in Panel B of Table 2, the percentage and cumulative percentage of the residuals' variations explained by the residual PCs. We find that the regression residuals, independent of the yield factors by construction, exhibit large common variations. The first PC explains 70% and the first two PCs together explain more than 90% of the variations in the regression residuals, with and without the squared PCs included in the regression. Therefore, TALL seems to capture highly nonlinear rare disaster risk, which is largely orthogonal to yield curve factors that capture market expectations of future levels of interest rates, .

4 Tail Risk Premium across Fixed-Income Markets

In this section, we estimate the magnitude of the tail risk premium on various fixed-income securities, including Treasury bonds, corporate bonds, CMBS, and fixed-income hedge funds. We first consider univariate regressions with TALL as the sole predictor, followed by multivariate regressions that include other well-established return predictors that have been employed in the literature.

4.1 Univariate Regressions

We estimate the tail risk premium by considering the following univariate predictive regression:

$$rx_{t+h}^{(i)} = \alpha_h^{(i)} + \beta_h^{(i)} \text{TALL}_t + \varepsilon_{t+h}^{(i)}, \quad (12)$$

where $rx_{t+h}^{(i)}$ denotes the h -period excess return on asset i , and TALL_t is the time- t tail risk measure of a specific tenor (1, 2, 5, 10, 20, and 30 years). We always report standardized regression results. That is, we de-mean all regressors and regressands and then divide them by their corresponding standard deviations. This standardization makes coefficients comparable across predictors and makes it possible to compare them in terms of both statistical and economic significance. We compute t-statistics using Newey and West (1987) standard errors. The sample period varies according to the availability of returns data, from June 1993 through January 2013 in general. For brevity, we do not report estimates of $\alpha_h^{(i)}$ in all our

¹⁷Following Trolle and Schwartz (2014), we also run regressions using a rolling window of 12 observations to accommodate possible time-variation in the relation between swap rates and TALL , which yields only slightly higher R^2 s.

results.

Treasury Bonds. Table 3 reports the regression results of excess returns over a one-year holding horizon of Treasury bonds with maturities of 2, 5, 10, and 20 years on TALL at each swap tenor, individually and collectively. For Treasury bonds, we use the interpolated zero-coupon yield data of Gürkaynak, Sack, and Wright (2006, GSW), available between June 1993 and January 2013 on the Fed webpage, to calculate annual Treasury bond excess returns at the monthly frequency. Specifically, let $p_t^{(\tau)}$ denote the time- t log price of a τ -year bond. The annual return on this τ -year bond is $r_{t+1}^{(\tau)} = p_{t+1}^{(\tau-1)} - p_t^{(\tau)}$, and the excess return is defined as $rx_{t+1}^{(\tau)} \equiv r_{t+1}^{(\tau)} - y_t^{(1)}$, where $y_t^{(1)}$ is the one year yield.¹⁸

In general, we find that TALLs of short and intermediate tenors have strong predictive power for excess returns on bonds of short and intermediate maturities. For example, $\text{TALL}^{(1)}$ and $\text{TALL}^{(2)}$ have the largest and most significant regression coefficients and highest R^2 s for 2-year bonds, while $\text{TALL}^{(5)}$ has the strongest predictive power for 5-year bonds. Although TALLs of longer tenors tend to have stronger predictive power for long-term bonds, $\text{TALL}^{(5)}$ tends to have the strongest predictive power for intermediate- and long-term bonds. TALLs of various tenors can explain a significant portion of excess returns on bonds at differing maturities. The highest R^2 achieved in univariate regressions for 2-, 5-, 10-, and 20-year bonds are 13%, 27%, 22%, and 6%, respectively. By combining TALLs at all tenors in one regression, the R^2 s can be significantly improved to 20%, 32%, 20%, and 6% for bonds at 2-, 5-, 10-, and 20-year maturities, respectively. These R^2 s are generally higher than those of the Fama and Bliss (1987) regressions with individual forward rates as regressors.

Corporate Bonds. Table 4 reports the regression results for monthly excess returns on Barclay indices of long- and intermediate-term BBB-rated corporate bonds over 3-, 6- and 12-month holding horizons on TALL at each swap tenor individually and collectively (results for B- and CCC-rated corporate bonds are provided in the Internet Appendix).

We observe that TALL has strong predictive power for corporate bond excess returns over 6- and 12-month holding horizons with negative regression coefficients throughout. Moreover, significant predictability comes mainly from TALLs of long tenors (10, 20, and 30 years), although TALLs of short

¹⁸The GSW dataset, consisting of smoothed fitted yields using an extended Nelson and Siegel (1987) model, may have some limitations (Le and Singleton, 2013). For robustness, we repeat the analysis using (i) the Fama-Bliss discount bond database from CRSP with maturities from one to five years and (ii) non-interpolated actual bond portfolios for differing maturity segments from the Fama database of CRSP. We confirm that the return predictability of TALL is robust to alternative bond databases and various ways of constructing bond returns. The results can be found in the Internet Appendix.

tenors have also significant predictive power for intermediate-term corporate bond returns. In addition, the regression R^2 s vary, for the most part, between 10% and 18%, indicating the robust explanatory power of individual TAILS for corporate bond returns. By including TAIL at all tenors in the regression, we significantly increase the R^2 , highlighting the information contained in the term structure of TAILS.

Commercial Mortgage-Backed Securities. Table 5 reports regression results for monthly excess returns on Barclays indices of AAA- and A-rated CMBS over 3-, 6- and 12-month holding horizons (results for BBB- and B-rated CMBS are provided in the Internet Appendix) on TAILS at each swap tenor individually and collectively. The sample period is between March 1997 and January 2013. We find significant negative coefficients on TAILS at all tenors over various horizons for each rating category, except for AAA-rated CMBS at the 3-month horizon. The predictability of TAIL at long tenors is particularly strong, with most R^2 s higher than 20%. The predictability of TAIL at short tenors is also impressive, with most R^2 s in the 10-15% range. By including TAILS at all tenors in the regression, we significantly increase the corresponding R^2 , highlighting the information contained in the term structure of TAILS.

Fixed-Income Hedge Funds. We obtain monthly hedge fund return indices from Hedge Fund Research, Inc. (HFR).¹⁹ We focus on three return indices associated with fixed-income securities trading (relative value, corporate, and yield alternatives), which are available from June 1993 through January 2013. Table 6 reports the regression results for the returns on three types of fixed-income hedge funds on TAIL at each tenor, individually and collectively. We also find significant negative coefficients on TAILS at 5-, 10-, 20-, and 30- year tenors for three hedge fund indices with R^2 s ranging between 3% and 9%. By including TAILS at all tenors in the regression, we significantly increase the corresponding R^2 , highlighting the information contained in the term structure of TAILS.

Overall, we document the strong predictive power of TAIL for excess returns on a wide range of fixed-income securities over various investment horizons, suggesting that tail risk is universally priced in all major fixed-income markets.

¹⁹According to the HFR website, funds are eligible for listing in the return index if they report monthly returns net of all fees, report assets and performance in USD, and have at least \$50 million in assets under management or have been actively trading for at least 12 months.

4.2 Multivariate Regressions

Given the strong return predictability of TALL at individual swap tenors, we study whether the term structure of TALL across all tenors has additional predictive power and whether the predictability is robust to other established return predictors in the literature. Instead of using individual TALL s at all tenors collectively, we use their principal components (PC) as factors in multivariate regressions, which parsimoniously summarize the information in the term structure of TALL s. The PCs of TALL s, orthogonal to each other by construction, circumvent the potential multicollinearity problem given the nontrivial correlations of TALL s at individual tenors.²⁰

In our multivariate regressions, we also consider other well established bond return predictors in the literature. We first consider the eight macro factors of Ludvigson and Ng (2009, 2011), \widehat{F}_j , $j = 1, \dots, 8$, extracted from more than 100 time series of macroeconomic variables. We obtain these macroeconomic data from Global Insight and the Federal Reserve Economic Data base (FRED) and update them to July 2012.²¹ We construct the tent-shaped factor of Cochrane and Piazzesi (2005) (CP) based on forward rates implied by GSW Treasury yields data (with maturities of 1 to 20 years).²²

Panel A.1 of Table 7 shows that the PCs of the term structure of TALL s have strong predictive power for excess returns on Treasury bonds of 2-, 5-, 10-, and 20-year maturities, with most R^2 s ranging between 20% and 35%. The first and sixth TALL PCs have significant predictive power for Treasury bond returns at almost all horizons with most t -statistics greater than 3. Moreover, the coefficients on the second, third, and fourth PCs are significant for Treasury bonds with maturities between 5 and 15 years, with a negative sign for the second PC but a positive sign for the other two.

²⁰Results in the Internet Appendix show that the first, second, and third PC explains roughly 77%, 20%, and 3% of the variations in TALL s, respectively. Moreover, TALL^{PC1} , TALL^{PC3} , and TALL^{PC4} load heavily on TALL s at short tenors (1, 2, and 5 years), TALL^{PC5} loads heavily on TALL at the 10-year tenor, and TALL^{PC2} and TALL^{PC6} load heavily on TALL s at long tenors (20 and 30 years). However, we need be cautions in interpreting the meanings of these PCs, because the leading loadings of TALL^{PC3} , TALL^{PC4} , and TALL^{PC6} have opposite signs.

²¹Ludvigson and Ng (2010) consider 131 monthly macroeconomic time series through December 2007. We exclude six series that are no longer available after 2007 and three after 2010, as well as financial time series of stock markets and interest rates. In total, we consider 101 variables. The Internet Appendix provides a detailed description of the macro data.

²²Following Cochrane and Piazzesi (2005), we run the following predictive regression,

$$\begin{aligned} \overline{r\bar{x}}_{t+1} &= \beta_0 + \sum_{i=1}^m \beta_i f_t^{(i)} + \bar{\varepsilon}_{t+1} \\ &= \beta_0 + \beta' \mathbf{f}_t + \bar{\varepsilon}_{t+1}, \end{aligned}$$

where $\overline{r\bar{x}}_{t+1}$ represents that average one-year excess returns on Treasury bonds with 2- to m -year maturity, and \mathbf{f}_t is vector of m forward rates with maturities of 1 to m years at time t . The CP return forecasting factor is constructed as $\beta' \mathbf{f}_t$.

Panel A.2 of Table 7 shows that by incorporating the two sets of control variables into the above regressions, we can significantly increase the R^2 s, confirming the predictive power of these established factors. The significance of the first T\AAILL PC is mostly driven out by the additional predictors due to their high correlation, along with the significance of the sixth PC for long-term bonds. However, both the second and sixth PCs are still highly significant for predicting short-term bond returns, whereas T\AAILL^{PC3} is significant for Treasury bond returns at all maturities. Overall, we conclude that the term structure of T\AAILL has strong predictive power for Treasury bond returns even after controlling for the established predictors in the literature.

The regression results in Panels B, C, and D of Table 7 for corporate bonds, CMBS, and fixed-income arbitrage funds, respectively, reveal patterns that are similar to the one we found for Treasury bonds. Basically, the PCs of the term structure of T\AAILL have strong predictive power for excess returns of these assets, which is robust to the inclusion of the well-known return predictors. It is worth mentioning, however, that more T\AAILL PCs remain highly significant for these three securities than for Treasury bonds in the presence of the macro and CP factors. For example, the first two PCs are highly significant in predicting (i) both long-term and short-term corporate bond returns over almost all horizons, (ii) returns on AAA- and A-rated CMBS over almost all horizons, and (iii) fixed-income arbitrage fund returns for all three investment styles, with most t-statistics greater than 5.

To summarize, the strong return predictability of the term structure of T\AAILL is remarkably robust to the inclusion of some well-known bond return predictors in the literature. In unreported results, we also show the robustness to the equity tail risk factors in Bollerslev and Todorov (2011) and Kelly and Jiang (2014). The reported return predictability is strong both statistically and economically. The results point to the universal role of tail risk in determining the expected returns for a wide range of fixed-income securities.

4.3 Out-of-Sample Predictability of T\AAILL

The univariate and multivariate regressions reported in the previous two sections are based on in-sample analysis. In this section, we study whether T\AAILL has out-of-sample predictability for excess returns on Treasury bonds, corporate bonds, and CMBS over a 12-month holding period. We run out-of-sample

predictive regressions on `TAILL` at all tenors directly instead of their PCs because only data up to time t can be used to forecast excess returns at time $t + 1$.

We split the entire sample into two subsamples, such that the earlier sample is used to estimate the predictive model and the later sample is used to evaluate the out-of-sample forecast of the predictive model. For Treasury and corporate bonds, the estimation sample is from June 1993 through January 2002 and the forecast sample is from January 2003 through January 2013. For CMBS, the estimation sample is from March 1997 through January 2005 and the forecast sample is from January 2006 through January 2013. We extend the estimation sample by one month and repeat the exercise of model estimation and out-of-sample forecast for the next month until we reach the end of the entire sample.

We first compare the predictive power of a model with `TAILL`s at 1-, 2-, 5-, 10-, 20-, and 30-year tenors collectively as predictors with that of a random walk model, where future expected return equals the historical average return. To examine whether `TAILL` has incremental predictive power over the CP factor, we compare the forecasting performance of a model with `TAILL` and the CP factor as predictors with that of a model with the CP factor and a constant as predictors.

We use two methods to evaluate the out-of-sample performance of the two pairs of models. First, we consider the out-of-sample R^2 of Campbell and Thompson (2008), defined as

$$R_{OOS}^2 = 1 - \frac{\sum_{t=1}^T (y_t - \hat{y}_t^i)^2}{\sum_{t=1}^T (y_t - \hat{y}_t^j)^2}, \quad (13)$$

where \hat{y}_t^i is the forecast of variable y_t under model i and \hat{y}_t^j is the forecast of variable y_t under model j . R_{OOS}^2 measures the proportional reduction in the mean-squared error by model i relative to model j . Positive values of R_{OOS}^2 imply that model i produces a lower mean-squared prediction error than model j . Second, we consider the encompassing test statistic ENC-NEW proposed by Clark and McCracken (2001). The null hypothesis of the ENC-NEW test is that the benchmark model encompasses all the predictability in excess returns and that `TAILL` cannot further improve the forecasting performance.²³

Table 8 reports the R_{OOS}^2 s and the ENC-NEW test statistics for the two pairs of models: (i) `TAILL` versus the random walk model, and (ii) (`TAILL`+CP) versus (Const+CP). We observe that the R_{OOS}^2 s for

²³As shown by Clark and McCracken (2005), the ENC-NEW test statistic has a non-standard distribution under the null hypothesis, and we obtain the critical values by bootstrap.

the first pair of models are positive for all assets, indicating that these excess returns are time varying and predictable by the term structure of $\mathbb{T}\mathbb{A}\mathbb{I}\mathbb{L}$. The ENC-NEW test statistics and corresponding bootstrap critical values further confirm the out-of-sample predictability of $\mathbb{T}\mathbb{A}\mathbb{I}\mathbb{L}$. For the second pair of models, we observe that the R_{OOS}^2 s are positive for Treasury bonds with 2- and 20-year maturities and negative for all other maturities. Along with the ENC-NEW test statistics and bootstrap critical values, these values of R_{OOS}^2 show that $\mathbb{T}\mathbb{A}\mathbb{I}\mathbb{L}$ has additional predictive power beyond the CP factor only for Treasury bonds of 2- and 20-year maturities. For both corporate bonds and CMBS, however, both the R_{OOS}^2 and ENC-NEW test statistics in Panels B and C show that the term structure of $\mathbb{T}\mathbb{A}\mathbb{I}\mathbb{L}$ has significant incremental forecasting power beyond the CP factor. Overall, the out-of-sample results confirm the in-sample evidence that tail risk premium is an important component of asset returns for all major fixed-income markets.

4.4 Predictive Power of $\mathbb{T}\mathbb{A}\mathbb{I}\mathbb{L}$ Over Different Horizons

Our analysis so far has relied on $\mathbb{T}\mathbb{A}\mathbb{I}\mathbb{L}$ constructed from swaptions with one month to maturity but at different swap tenors (1, 2, 5, 10, 20, and 30 years), which captures tail risk over the next month. However, investors with other investment horizons might be interested in tail risk constructed from swaptions with corresponding time to maturity.

Table 9 reports the regression results for excess returns on Treasury bonds, corporate bonds, and CMBS over a 12-month holding period on the PCs of $\mathbb{T}\mathbb{A}\mathbb{I}\mathbb{L}$ s constructed from swaptions with one year to maturity at varying swap tenors. We find that the one-year $\mathbb{T}\mathbb{A}\mathbb{I}\mathbb{L}$ seems to have stronger predictive power than the one-month $\mathbb{T}\mathbb{A}\mathbb{I}\mathbb{L}$ for the 12-month holding period returns for all three asset classes. The regression R^2 s increase more than 10% for Treasury bonds, more than 30% for corporate bonds, and between 20% and 40% for CMBS. For Treasury bonds, the explanatory power of the one-year $\mathbb{T}\mathbb{A}\mathbb{I}\mathbb{L}$ is much larger than that of the macro factors and about half of that of the CP factor (see the Internet Appendix for return predictability results by macro and CP factors individually). Therefore, the $\mathbb{T}\mathbb{A}\mathbb{I}\mathbb{L}$ constructed from swaptions with a given time-to-maturity is especially informative for gauging the future state of the economy over corresponding investment horizons.

4.5 Tail or Volatility Risk?

To ensure that the tail risk premium we identify in fixed-income returns is not due to volatility risk, we incorporate two volatility measures in the predictive regression. The first is simply the at-the-money implied volatility (IV^{ATM}) of one-month swaptions with 2-, 5-, and 10-year swap tenors. The second is a model-free measure of implied volatility (MFIV) based on equation (8), which is similar to the way in which the CBOE constructs VIX (Carr and Wu, 2009). Differing from IV^{ATM} , MFIV contains information pertaining to both ATM and OTM swaptions.

We examine the robustness of the individual predictive power of one-month $TAIL$ s at 2-, 5-, and 10-year tenors, controlling for the corresponding IV^{ATM} and $MFIV$ individually. Panels A, B, C, and D of Table 10 report the results for Treasury bonds, corporate bonds, CMBS, and hedge funds, respectively. The results show that the volatility factors have predictive power for certain asset returns: short-term volatility for Treasury bonds, medium- to long-term volatility for corporate bonds and CMBS, and medium-term volatility for relative-value hedge funds. However, the return predictability of $TAIL$ is remarkably robust to the inclusion of either IV^{ATM} or $MFIV$, indicating that the tail risk premium we identify is not due to volatility risk.²⁴

4.6 Tail Risk Premium in Equity Returns

Given the systematic impact of interest rates on financial markets, we examine whether interest rate tail risk is priced in equity returns. We use the value-weighted return index from CRSP to proxy the market portfolio and construct growth and value portfolio returns using the six portfolios formed on size and book-to-market from Ken French's data library. The growth (value) portfolio return is the average of the returns on small and large growth (value) portfolios. We compute the excess returns on each portfolio over three, six, and twelve month horizons.

Table 11 reports the regression results of equity portfolio returns on $TAIL$, with standard predictors, the log dividend yield (DY) and log earnings/price ratio (E/P), and the net equity expansion (NTIS) factor proposed by Goyal and Welch (2008) that we obtain from Amit Goyal's webpage, as control variables.

²⁴We also find that the return predictability of $TAIL$ is robust to the variance risk premium of swap rates, computed as the difference between $MFIV$ and realized variance (using daily time series of swap rates), as well as equity variance risk premium in Carr and Wu (2009).

We observe impressive predictive power of TAILL for equity portfolio returns, with R^2 s ranging between 6% and 12% when no controlling variables are included. Adding the established equity return predictors improves the regression R^2 s significantly for most equity portfolio returns. However, for the most part, the significance of TAILL survives. For example, TAILL s at both the 1- and 30-year tenors are statistically significant when predicting 12-month excess returns of all three equity portfolios. These results point to the economy-wide impact of interest rate tail risk.

5 Tail Risk and Economic Fundamentals

To better understand the economic nature of TAILL , we investigate the relation between TAILL and a spectrum of fundamental economic variables. Given the forward-looking nature of TAILL , we also show that TAILL can predict the future values of these fundamental factors over multiple horizons.

5.1 Economic Variables

We consider the following five groups of data that measure various aspects of the economy.²⁵

Macroeconomic Variables. We use the first two PCs (\hat{F}_1 and \hat{F}_2) of Ludvigson and Ng (2009, 2010) and the Chicago Fed National Activity Index ($CFNAI$) to measure macroeconomic risk. $CFNAI$ is a weighted average of 85 existing monthly indicators of national economic activity.²⁶ We interpret $CFNAI$ as a growth indicator because a positive $CFNAI$ reading corresponds to above-trend growth. We use the monthly unsmoothed version of the index from June 1993 through January 2013, but we have verified that our results are robust to using the 3-month moving average version.

Liquidity Measures. We consider three liquidity factors that measure several aspects of liquidity. The first is a measure of funding liquidity for fixed-income markets extracted from on- and off-the-run Treasury bonds by Fontaine and Garcia (2011) (ILQ^{FG}). We obtain the monthly ILQ^{FG} measure between June 1993 and March 2012 from Jean-Sebastien Fontaine’s webpage. The second is the noise measure calculated using Treasury securities in Hu, Pan, and Wang (2012) (ILQ^{HPW}), which is associated with

²⁵In the Internet Appendix (Section 3), we show that TAILL is also affected by macroeconomic news and announcements of the Federal Reserve on monetary policies.

²⁶These indicators include (i) production and income, (ii) employment, unemployment, and hours, (iii) personal consumption and housing, and (iv) sales, orders, and inventories. The index is constructed to have an average value of zero and a standard deviation of one.

the abundance of arbitrage capital for the whole financial market. We download the daily ILQ^{HPW} measure between June 1993 and December 2011 from Jun Pan’s webpage and use end-of-month values as monthly time series. The third captures the liquidity of the collateral channel. We use the volume of repo failures involving Treasury bonds as collateral (ILQ^{COL}). We obtain the monthly dollar amounts of settlement failures (value at the fourth week of each month) for Treasury bonds from the New York Fed from June 1993 through January 2013.

Credit Risk Measures. We consider three variables for credit risk, the swap spread (SWS), the credit spread (CS), and the TED spread. SWS is the difference between the ten-year swap rate and the corresponding constant maturity Treasury yields from the H.15 statistical release of the Fed. CS is the difference between Moody’s AAA and BAA corporate bond yields, both obtained from FRED. The TED spread is the difference between the three-month LIBOR and the 3-month Treasury bill rate with the latter obtained from FRED. Data pertaining to all three variables are available for the period between June 1993 and January 2013.

Mortgage Refinancing Activities. We consider mortgage convexity (CV), duration (DUR), and the MBA refinancing index (REF) to capture MBS hedging activities, which are intimately associated with the interest rate derivatives market (see Duarte (2008) and Perli and Sack (2003) for details). We obtain monthly time series of CV and DUR from Merrill Lynch and REF , the Mortgage Bankers Association (MBA) refinancing index, from J.P. Morgan. The sample period is between June 1997 and January 2013.

Consensus Forecasts. In addition, we obtain monthly survey data on macro variables from BlueChip Economic Indicators (BCEI) and BlueChip Financial Forecasts (BCFF) that are regularly used by the Fed to gauge market expectations.²⁷ For each month, we compute the consensus forecast for each variable as the median of individual forecasts and the forecast uncertainty as the cross sectional standard deviation. We consider forecasts of the unemployment rate (UNEM), the ten-year constant maturity Treasury yield (LR), industrial production (IP), housing starts (HOUST), the consumer price index (CPI), and total US auto and truck Sales (AS) for the subsequent calendar year from BCEI, and of the federal funds rate (FFR) and real GDP (RGDP) one-year ahead from BCFF. The BCEI forecast series are adjusted using

²⁷BCEI collects monthly forecasts of twelve key economic variables from about fifty leading financial and economic advisory firms, while the BCFF survey contains monthly forecasts of yields, inflation, and GDP growth from approximately 45 leading financial institutions.

the x-12 ARIMA filter. These forecast data are available for the period between June 1993 and January 2013.

5.2 Tail Risk and Contemporaneous Economic Variables

To investigate the link between TALL and economic fundamentals, we run contemporaneous regressions of TALL (with tenors of 1, 2, 5, 10, 20, and 30 years) on the aforementioned economic variables. The sample period for most of the regressions is from June 1993 to January 2013 but is from June 1997 to January 2013 for mortgage factors due to limited data availability.

The top two panels of Figure 4 report the regression results of TALL on the macro factors \hat{F}_1 , \hat{F}_2 and $CFNAI$: the left panel reports the t -statistics calculated using the Newey and West (1987) standard errors and the right panel reports the adjusted R^2 s of the regressions. The left panel shows that the coefficients on \hat{F}_1 and \hat{F}_2 (not $CFNAI$) are statistically significant mostly for TALL s at long tenors (10, 20, and 30 years), except that the coefficient on \hat{F}_2 is borderline significant for TALL at the 1-year tenor. Moreover, the right panel shows that the macro factors have the strongest predictive power for TALL s at long tenors with R^2 s between 40% and 50%. Given that \hat{F}_1 and \hat{F}_2 combine information from more than 100 macro variables, the large R^2 s of these contemporaneous regressions suggest that TALL s at long tenors are closely related to macroeconomic fundamentals.

The second two panels of Figure 4 report the regression results for TALL on credit factors TED , SWS and CS . The left panel shows that the coefficients on TED , SWS and CS are all statistically significant for TALL s at long-term tenors (10, 20, and 30 years). The coefficients on SWS and CS are positive and negative, respectively, both highly significant (t -statistics larger than 3). Furthermore, the right panel shows that the regression R^2 s are between 25% and 40% for TALL s at long-term tenors while they are only approximately 10% for TALL s at short-term tenors.

The third two panels of Figure 4 report the regression results for TALL on mortgage factors $CONVX$, DUR , and REF . The left panel shows that the coefficient on REF ($CONVX$) is positive (negative) and statistically significant only for TALL s at medium horizons. Moreover, the right panel shows that the mortgage factors have the greatest explanatory power for TALL s at medium tenors, although the R^2 s are no more than 18%.

The last two panels of Figure 4 report the regression results for TALL on the liquidity factors ILQ^{HPW} , ILQ^{FG} , and ILQ^{COL} . We find that ILQ^{FG} and ILQ^{COL} are statistically significant mainly for TALL s at short tenors (1, and 2 years), whereas ILQ^{HPW} is highly significant for both short and long tenors. Such a pattern is consistent with the fact that both ILQ^{FG} and ILQ^{COL} capture the funding liquidity of financial intermediaries, which depend on short-term funding to finance their positions, while ILQ^{HPW} captures “collective information over the entire yield curve” (Hu, Pan, and Wang, 2012). Furthermore, we observe from the right panel that the regression R^2 s are more than 40% at all tenors and slightly higher than 50% at short tenors.

Table 12 reports the regression results for TALL at 1-, 2-, 5-, 10-, 20-, and 30-year tenors on contemporaneous consensus on and uncertainty in the forecasts for UNEM, LR, IP, HOUST, CPI, and AS, FFR, and RGDP. The regression coefficients, t -statistics based on the Newey and West (1987) standard errors, and R^2 s are reported in the first, second (in parentheses), and third columns of each block, respectively. In general, there is a significant and negative relation between the consensus measures (except for \hat{E}_{UNEM}) and TALL s at long tenors with R^2 s around 20%. This suggests that better forecasts of future economic variables are associated with lower tail risk of falling interest rate; intuitively, a better economic outlook is consistent with a higher rate of return on investments and a lower likelihood that the central bank will cut the rates further to stimulate economic growth. On the other hand, there is a significant and positive relation between the uncertainty measures (except for \hat{U}_{HOUST}) and TALL s at short tenors with R^2 s also around 20%. This suggests that higher uncertainty in the forecasts of future economic variables is associated with higher tail risk of falling interest rates, potentially because the depressing economic outlook caused by high levels of uncertainty will lead to deflation risk and the central bank may cut the rates accordingly. These results show that TALL , a leading indicator of tail risk in fixed-income markets, incorporates important information about macroeconomic uncertainty.²⁸

In summary, the results reported in this section point to strong links between TALL and various aspects of the economy: the macro and credit factors are closely linked to TALL at long tenors, the mortgage factors to TALL at medium tenors, the funding liquidity factor to TALL at short tenors, and the market liquidity factor to TALL at all tenors. That is, TALL captures information pertaining to tail

²⁸To address the concern that our results are driven mainly by the recent global financial crisis, we repeat our analysis using data only through December 2007 and obtain very similar results.

risk associated with macroeconomic risk, credit risk, mortgage hedging activity, and funding liquidity. Given that the R^2 s of most regressions are less than 50%, T\AAILL still contains information about tail risk in fixed-income markets beyond that contained in these economic variables.

5.3 Tail Risk and Future Economic Variables

Given that T\AAILL captures *ex ante* market expectations of a tail event, we test the ability of this forward-looking measure for predicting the 12 fundamental economic variables over a wide range of horizons.

Figure 5 reports the regression R^2 s of the macro, credit, mortgage, and liquidity factors over 1-, 3-, 6-, 12-, and 18-month horizons on T\AAILL s at 1-, 2-, 5-, 10-, 20-, and 30-year tenors (Table 7 of the Internet Appendix reports regression coefficients and t-statistics). Over the 1-month horizon, we observe fairly large regression R^2 s, over 40% for the market liquidity and macro factors, around 30% for the credit and mortgage factors, and about 15% for the funding liquidity factors. Interestingly, while T\AAILL has significant predictive power for future values of $CFNAI$, the reverse is not true. Furthermore, the predictive power of T\AAILL for the credit factors (SWS and CS), mortgage factors (REF and $CONVX$), and macro factors (\widehat{F}_2) remains strong for up to 12-month horizons with R^2 s more than 20%. For the rest of these factors, the predictive power weakens with forecasting horizon: The R^2 for ILQ^{HPW} at the 12-month horizon is less than half of that at the 1-month horizon. Overall, T\AAILL can be viewed as a leading indicator for various aspects of the economy associated with the tail risk in fixed-income markets.

6 Conclusion

In this paper, we have provided one of the first comprehensive studies of tail risk in fixed-income markets. Using a unique database of interest rate swaptions from 1993 through 2013, we construct a model-free measure of tail risk for fixed-income markets, which captures the price of insuring against extreme movements in interest rate swap rates over a given investment horizon. We show that our tail risk measure closely tracks the variations of tail risks in the economy and has strong and robust predictive power for returns on Treasury bonds, corporate bonds, CMBS, fixed-income hedge funds, and even equity returns, suggesting that interest rate tail risk is universally priced in all major financial markets. We also find strong links between T\AAILL and a wide range of fundamental economic variables.

Appendix: Proofs

Proof of Proposition 1: Applying the Ito's formula to model (9), we have

$$\ln S_{m,n}(T_m) = \ln S_{m,n}(t) + \int_t^{T_m} \frac{dS_{m,n}(s)}{S_{m,n}(s-)} - \frac{1}{2} \int_t^{T_m} \sigma_{s-}^2 ds + \int_t^{T_m} \int_{R_0} (1+x-e^x) \omega [dx, dt]. \quad (\text{A.1})$$

Then the quadratic variation of the process $\ln S_{m,n}(t)$ over $[t, T_m]$ is

$$\begin{aligned} QV_{m,n}(t) &= \int_t^{T_m} \sigma_{s-}^2 ds + \int_t^{T_m} x^2 \omega [dx, dt] \\ &= 2 \left[\int_t^{T_m} \frac{dS_{m,n}(s)}{S_{m,n}(s-)} - \ln \frac{S_{m,n}(T_m)}{S_{m,n}(t)} \right] + 2 \int_t^{T_m} \int_{R_0} \left(1+x+\frac{x^2}{2} - e^x \right) \omega [dx, dt]. \end{aligned}$$

Taking expectations of $QV_{m,n}(t)$ under the annuity measure $\mathbb{A}^{m,n}$ gives

$$\begin{aligned} \mathbb{V}\mathbb{P}_{m,n}(t) &= 2\mathbb{E}_t^{\mathbb{A}^{m,n}} \left[\frac{S_{m,n}(T_m)}{S_{m,n}(t)} - 1 - \ln \frac{S_{m,n}(T_m)}{S_{m,n}(t)} \right] \\ &\quad + 2\mathbb{E}_t^{\mathbb{A}^{m,n}} \int_t^{T_m} \int_{R_0} \left(1+x+\frac{x^2}{2} - e^x \right) \omega [dx, dt] \\ &= -2\mathbb{E}_t^{\mathbb{A}^{m,n}} \left[\ln \frac{S_{m,n}(T_m)}{S_{m,n}(t)} \right] + 2\mathbb{E}_t^{\mathbb{A}^{m,n}} \int_t^{T_m} \int_{R_0} \left(1+x+\frac{x^2}{2} - e^x \right) \omega [dx, dt]. \quad (\text{A.2}) \end{aligned}$$

Similar to Bakshi and Madan (2000) and Bakshi, Kapadia, and Madan (2003), we have

$$\begin{aligned} \ln S_{m,n}(T_m) &= \ln S_{m,n}(t) + \frac{S_{m,n}(T_m) - S_{m,n}(t)}{S_{m,n}(t)} \\ &\quad - \int_{S_{m,n}(t)}^{\infty} \frac{1}{K^2} (S_{m,n}(T_m) - K)^+ dK - \int_0^{S_{m,n}(t)} \frac{1}{K^2} (K - S_{m,n}(T_m))^+ dK. \end{aligned}$$

Taking expectations of $\ln S_{m,n}(T_m)$ under the annuity measure $\mathbb{A}^{m,n}$ gives

$$\begin{aligned} &\mathbb{E}_t^{\mathbb{A}^{m,n}} [\ln S_{m,n}(T_m)] \\ &= \ln S_{m,n}(t) - \frac{1}{A_{m,n}(t)} \left\{ \int_{S_{m,n}(t)}^{\infty} \frac{1}{K^2} \mathcal{P}_{m,n}(t; K) dK + \int_0^{S_{m,n}(t)} \frac{1}{K^2} \mathcal{R}_{m,n}(t; K) dK \right\}. \quad (\text{A.3}) \end{aligned}$$

Combining equations (A.2) and (A.3) delivers

$$\begin{aligned}
\mathbb{V}\mathbb{P}_{m,n}(t) &= \frac{2}{A_{m,n}(t)} \left\{ \int_{S_{m,n}(t)}^{\infty} \frac{1}{K^2} \mathcal{P}_{m,n}(t; K) dK + \int_0^{S_{m,n}(t)} \frac{1}{K^2} \mathcal{R}_{m,n}(t; K) dK \right\} \\
&\quad + 2\mathbb{E}_t^{\mathbb{A}^{m,n}} \int_t^{T_m} \int_{R_0} \left(1 + x + \frac{x^2}{2} - e^x \right) \omega [dx, dt] \\
&= \mathbb{I}\mathbb{V}_{m,n}(t) + 2\mathbb{E}_t^{\mathbb{A}^{m,n}} \int_t^{T_m} \int_{R_0} \left(1 + x + \frac{x^2}{2} - e^x \right) \omega [dx, dt].
\end{aligned}$$

This completes the proof of part (i). The result of part (ii) can be derived given that the last term is zero for a continuous process of $S_{m,n}(t)$. **Q.E.D.**

Proof of Proposition 2: By a Taylor series expansion with the remainder term, we have

$$\begin{aligned}
\ln^2 S_{m,n}(T_m) &= \ln^2 S_{m,n}(t) + 2 \ln S_{m,n}(t) \frac{S_{m,n}(T_m) - S_{m,n}(t)}{S_{m,n}(t)} \\
&\quad + 2 \int_{S_{m,n}(t)}^{\infty} \frac{1 - \ln K}{K^2} (S_{m,n}(T_m) - K)^+ dK + 2 \int_0^{S_{m,n}(t)} \frac{1 - \ln K}{K^2} (K - S_{m,n}(T_m))^+ dK. \quad (\text{A.4})
\end{aligned}$$

Taking expectations of $\ln^2 S_{m,n}(T_m)$ under the annuity measure $\mathbb{A}^{m,n}$ gives

$$\begin{aligned}
&\mathbb{E}_t^{\mathbb{A}^{m,n}} \ln^2 S_{m,n}(T_m) \\
&= \ln^2 S_{m,n}(t) + \frac{2}{A_{m,n}(t)} \left\{ \int_{S_{m,n}(t)}^{\infty} \frac{1 - \ln K}{K^2} \mathcal{P}_{m,n}(t; K) dK + \int_0^{S_{m,n}(t)} \frac{1 - \ln K}{K^2} \mathcal{R}_{m,n}(t; K) dK \right\}. \quad (\text{A.5})
\end{aligned}$$

Therefore, we have

$$\begin{aligned}
&Var_t^{\mathbb{A}^{m,n}} \left[\ln \frac{S_{m,n}(T_m)}{S_{m,n}(t)} \right] = \mathbb{E}_t^{\mathbb{A}^{m,n}} \ln^2 S_{m,n}(T_m) - \left[\mathbb{E}_t^{\mathbb{A}^{m,n}} \ln S_{m,n}(T_m) \right]^2 \\
&= \frac{2}{A_{m,n}(t)} \left\{ \int_{K > S_{m,n}(t)} \frac{1 - \ln(K/S_{m,n}(t))}{K^2} \mathcal{P}_{m,n}(t; K) dK + \int_{K < S_{m,n}(t)} \frac{1 - \ln(K/S_{m,n}(t))}{K^2} \mathcal{R}_{m,n}(t; K) dK \right\} \\
&\quad - \frac{1}{(A_{m,n}(t))^2} \left\{ \int_{K > S_{m,n}(t)} \frac{1}{K^2} \mathcal{P}_{m,n}(t; K) dK + \int_{K < S_{m,n}(t)} \frac{1}{K^2} \mathcal{R}_{m,n}(t; K) dK \right\}^2,
\end{aligned}$$

where (A.3) and (A.5) are used in the last equality. This completes the proof of part (i). The proof of part (ii) is similar to Du and Kapadia (2012; Proposition 1) with the annuity measure $\mathbb{A}^{m,n}$ replacing the risk-neutral measure. **Q.E.D.**

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Table 1 Summary Statistics of the TAIL Estimates

This table reports basic summary statistics and correlations of the monthly TAILs at swap tenors of 1, 2, 5, 10, 20, and 30 years, from June 1993 through January 2013. The maturity of swaptions used to calculate TAIL is one month.

Panel A: Basic Statistics						
	TAIL ⁽¹⁾	TAIL ⁽²⁾	TAIL ⁽⁵⁾	TAIL ⁽¹⁰⁾	TAIL ⁽²⁰⁾	TAIL ⁽³⁰⁾
Mean	0.002	0.010	0.013	0.013	0.012	0.012
StDev	0.037	0.035	0.031	0.023	0.019	0.016
Min	-0.199	-0.099	-0.094	-0.071	-0.047	-0.030
Max	0.088	0.114	0.110	0.097	0.098	0.101
Skewness	-1.811	-0.145	0.112	0.259	0.833	1.632
Kurtosis	9.035	4.684	5.087	6.194	6.741	7.786
AC(1)	0.856	0.924	0.932	0.909	0.903	0.901
Panel B: Correlation Matrix						
	TAIL ⁽¹⁾	TAIL ⁽²⁾	TAIL ⁽⁵⁾	TAIL ⁽¹⁰⁾	TAIL ⁽²⁰⁾	TAIL ⁽³⁰⁾
TAIL ⁽¹⁾	1.000	0.873	0.749	0.599	0.466	0.311
TAIL ⁽²⁾		1.000	0.942	0.833	0.713	0.576
TAIL ⁽⁵⁾			1.000	0.956	0.872	0.758
TAIL ⁽¹⁰⁾				1.000	0.967	0.889
TAIL ⁽²⁰⁾					1.000	0.971
TAIL ⁽³⁰⁾						1.000

Table 2: Evidence for Unspanned TAIL

Panel A reports the R²s from regressing monthly TAIL changes on the first three principal components (and the principal components squared) of the forward swap rates in the first (second) row. Panel B reports principal components of the six regression residuals (corresponding to the TAIL of 1-, 2-, 5-, 10-, 20-, and 30-year tenors), in particular the percentage and cumulative percentage of the explained variation in the first and second rows, respectively, in each block. The block labeled as ' $S_{m,n}^{PC}$ ' ($S_{m,n}^{PC}$ and $(S_{m,n}^{PC})^2$) refers to the regression on the first three principal components (and the principal components squared) of the forward swap rates. Data are monthly and the sample spans the period of June 1993–January 2013.

Panel A: Regression R ²						
	TAIL ⁽¹⁾	TAIL ⁽²⁾	TAIL ⁽⁵⁾	TAIL ⁽¹⁰⁾	TAIL ⁽²⁰⁾	TAIL ⁽³⁰⁾
$S_{m,n}^{PC}$	0.0031	0.0557	0.0613	0.0635	0.0732	0.0780
$S_{m,n}^{PC}$ and $(S_{m,n}^{PC})^2$	0.0070	0.0931	0.1202	0.1245	0.1231	0.1214
Panel B: PCs of Residuals						
	TAIL ^{PC1}	TAIL ^{PC2}	TAIL ^{PC3}	TAIL ^{PC4}	TAIL ^{PC5}	TAIL ^{PC6}
$S_{m,n}^{PC}$	0.6955	0.2139	0.0678	0.0179	0.0040	0.0009
	0.6955	0.9093	0.9772	0.9951	0.9991	1.0000
$S_{m,n}^{PC}$ and $(S_{m,n}^{PC})^2$	0.6904	0.2151	0.0706	0.0187	0.0042	0.0009
	0.6904	0.9056	0.9762	0.9948	0.9991	1.0000

Table 3: Regression of Treasury Bond Excess Returns on TAILL

This table reports the predictive regression results, using TAILL as predictors, of one-year excess returns on Treasury bonds of 2-, 5-, 10-, and 20-year maturities. The first six columns of each panel reports the OLS estimates of regression coefficients using individual TAILLs as the single predictor, with t-statistics based on Newey and West (1987) standard errors in parentheses, whereas the last column reports those of multivariate regressions using all the six TAILLs jointly as predictors. The adjusted R^2 s are reported in the last row of each panel. For brevity, we do not report the regression intercepts. All regressions are standardized, with all variables de-meanded and divided by their respective standard deviations. All data are monthly, and the sample period is June 1993–January 2013.

A: 2-year Treasury Bond								B: 5-year Treasury Bond						
$TAIL^{(1)}$	0.37						0.35	0.35						0.35
	(3.68)						(0.93)	(2.81)						(1.33)
$TAIL^{(2)}$		0.36					-0.03		0.42					-0.37
		(3.75)					(-0.07)		(3.60)					(-1.19)
$TAIL^{(5)}$			0.25				0.00			0.53				0.89
			(2.11)				(0.00)		(5.50)					(2.90)
$TAIL^{(10)}$				0.10			1.61			0.40				0.75
				(0.94)			(2.28)			(3.95)				(1.41)
$TAIL^{(20)}$					-0.01		-4.60				0.31			-5.27
					(-0.12)		(-2.82)			(3.75)				(-3.68)
$TAIL^{(30)}$						-0.03	3.05					0.28		4.32
						(-0.40)	(2.59)					(3.62)		(3.42)
R^2	0.13	0.13	0.06	0.01	0.00	0.00	0.20	0.12	0.18	0.27	0.15	0.09	0.08	0.32
C: 10-year Treasury Bond								D: 20-year Treasury Bond						
$TAIL^{(1)}$	0.26						0.50	0.17						0.64
	(1.96)						(2.51)	(1.85)						(2.89)
$TAIL^{(2)}$		(0.33)					-0.65		0.17					-0.80
		(2.51)					(-2.27)		(2.01)					(-2.18)
$TAIL^{(5)}$			0.48				1.03			0.21				0.74
			(5.91)				(3.15)		(2.90)					(2.08)
$TAIL^{(10)}$				0.40			0.26			0.15				0.04
				(5.27)			(0.61)			(2.14)				(0.11)
$TAIL^{(20)}$					0.35		-4.39				0.12			-2.48
					(5.15)		(-3.12)			(1.90)				(-1.57)
$TAIL^{(30)}$						0.33	3.91					0.11		2.20
						(5.19)	(3.09)					(1.80)		(1.59)
R^2	0.06	0.10	0.22	0.16	0.12	0.11	0.26	0.02	0.02	0.04	0.02	0.01	0.01	0.06

Table 4: Corporate Bond Returns and TAIL

This table reports the predictive regression results, using TAIL as predictors, of both long- and intermediate-term corporate bond excess returns, with 3-, 6-, and 12-month holding periods and ratings of BBB. The first six columns of each block reports OLS estimates of regression coefficients using individual TAILS as the single predictor, with t-statistics based on the Newey and West (1987) standard errors reported below regression coefficients, whereas the last column reports those of multivariate regressions using all the six TAILS jointly as predictors. The adjusted R²s are reported in the last row of each block. For brevity, we do not report estimates of the regression intercepts. Regressions are standardized, with all variables de-meaned and divided by their respective standard deviation. All data are monthly, and the sample period is June 1993–January 2013.

A: Long- & BBB- Corporate Bonds																					
3m								6m								12m					
TAIL ⁽¹⁾	-0.22						-1.00	-0.23						-1.29	-0.14						0.11
	-1.49						-1.71	-1.39						-1.69	-0.80						0.26
TAIL ⁽²⁾		-0.19					1.11		-0.18					1.38		-0.18					-0.47
		-1.13					1.29		-1.04					1.43		-1.06					-0.92
TAIL ⁽⁵⁾			-0.24				-0.32		-0.29					-0.14		-0.27					0.71
			-1.39				-0.68		-1.77					-0.35		-1.94					1.13
TAIL ⁽¹⁰⁾				-0.25			1.41		-0.37					0.41		-0.36					-0.57
				-1.59			1.32		-2.90					0.46		-3.79					-0.64
TAIL ⁽²⁰⁾					-0.26		-6.11		-0.37					-4.09		-0.35					-1.77
					-1.70		-1.95		-3.35					-1.81		-4.24					-0.66
TAIL ⁽³⁰⁾						-0.24	4.55						-0.35	3.21						-0.33	1.58
						-1.5	1.94						-2.99	1.75						-3.87	0.59
R ²	0.05	0.03	0.05	0.06	0.07	0.05	0.23	0.05	0.03	0.08	0.13	0.13	0.12	0.29	0.02	0.03	0.07	0.12	0.12	0.11	0.14
B: Intermediate- & BBB- Corporate Bonds																					
3m								6m								12m					
TAIL ⁽¹⁾	-0.28						-0.84	-0.35						-1.36	-0.33						-0.16
	-1.93						-2.05	-2.15						-1.87	-2.02						-0.43
TAIL ⁽²⁾		-0.25					0.88		-0.30					1.34		-0.37					-0.32
		-1.55					1.45		-1.71					1.47		-2.20					-0.71
TAIL ⁽⁵⁾			-0.27				-0.33		-0.36					-0.16		-0.41					0.55
			-1.52				-0.85		-2.07					-0.41		-2.86					0.98
TAIL ⁽¹⁰⁾				-0.23			1.84		-0.39					0.83		-0.43					-0.30
				-1.33			1.71		-2.51					1.01		-3.44					-0.38
TAIL ⁽²⁰⁾					-0.24		-7.48		-0.37					-5.58		-0.39					-2.19
					-1.31		-2.27		-2.51					-2.79		-3.16					-0.84
TAIL ⁽³⁰⁾						-0.21	5.59						-0.34	4.31						-0.37	1.80
						-1.13	2.27						-2.18	2.76						-2.72	0.69
R ²	0.08	0.06	0.07	0.05	0.05	0.04	0.26	0.12	0.09	0.13	0.15	0.14	0.12	0.39	0.11	0.13	0.17	0.18	0.15	0.13	0.25

Table 5: CMBS Returns and TAIL

This table reports the predictive regression results, using TAILS as predictors, of CMBS excess returns with 3-, 6-, and 12-month holding periods and ratings of AAA and A. The first six columns of each block reports OLS estimates of regression coefficients using individual TAILS as the single predictor, with t-statistics based on the Newey and West (1987) standard errors reported below regression coefficients, whereas the last column reports those of multivariate regressions using all the six TAILS jointly as predictors. The adjusted R²s are reported in the last row of each block. For brevity, we do not report estimates of the regression intercepts. Regressions are standardized, with all variables de-meaned and divided by their respective standard deviation. All data are monthly, and the sample period is December 1997–December 2011.

A: AAA-CMBS																					
3m								6m								12m					
TAIL ⁽¹⁾	-0.26						-0.81	-0.27					-1.06	-0.23							0.38
	-1.38						-1.39	-1.48					-1.32	-1.35							0.76
TAIL ⁽²⁾		-0.25					0.93		-0.4				0.97		-0.30						-0.99
		-1.18					1.17		-1.24				1.02		-1.68						-1.63
TAIL ⁽⁵⁾			-0.35				-0.47		-0.35				-0.28		-0.40						0.62
			-1.42				-1.06		-1.58				-0.72		-2.16						0.98
TAIL ⁽¹⁰⁾				-0.35			3.01		-0.42				1.97		-0.48						0.36
				-1.48			3.47		-1.99				3.65		-2.70						0.43
TAIL ⁽²⁰⁾					-0.39		-10.86		-0.47				-6.11		-0.49						-2.25
					-1.66		-5.02		-2.33				-2.91		-2.91						-0.78
TAIL ⁽³⁰⁾						-0.36	7.78		-0.45				3.81							-0.47	1.17
						-1.51	5.04		-2.17				1.86							-2.68	0.40
R ²	0.06	0.06	0.11	0.12	0.15	0.12	0.50	0.07	0.05	0.12	0.17	0.22	0.20	0.43	0.04	0.08	0.15	0.22	0.23	0.22	0.31
B: A-CMBS																					
3m								6m								12m					
TAIL ⁽¹⁾	-0.41						-0.54	-0.30					-0.70	-0.26							0.66
	-1.50						-1.55	-1.59					-1.28	-1.60							1.07
TAIL ⁽²⁾		-0.35					0.47		-0.33				0.33		-0.36						-1.48
		-1.44					0.98		-1.57				0.55		-1.93						-1.75
TAIL ⁽⁵⁾			-0.52				-0.34		-0.48				-0.20		-0.45						0.68
			-1.92				-1.02		-2.18				-0.52		-2.47						1.03
TAIL ⁽¹⁰⁾				-0.55			2.54		-0.57				1.38		-0.52						-0.01
				-2.17			5.47		-2.87				2.66		-3.01						-0.01
TAIL ⁽²⁰⁾					-0.58		-9.01		-0.62				-2.04		-0.52						0.51
					-2.42		-7.40		-3.24				-0.68		-3.17						0.16
TAIL ⁽³⁰⁾						-0.54	6.18		-0.61				0.18							-0.51	-1.25
						-2.23	5.96		-2.96				0.06								-2.89
R ²	0.09	0.11	0.26	0.29	0.33	0.29	0.60	0.08	0.10	0.22	0.32	0.38	0.37	0.54	0.06	0.12	0.20	0.26	0.27	0.26	0.41

Table 6: Fixed-Income Hedge Funds and TAIL

This table reports the predictive regression results, using TAILs as predictors, of one-month fixed-income hedge fund index returns, with trading styles as relative value, corporate, and yield alternative,. The first six columns of each block reports OLS estimates of regression coefficients using individual TAILs as the single predictor, with t-statistics based on the Newey and West (1987) standard errors reported below regression coefficients, whereas the last column reports those of multivariate regressions using all the six TAILs jointly as predictors. The adjusted R²s are reported in the last row of each block. For brevity, we do not report estimates of the regression intercepts. Regressions are standardized, with all variables de-meaned and divided by their respective standard deviation. All data are monthly, and the sample period is January 1994–April 2012.

	A: Fixed Income–Relative Value						B: Fixed Income–Corporate						C: Yield Alternative											
TAIL ⁽¹⁾	-0.24						-0.95	-0.18						-0.92	-0.19									-0.54
	-1.49						-3.34	-1.12						-2.57	-1.68									-3.68
TAIL ⁽²⁾		-0.24					1.24		-0.17					1.13		-0.19								0.59
		-1.49					2.94		-0.94					2.31		-1.59								2.81
TAIL ⁽⁵⁾			-0.31				-1.00		-0.23					-0.70		-0.22								-0.56
			-2.31				-2.18		-1.39					-1.74		-2.02								-2.14
TAIL ⁽¹⁰⁾				-0.26			1.40		-0.23					1.73		-0.19								1.36
				-2.28			2.21		-1.77					2.91		-1.99								2.05
TAIL ⁽²⁰⁾					-0.25		-4.17		-0.25					-5.63		-0.19								-3.49
					-2.25		-3.41		-2.11					-4.10		-2.09								-2.16
TAIL ⁽³⁰⁾						-0.22	3.08						-0.24	4.02									-0.18	2.31
						-2.05	3.19						-1.97	3.76									-1.97	1.95
R ²	0.05	0.05	0.09	0.06	0.06	0.05	0.19	0.03	0.02	0.05	0.05	0.06	0.05	0.19	0.03	0.03	0.04	0.03	0.03	0.03	0.03	0.03	0.03	0.09

Table 7: TAILL and Control Variables

Panel A: This panel reports the predictive regression results, using the TAILL PCs as predictors and controlling for the macroeconomic and CP factors, of one-year excess returns of Treasury bonds with 2-, 5-, 10-, and 20-year maturities. The macro factors \widehat{F}_j , $j = 1, 2, \dots, 8$, are estimated (following Ludvigson and Ng (2009, 2010)) as the first eight principal components from a dataset of 104 macro variables updated until July 2012. The CP return predictor of Cochrane and Piazzesi (2005) is constructed using the Gürkaynak, Sack, and Wright (2006) Treasury yield data obtained from the Fed website, with maturities of 1, 2, 3, 4, 5, 7, 10, 15, and 20 years. Panel A.1 reports results using TAILL PCs as predictors alone, while Panel A.2 reports results controlling for the macro and CP factors. We report the OLS estimates of regression coefficients, t-statistics based on the Newey and West (1987) standard errors (in parentheses), and adjusted R^2 s. For brevity, we do not report estimates of the regression intercepts. Regressions are standardized, with all variables de-measured and divided by their respective standard deviation. All data are monthly, and the sample period is June 1993–July 2012.

Panel A.1: Regression on TAILL PCs								
	2y		5y		10y		20y	
TAILL^{PC1}	0.33	(3.28)	0.44	(5.52)	0.36	(5.09)	0.18	(2.59)
TAILL^{PC2}	0.07	(1.03)	-0.25	(-3.86)	-0.31	(-3.95)	-0.10	(-1.07)
TAILL^{PC3}	0.04	(0.35)	0.17	(2.05)	0.11	(1.67)	0.00	(0.07)
TAILL^{PC4}	0.11	(1.06)	0.13	(1.91)	0.14	(2.10)	0.15	(1.74)
TAILL^{PC5}	0.15	(1.73)	-0.05	(-0.63)	-0.12	(-1.72)	-0.08	(-1.45)
TAILL^{PC6}	0.20	(2.85)	0.23	(3.68)	0.19	(3.24)	0.11	(1.56)
R^2	0.19		0.34		0.29		0.06	
Panel A.2: Regression on TAILL PCs Controlling for Other Predictors								
	2y		5y		10y		20y	
TAILL^{PC1}	-0.22	(-2.52)	0.04	(0.91)	0.05	(1.32)	-0.03	(-0.51)
TAILL^{PC2}	0.30	(3.67)	0.12	(3.23)	-0.03	(-0.43)	0.06	(0.53)
TAILL^{PC3}	-0.21	(-2.59)	-0.06	(-1.97)	-0.09	(-1.85)	-0.17	(-2.40)
TAILL^{PC4}	-0.04	(-0.85)	-0.01	(-0.43)	0.01	(0.12)	0.02	(0.30)
TAILL^{PC5}	0.07	(1.28)	0.00	(-0.15)	-0.06	(-1.26)	-0.03	(-0.50)
TAILL^{PC6}	-0.12	(-2.12)	-0.06	(-1.74)	0.00	(-0.03)	0.00	(-0.05)
\widehat{F}_1	-0.04	(-0.57)	0.11	(3.46)	0.03	(0.60)	-0.07	(-0.88)
\widehat{F}_2	-0.14	(-1.93)	0.08	(2.10)	0.05	(0.74)	-0.02	(-0.21)
\widehat{F}_3	0.01	(0.26)	-0.03	(-0.99)	-0.09	(-1.79)	-0.13	(-2.30)
\widehat{F}_4	-0.03	(-0.40)	0.09	(2.32)	0.13	(2.96)	0.19	(3.29)
\widehat{F}_5	0.00	(-0.01)	-0.05	(-1.81)	-0.05	(-1.60)	-0.06	(-1.28)
\widehat{F}_6	0.04	(1.08)	0.06	(1.50)	0.02	(0.31)	0.00	(0.03)
\widehat{F}_7	-0.01	(-0.30)	-0.03	(-0.82)	-0.06	(-1.22)	-0.04	(-0.85)
\widehat{F}_8	0.03	(0.59)	0.06	(2.0)	0.16	(3.31)	0.16	(3.13)
CP	1.09	(10.9)	0.85	(15.4)	0.69	11.3	0.57	(7.43)
R^2	0.69		0.73		0.55		0.24	

Panel B: This panel reports the predictive regression results, using the **TAILL** PCs as predictors and controlling for the macroeconomic and *CP* factors, of long-term and intermediate-term BBB corporate bond excess returns with 3-, 6-, and 12-month holding periods. The macro factors \widehat{F}_j , $j = 1, 2, \dots, 8$, are estimated (following Ludvigson and Ng (2009, 2010)) as the first eight principal components from a dataset of 104 macro variables updated until July 2012. The *CP* return predictor of Cochrane and Piazzesi (2005) is constructed using the Gürkaynak, Sack, and Wright (2006) Treasury yield data obtained from the Fed website, with maturities of 1, 2, 3, 4, 5, 7, 10, 15, and 20 years. Panels B.1 reports results using **TAILL** PCs as predictors alone, while Panels B.2 reports results controlling for the macro and *CP* factors. We report the OLS estimates of regression coefficients, t-statistics based on the Newey and West (1987) standard errors (in parentheses), and adjusted R^2 s. For brevity, we do not report estimates of the regression intercepts. Regressions are standardized, with all variables de-meaned and divided by their respective standard deviation. All data are monthly, and the sample period is June 1993–July 2012.

	Long-term						Intermediate-term					
	3m		6m		12m		3m		6m		12m	
B.1: Regression on TAILL PCs												
TAILL ^{PC1}	-0.15	(-1.31)	-0.14	(-1.03)	-0.12	(-0.85)	-0.25	(-2.27)	-0.28	(-2.25)	-0.31	(-2.44)
TAILL ^{PC2}	0.19	(2.18)	0.30	(4.21)	0.30	(3.79)	0.14	(1.73)	0.26	(3.57)	0.30	(3.75)
TAILL ^{PC3}	0.30	(3.81)	0.40	(2.42)	0.16	(1.47)	0.21	(3.34)	0.40	(2.45)	0.18	(1.87)
TAILL ^{PC4}	-0.20	(-1.44)	-0.26	(-1.76)	0.06	(0.73)	-0.13	(-1.25)	-0.24	(-1.8)	0.03	(0.40)
TAILL ^{PC5}	0.11	(0.88)	-0.01	(-0.08)	-0.16	(-0.86)	0.15	(1.31)	0.03	(0.26)	-0.11	(-0.67)
TAILL ^{PC6}	0.24	(1.71)	0.15	(1.57)	0.07	(0.52)	0.32	(2.14)	0.22	(2.62)	0.08	(0.67)
R^2	0.25		0.33		0.15		0.26		0.42		0.24	
B.2: Regression on TAILL PCs Controlling for Other Predictors												
TAILL ^{PC1}	-0.45	(-4.09)	-0.48	(-4.05)	-0.26	(-2.31)	-0.44	(-3.72)	-0.51	(-4.47)	-0.33	(-4.28)
TAILL ^{PC2}	0.50	(5.83)	0.67	(6.65)	0.66	(3.57)	0.35	(3.10)	0.50	(5.53)	0.52	(3.16)
TAILL ^{PC3}	0.24	(3.58)	0.26	(2.13)	-0.10	(-1.34)	0.16	(2.34)	0.25	(1.94)	-0.07	(-1.13)
TAILL ^{PC4}	-0.17	(-2.02)	-0.29	(-2.98)	-0.04	(-0.64)	-0.12	(-1.65)	-0.29	(-2.80)	-0.10	(-1.65)
TAILL ^{PC5}	0.06	(0.56)	-0.02	(-0.21)	-0.13	(-2.70)	0.09	(0.89)	0.00	(-0.04)	-0.10	(-2.35)
TAILL ^{PC6}	-0.02	(-0.18)	-0.18	(-2.25)	-0.25	(-2.99)	0.12	(0.94)	-0.04	(-0.51)	-0.16	(-2.14)
\widehat{F}_1	0.31	(3.13)	0.24	(2.79)	-0.06	(-0.45)	0.16	(1.24)	0.05	(0.60)	-0.26	(-2.33)
\widehat{F}_2	0.14	(1.35)	0.24	(1.60)	0.40	(2.07)	0.12	(1.18)	0.18	(1.50)	0.35	(2.08)
\widehat{F}_3	0.17	(4.31)	0.12	(3.07)	0.18	(3.91)	0.17	(3.84)	0.12	(3.01)	0.17	(4.33)
\widehat{F}_4	-0.28	(-3.66)	-0.20	(-4.00)	0.04	(0.59)	-0.21	(-2.54)	-0.13	(-2.27)	0.12	(2.18)
\widehat{F}_5	0.16	(2.66)	0.11	(2.25)	0.13	(3.25)	0.14	(2.72)	0.11	(1.89)	0.10	(2.78)
\widehat{F}_6	-0.04	(-0.56)	0.02	(0.88)	-0.06	(-1.62)	-0.06	(-0.75)	0.01	(0.35)	-0.06	(-2.18)
\widehat{F}_7	0.01	(0.30)	0.03	(0.86)	0.04	(0.85)	0.01	(0.40)	0.03	(1.01)	0.04	(0.98)
\widehat{F}_8	-0.14	(-2.80)	-0.11	(-3.07)	-0.14	(-3.17)	-0.16	(-3.25)	-0.13	(-3.79)	-0.15	(-3.77)
<i>CP</i>	0.34	(3.56)	0.55	(5.82)	0.57	(4.03)	0.25	(2.42)	0.48	(5.12)	0.50	(3.94)
R^2	0.48		0.65		0.58		0.41		0.64		0.63	

Panel C: This panel reports the predictive regression results, using the **TAIL** PCs as predictors and controlling for the macroeconomic and *CP* factors, of AAA and A rated CMBS excess returns with 3-, 6-, and 12-month holding periods. The macro factors \widehat{F}_j , $j = 1, 2, \dots, 8$, are estimated (following Ludvigson and Ng (2009, 2010)) as the first eight principal components from a dataset of 104 macro variables updated until July 2012. The *CP* return predictor of Cochrane and Piazzesi (2005) is constructed using the Gürkaynak, Sack, and Wright (2006) Treasury yield data obtained from the Fed website, with maturities of 1, 2, 3, 4, 5, 7, 10, 15, and 20 years. Panel C.1 reports results using **TAIL** PCs as predictors alone, while Panel C.2 reports results controlling for the macro and *CP* factors. We report the OLS estimates of regression coefficients, t-statistics based on the Newey and West (1987) standard errors (in parentheses), and adjusted R^2 s. For brevity, we do not report estimates of the regression intercepts. Regressions are standardized, with all variables de-meaned and divided by their respective standard deviation. All data are monthly, and the sample period is December 1997–December 2011.

	AAA						A					
	3m		6m		12m		3m		6m		12m	
C.1: Regression on TAIL PCs												
TAIL ^{PC1}	-0.29	(-2.96)	-0.25	(-2.9)	-0.30	(-4.30)	-0.39	(-5.74)	-0.35	(-4.64)	-0.37	(-4.03)
TAIL ^{PC2}	0.26	(3.45)	0.33	(4.31)	0.39	(3.73)	0.43	(5.45)	0.48	(6.73)	0.43	(4.45)
TAIL ^{PC3}	0.21	(1.87)	0.37	(1.76)	0.09	(0.88)	0.17	(2.27)	0.31	(2.03)	0.03	(0.25)
TAIL ^{PC4}	-0.08	(-0.63)	-0.07	(-0.48)	0.25	(2.75)	0.00	(0.05)	0.07	(0.75)	0.35	(2.64)
TAIL ^{PC5}	0.30	(2.53)	0.23	(2.07)	0.01	(0.05)	0.27	(3.56)	0.23	(1.98)	0.01	(0.07)
TAIL ^{PC6}	0.46	(5.53)	0.24	(2.45)	0.10	(0.69)	0.38	(7.76)	0.06	(0.42)	-0.02	(-0.16)
R^2	0.51		0.44		0.31		0.61		0.53		0.42	
C.2: Regression on TAIL PCs Controlling for Other Predictors												
TAIL ^{PC1}	-0.32	(-4.24)	-0.26	(-3.44)	-0.22	(-1.97)	-0.38	(-7.00)	-0.31	(-3.08)	-0.25	(-1.79)
TAIL ^{PC2}	0.51	(9.25)	0.62	(5.67)	0.65	(3.19)	0.56	(9.66)	0.66	(5.52)	0.41	(2.87)
TAIL ^{PC3}	0.17	(1.96)	0.24	(1.76)	-0.14	(-1.22)	0.11	(1.95)	0.15	(1.79)	-0.15	(-1.15)
TAIL ^{PC4}	-0.05	(-0.71)	-0.10	(-0.95)	0.12	(1.87)	0.00	(0.06)	0.00	(0.04)	0.22	(2.21)
TAIL ^{PC5}	0.30	(3.98)	0.28	(3.96)	0.07	(0.91)	0.28	(4.48)	0.27	(3.86)	-0.01	(-0.08)
TAIL ^{PC6}	0.26	(4.69)	0.01	(0.20)	-0.11	(-1.30)	0.21	(5.94)	-0.13	(-1.20)	-0.10	(-1.13)
\widehat{F}_1	0.15	(1.83)	0.05	(0.47)	-0.22	(-1.59)	-0.04	(-0.64)	-0.15	(-1.48)	-0.44	(-2.83)
\widehat{F}_2	0.39	(4.30)	0.50	(3.33)	0.51	(2.96)	0.39	(4.97)	0.44	(3.19)	0.22	(1.33)
\widehat{F}_3	0.14	(3.14)	0.08	(2.28)	0.14	(2.63)	0.06	(1.42)	0.09	(1.88)	0.10	(2.15)
\widehat{F}_4	-0.22	(-2.25)	-0.12	(-2.32)	0.10	(1.08)	-0.20	(-2.07)	-0.04	(-0.99)	0.14	(1.38)
\widehat{F}_5	0.19	(2.31)	0.11	(1.66)	0.08	(1.52)	0.06	(1.56)	0.06	(1.47)	0.02	(0.42)
\widehat{F}_6	-0.03	(-0.99)	0.04	(0.88)	-0.06	(-1.90)	0.06	(1.12)	0.03	(0.68)	-0.06	(-2.07)
\widehat{F}_7	-0.03	(-0.65)	0.00	(0.04)	-0.05	(-1.28)	0.04	(1.45)	0.02	(0.64)	-0.06	(-1.31)
\widehat{F}_8	-0.11	(-3.05)	-0.09	(-3.44)	-0.14	(-2.71)	-0.12	(-3.44)	-0.11	(-3.76)	-0.11	(-2.10)
<i>CP</i>	0.03	(0.35)	0.15	(2.09)	0.30	(2.46)	0.10	(2.07)	0.23	(2.62)	0.27	(2.30)
R^2	0.68		0.64		0.65		0.75		0.75		0.64	

Panel D: This panel reports the predictive regression results, using the T\AAILL PCs as predictors and controlling for the macroeconomic and CP factors, of one-month index excess returns of fixed-income hedge funds with strategy styles as relative value, corporate, and yield alternative. The macro factors \widehat{F}_j , $j = 1, 2, \dots, 8$, are estimated (following Ludvigson and Ng (2009, 2010)) as the first eight principal components from a dataset of 104 macro variables updated until July 2012. The CP return predictor of Cochrane and Piazzesi (2005) is constructed using the Gürkaynak, Sack, and Wright (2006) Treasury yield data obtained from the Fed website, with maturities of 1, 2, 3, 4, 5, 7, 10, 15, and 20 years. Panel D.1 reports results using T\AAILL PCs as predictors alone, while Panel D.2 reports results controlling for the macro and CP factors. We report the OLS estimates of regression coefficients, t-statistics based on the Newey and West (1987) standard errors (in parentheses), and adjusted R^2 s. For brevity, we do not report estimates of the regression intercepts. Regressions are standardized, with all variables de-measured and divided by their respective standard deviation. All data are monthly, and the sample period is January 1994-April 2012.

	Fixed Income–Relative Value		Fixed Income–Corporate		Yield Alternative	
	Panel D.1 Regression on PCs of T\AAILL					
T\AAILL^{PC1}	-0.27	(-2.75)	-0.19	(-1.95)	-0.22	(-2.83)
T\AAILL^{PC2}	0.15	(1.68)	0.16	(2.67)	0.10	(1.65)
T\AAILL^{PC3}	0.07	(1.38)	0.16	(2.97)	0.07	(1.46)
T\AAILL^{PC4}	-0.23	(-2.70)	-0.16	(-1.77)	-0.08	(-1.90)
T\AAILL^{PC5}	0.16	(1.83)	0.17	(2.21)	0.15	(2.03)
T\AAILL^{PC6}	0.19	(3.34)	0.25	(4.00)	0.15	(2.10)
R^2		0.19		0.19		0.09
	Panel D.2. Regression on PCs of T\AAILL Controlling Other Predictors					
T\AAILL^{PC1}	-0.27	(-3.58)	-0.34	(-4.86)	-0.31	(-5.92)
T\AAILL^{PC2}	0.28	(3.67)	0.30	(4.91)	0.14	(2.34)
T\AAILL^{PC3}	0.05	(1.16)	0.10	(1.68)	0.03	(0.63)
T\AAILL^{PC4}	-0.18	(-3.33)	-0.14	(-2.63)	-0.08	(-2.10)
T\AAILL^{PC5}	0.10	(1.62)	0.09	(1.92)	0.06	(1.24)
T\AAILL^{PC6}	0.10	(1.68)	0.09	(1.60)	0.06	(0.67)
\widehat{F}_1	0.09	(0.89)	0.08	(0.93)	-0.02	(-0.21)
\widehat{F}_2	0.07	(0.93)	0.01	(0.08)	-0.05	(-0.76)
\widehat{F}_3	0.25	(3.67)	0.19	(3.94)	0.13	(2.16)
\widehat{F}_4	-0.09	(-0.89)	-0.12	(-1.40)	-0.08	(-0.98)
\widehat{F}_5	0.19	(3.08)	0.16	(2.70)	0.20	(3.26)
\widehat{F}_6	-0.27	(-1.88)	-0.24	(-2.20)	-0.11	(-0.94)
\widehat{F}_7	0.04	(0.65)	0.08	(1.58)	-0.06	(-0.71)
\widehat{F}_8	-0.06	(-1.52)	-0.15	(-2.56)	-0.06	(-0.92)
CP	0.01	(0.06)	0.26	(2.88)	0.18	(1.77)
R^2		0.34		0.37		0.15

Table 8: Out-of-Sample Predictability

This table reports the out-of-sample prediction results, using the `TAILL` (of 1-, 2-, 5-, 10-, 20-, and 30-year tenors jointly), for one-year excess returns of Treasury bonds, corporate bonds, and CMBS. ‘`TAILL vs Const`’ reports forecast comparisons of an unrestricted model with `TAILL` versus a restricted constant expected return benchmark model, while ‘`TAILL + CP vs Const+CP`’ of an unrestricted model with `TAILL` and the `CP` factor versus a restricted benchmark including a constant and the `CP` factor. R_{OOS}^2 is the out-of-sample R^2 of Campbell and Thompson (2008), with a positive value indicating that the unrestricted model has a lower forecast error than the restricted benchmark. ENC-NEW denotes the test statistic of Clark and McCracken (2001) for the null hypothesis that the benchmark model encompasses the unrestricted model. The ‘*’ indicates significance for the ENC-NEW test statistic at minimally the 95% level using the bootstrapped critical values (BCV). The initial estimation period and out-of-sample evaluation period are 1993:06–2002:01 and 2003:01–2013:01 for Treasury and corporate bonds, and 1997:12–2005:01 and 2006:01–2011:12 for CMBS. All data are monthly.

		Panel A: Treasury Bonds					
		2y	5y	10y	20y		
<code>TAILL vs Const</code>	R_{OOS}^2	0.12	0.20	0.20	0.16		
	ENC-New	53.7*	55.2*	48.22*	26.92*		
	95% BCV	11.28	14.15	18.90	12.16		
<code>(TAILL + CP) vs (Const+CP)</code>	R_{OOS}^2	0.29	-0.23	-0.19	0.00		
	ENC-New	97.98*	2.82	10.98	10.51*		
	95% BCV	64.57	10.73	15.36	8.71		
		Panel B: Corporate Bonds					
		BBB		BB		CCC	
		Long	Intermediate	Long	Intermediate	Long	Intermediate
<code>TAILL vs Const</code>	R_{OOS}^2	0.39	0.29	0.36	0.20	0.16	0.10
	ENC-New	86.84*	83.65*	85.09*	82.13*	57.09*	58.51*
	95% BCV	8.55	10.20	10.23	11.28	9.64	15.76
<code>(TAILL + CP) vs (Const+CP)</code>	R_{OOS}^2	0.35	0.40	0.23	0.23	0.10	0.11
	ENC-New	73.90*	97.75*	79.85*	82.00*	55.67*	59.95*
	95% BCV	10.13	13.57	9.65	13.70	13.20	16.00
		Panel B: CMBS					
		AAA	A	BBB	B		
<code>TAILL vs Const</code>	R_{OOS}^2	0.30	0.42	0.11	0.25		
	ENC-New	58.13*	70.55*	21.69*	61.73*		
	95% BCV	8.23	7.23	14.20	9.80		
<code>(TAILL + CP) vs (Const+CP)</code>	R_{OOS}^2	0.39	0.54	0.42	0.48		
	ENC-New	73.28*	98.33*	50.14*	97.68*		
	95% BCV	7.96	7.25	17.16	9.86		

Table 9: Return Predictability of TAIL over Various Horizons

This table reports the results of predictive regressions, using PCs of the one-year TAIL (of 1-, 2-, 5-, 10-, 20-, and 30-year tenors jointly), for one-year excess returns of Treasury bonds, corporate bonds, and CMBS. We report the OLS estimates of regression coefficients, t-statistics based on the Newey and West (1987) standard errors (in parentheses), and adjusted R²s. For brevity, we do not report estimates of the regression intercepts. Regressions are standardized, with all variables de-measured and divided by their respective standard deviation. All data are monthly, and the sample period is from June 1993 through January 2013 for Treasury and corporate bonds, and from December 1997 through December 2011 for CMBS.

Panel A: 12-month Treasury bond excess returns												
	2y		5y		10y		20y					
TAIL ^{PC1}	0.23	(2.49)	0.24	(3.69)	0.16	(2.67)	0.08	(0.88)				
TAIL ^{PC2}	-0.12	(-1.55)	-0.49	(-7.84)	-0.51	(-8.27)	-0.27	(-3.29)				
TAIL ^{PC3}	0.36	(4.15)	0.29	(4.09)	0.22	(3.15)	0.23	(3.54)				
TAIL ^{PC4}	0.06	(0.60)	-0.14	(-2.09)	-0.11	(-2.24)	0.00	(0.06)				
TAIL ^{PC5}	0.20	(2.19)	0.06	(1.00)	-0.01	(-0.27)	-0.1	(-1.46)				
TAIL ^{PC6}	0.22	(2.28)	0.11	(1.69)	0.06	(1.13)	0.03	(0.56)				
R ²	0.29		0.44		0.37		0.12					

Panel B: 12-month corporate bond excess returns												
	Long						Intermediate					
	BBB		B		CCC		BBB		B		CCC	
TAIL ^{PC1}	-0.03	(-1.92)	-0.01	(-0.55)	-0.04	(-0.92)	-0.03	(-3.58)	-0.04	(-2.91)	-0.04	(-1.49)
TAIL ^{PC2}	0.01	(1.12)	0.02	(1.08)	0.04	(1.09)	0.01	(1.40)	0.02	(1.35)	0.02	(0.69)
TAIL ^{PC3}	0.08	(4.48)	0.11	(4.86)	0.12	(3.03)	0.05	(4.38)	0.06	(4.19)	0.11	(3.69)
TAIL ^{PC4}	-0.06	(-2.35)	-0.07	(-2.74)	-0.05	(-1.04)	-0.03	(-2.16)	-0.03	(-1.92)	-0.06	(-1.73)
TAIL ^{PC5}	0.00	(0.09)	0.00	(0.16)	0.03	(0.74)	-0.01	(-1.42)	-0.03	(-2.75)	-0.04	(-1.86)
TAIL ^{PC6}	0.00	(0.53)	0.02	(1.35)	0.01	(0.28)	0.01	(1.64)	0.01	(1.13)	0.01	(0.80)
R ²	0.48		0.50		0.15		0.55		0.49		0.39	

Panel C: 12-month CMBS excess returns												
	AAA		A		BBB		B					
TAIL ^{PC1}	-0.27	(-2.77)	-0.29	(-2.75)	-0.28	(-4.09)	-0.16	(-2.31)				
TAIL ^{PC2}	0.19	(1.46)	0.27	(1.96)	-0.03	(-0.32)	-0.73	(-7.91)				
TAIL ^{PC3}	0.57	(3.67)	0.52	(3.91)	0.57	(6.88)	-0.04	(-0.60)				
TAIL ^{PC4}	-0.23	(-1.37)	-0.04	(-0.20)	-0.39	(-2.79)	-0.30	(-3.85)				
TAIL ^{PC5}	0.02	(0.25)	-0.06	(-0.53)	-0.04	(-0.39)	-0.25	(-5.22)				
TAIL ^{PC6}	0.04	(0.60)	-0.02	(-0.13)	0.03	(0.45)	0.09	(1.16)				
R ²	0.52		0.41		0.64		0.75					

Table 10: TAIL and Volatility Factors

This table reports the results of predictive regressions, using the TAIL of 5-, 10-, and 20-year tenors controlling for swaption-based volatility factors of corresponding tenors respectively, for one-year excess returns of Treasury bonds, corporate bonds, and CMBS, and monthly index excess returns of fixed-income hedge funds. We consider the at-the-money swaption implied volatility (*ATMV*) and model-free implied volatility (*MFIV*) defined in (8). We report estimates of regression coefficients, Newey and West (1987) t-statistics (in parentheses), and adjusted R²s. For brevity, we do not report estimates of the regression intercepts. Regressions are standardized, with all variables de-meaned and divided by their respective standard deviation. All data are monthly, and the sample period is June 1993-January 2013 for Treasury and corporate bonds, December 1997-December 2011 for CMBS, and January 1994-April 2012 for hedge fund index returns.

	A: 10y Treasury						B: Long-term BBB Corporate Bond					
TAIL ⁽⁵⁾	0.48	0.55					-0.41	-0.29				
	(7.82)	(8.32)					(-2.09)	(-1.89)				
ATMV ⁽⁵⁾	0.30						0.24					
	(5.68)						(1.11)					
MFIV ⁽⁵⁾		0.32						0.03				
		(5.64)						(0.19)				
TAIL ⁽¹⁰⁾			0.34	0.40					-0.90	-1.12		
			(4.55)	(5.50)					(-3.35)	(-2.80)		
ATMV ⁽¹⁰⁾			0.18						0.69			
			(1.75)						(2.29)			
MFIV ⁽¹⁰⁾				0.18						0.85		
				(2.47)						(2.00)		
TAIL ⁽²⁰⁾					0.26	0.33					-0.81	-1.16
					(3.99)	(5.35)					(-3.65)	(-3.32)
ATMV ⁽²⁰⁾					0.19						0.60	
					(1.52)						(2.35)	
MFIV ⁽²⁰⁾						0.15						0.90
						(1.81)						(2.51)
R ²	0.31	0.32	0.18	0.19	0.14	0.14	0.10	0.07	0.30	0.27	0.26	0.26
	C: AAA CMBS						D: Fixed Income-Relative Value					
TAIL ⁽⁵⁾	-0.43	-0.32					-0.51	-0.54				
	(-2.43)	(-2.60)					(-2.15)	(-1.87)				
ATMV ⁽⁵⁾	0.07						0.24					
	(0.49)						(1.07)					
MFIV ⁽⁵⁾		-0.14						0.29				
		(-0.89)						(1.05)				
TAIL ⁽¹⁰⁾			-0.84	-0.90					-0.35	-0.86		
			(-3.26)	(-2.90)					(-3.01)	(-2.60)		
ATMV ⁽¹⁰⁾			0.48						0.11			
			(2.35)						(0.78)			
MFIV ⁽¹⁰⁾				0.48						0.64		
				(1.59)						(1.88)		
TAIL ⁽²⁰⁾					-0.85	-1.12					-0.26	-0.66
					(-4.00)	(-4.80)					(-2.83)	(-2.74)
ATMV ⁽²⁰⁾					0.48						0.02	
					(3.18)						(0.16)	
MFIV ⁽²⁰⁾						0.70						0.45
						(3.08)						(1.54)
R ²	0.15	0.16	0.32	0.27	0.33	0.32	0.11	0.12	0.06	0.12	0.05	0.08

Table 11: Equity Returns and TAIL

This table reports the predictive regression results, using the TAIL (of 1-, 2-, 5-, 10-, 20-, and 30-year tenors jointly) and controlling for other predictors, of 3-, 6- and 12-month excess returns on the equity market (value-weighted CRSP index). The control variables we use are log dividend yield (DY), log earnings/price ratio (E/P), and net equity expansion (NTIS) used by Goyal and Welch (2008) and obtained from Amit Goyal's webpage. We report the OLS estimates of regression coefficients, t-statistics based on the Newey and West (1987) standard errors (in parentheses), and adjusted R²s. For brevity, we do not report estimates of the regression intercepts. Regressions are standardized, with all variables de-meaned and divided by their respective standard deviation. All data are monthly, and the sample period is June 1993–December 2011.

	Market					
	3m		6m		12m	
TAIL ⁽¹⁾	-0.10	-0.14	0.04	0.02	0.28	0.23
	(-0.69)	(-0.86)	(0.31)	(0.10)	(2.24)	(2.38)
TAIL ⁽²⁾	-0.93	-0.88	-0.80	-0.84	-0.65	-0.72
	(-3.19)	(-2.45)	(-2.01)	(-1.68)	(-1.72)	(-1.58)
TAIL ⁽⁵⁾	1.84	1.62	1.49	1.40	0.94	1.09
	(5.44)	(2.31)	(5.67)	(1.85)	(2.26)	(1.31)
TAIL ⁽¹⁰⁾	-1.71	-1.84	-1.21	-1.61	0.68	0.07
	(-1.49)	(-1.47)	(-1.32)	(-1.55)	(0.68)	(0.08)
TAIL ⁽²⁰⁾	0.83	2.00	-0.95	0.73	-4.38	-2.72
	(0.51)	(1.06)	(-0.65)	(0.45)	(-1.69)	(-1.70)
TAIL ⁽³⁰⁾	-0.26	-1.15	1.29	0.16	3.25	2.20
	(-0.26)	(-0.83)	(1.30)	(0.14)	(1.96)	(2.10)
DY		0.21		0.25		0.36
		(1.65)		(1.50)		(1.92)
E/P		-0.08		-0.08		0.01
		(-0.48)		(-0.43)		(0.05)
NTIS		0.19		0.38		0.45
		(1.40)		(1.89)		(2.23)
R ²	0.10	0.16	0.10	0.26	0.07	0.37

Table 12: TAIL and Macroeconomic Uncertainty

This table reports the results from regressing TAILS of 1-, 2-, 5-, 10-, 20-, and 30-year tenors on contemporaneous consensus and uncertainty measures of macroeconomic forecast surveys. Forecast data of the unemployment (UNEM), ten-year constant maturity Treasury yield (LR), industrial rial production (IP), housing starts (HOUST), consumer price index (CPI), and total US auto and truck Sales (AS) for the subsequent calendar year are from BCEI, whereas those of the federal funds rate (FFR) and real GDP (RGDP) one-year ahead are from BCFF. The BCEI forecast series are adjusted using the x-12 ARIMA filter. \hat{E} denotes the consensus measure, defined as a median forecast, and \hat{U} proxies for the uncertainty measure, computed as the cross sectional standard deviation of individual forecasts. Regressions are standardized, meaning all variables are de-meant and divided by their respective standard deviation. The OLS estimators of regression coefficients, t-statistics based on the Newey and West (1987) standard errors, and regression R^2 are reported in the first, second (in parentheses), and third columns of each block, respectively. All data are monthly, and the sample period is June 1993–January 2013.

	TAIL ⁽¹⁾			TAIL ⁽²⁾			TAIL ⁽⁵⁾			TAIL ⁽¹⁰⁾			TAIL ⁽²⁰⁾			TAIL ⁽³⁰⁾		
\hat{E}_{FFR}	0.15	(0.69)	0.02	0.01	(0.05)	0.00	-0.39	(-2.16)	0.15	-0.55	(-4.51)	0.30	-0.53	(-4.18)	0.28	-0.55	(-4.18)	0.29
\hat{U}_{FFR}	0.55	(3.86)	0.29	0.55	(3.38)	0.29	0.35	(2.28)	0.12	0.00	(0.02)	0.00	-0.10	(-0.75)	0.01	-0.15	(-1.11)	0.02
\hat{E}_{RGDP}	0.14	(0.66)	0.02	0.00	(0.02)	0.00	-0.39	(-2.22)	0.15	-0.55	(-4.52)	0.29	-0.53	(-4.14)	0.27	-0.54	(-4.13)	0.29
\hat{U}_{RGDP}	0.57	(4.44)	0.31	0.57	(3.75)	0.31	0.34	(2.21)	0.11	-0.02	(-0.13)	0.00	-0.13	(-0.91)	0.01	-0.17	(-1.28)	0.03
\hat{E}_{UNEM}	-0.27	(-1.39)	0.07	-0.12	(-0.62)	0.01	0.30	(2.63)	0.09	0.47	(3.39)	0.22	0.50	(3.40)	0.25	0.53	(3.46)	0.27
\hat{U}_{UNEM}	0.38	(2.59)	0.14	0.37	(2.41)	0.13	0.18	(1.13)	0.03	-0.03	(-0.25)	0.00	-0.08	(-0.69)	0.00	-0.11	(-0.96)	0.01
\hat{E}_{LR}	0.25	(1.16)	0.06	0.14	(0.58)	0.01	-0.25	(-1.25)	0.06	-0.46	(-3.41)	0.20	-0.49	(-3.60)	0.23	-0.51	(-3.67)	0.25
\hat{U}_{LR}	0.06	(0.50)	0.00	0.07	(0.49)	0.00	0.07	(0.48)	0.00	0.07	(0.65)	0.00	0.02	(0.22)	0.00	0.02	(0.22)	0.00
\hat{E}_{IP}	-0.09	(-0.51)	0.00	-0.10	(-0.49)	0.01	-0.19	(-0.69)	0.03	-0.36	(-1.36)	0.13	-0.43	(-1.61)	0.18	-0.43	(-1.63)	0.18
\hat{U}_{IP}	0.41	(3.70)	0.16	0.48	(5.34)	0.23	0.53	(4.44)	0.28	0.51	(2.83)	0.25	0.45	(2.05)	0.20	0.43	(1.93)	0.19
\hat{E}_{HOUST}	-0.54	(-7.52)	0.27	-0.60	(-10.08)	0.34	-0.37	(-5.94)	0.12	0.01	(0.19)	0.00	0.11	(1.61)	0.01	0.16	(2.23)	0.02
\hat{U}_{HOUST}	-0.59	(-8.11)	0.32	-0.64	(-12.31)	0.38	-0.34	(-4.24)	0.10	0.08	(0.91)	0.00	0.18	(2.00)	0.03	0.23	(2.52)	0.05
\hat{E}_{CPI}	0.06	(0.37)	0.00	-0.08	(-0.57)	0.00	-0.38	(-3.52)	0.14	-0.45	(-3.72)	0.19	-0.42	(-3.22)	0.18	-0.43	(-3.10)	0.18
\hat{U}_{CPI}	0.25	(3.18)	0.06	0.39	(5.29)	0.15	0.63	(8.18)	0.39	0.64	(5.36)	0.41	0.64	(4.39)	0.40	0.63	(4.23)	0.39
\hat{E}_{AS}	0.25	(1.37)	0.06	0.12	(0.63)	0.01	-0.22	(-1.47)	0.04	-0.37	(-2.31)	0.14	-0.44	(-2.68)	0.19	-0.47	(-2.81)	0.22
\hat{U}_{AS}	0.41	(2.78)	0.16	0.48	(3.47)	0.23	0.43	(3.36)	0.18	0.30	(1.62)	0.08	0.27	(1.28)	0.07	0.26	(1.19)	0.06

Figure 1: Swaption Implied Volatility

The top left panel plots the average (over time series) Black's implied volatility of 1-month swaptions against the swap tenor and moneyness, whereas the top right panel plots at-the-money Black's implied volatility across the swap tenor. The bottom panels provide time series of implied volatility, across various moneyness levels, of 1-month swaptions with 2-year and 10-year swap tenors. The sample period is from June 1993 through January 2013.

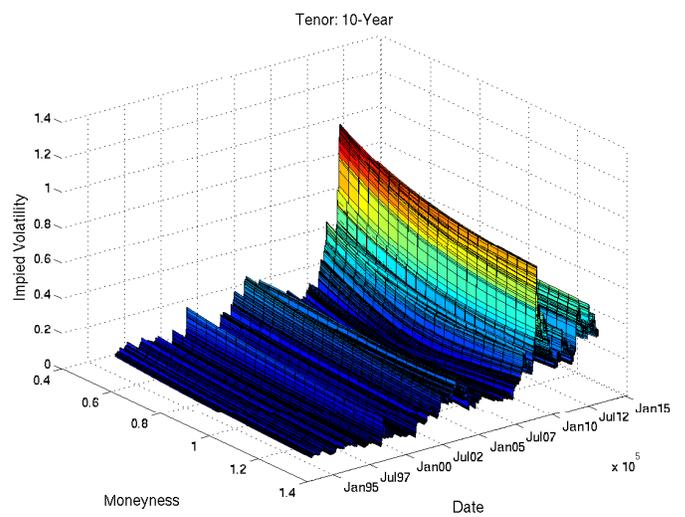
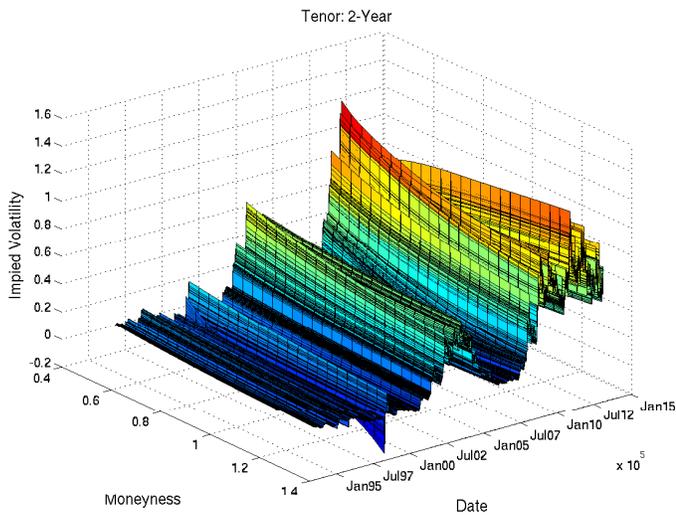
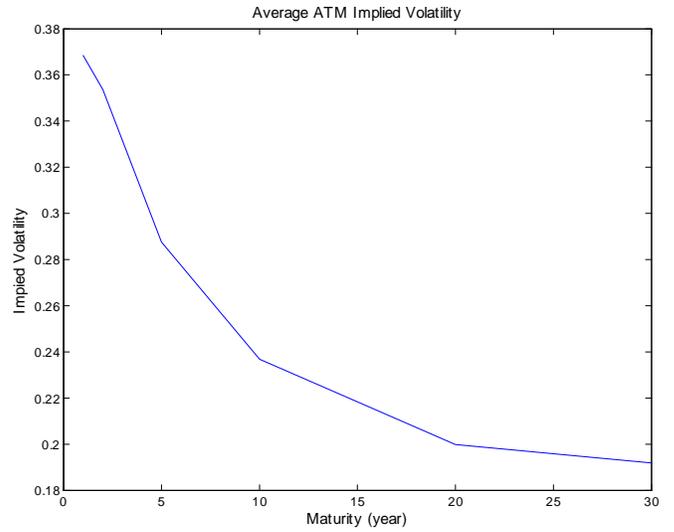
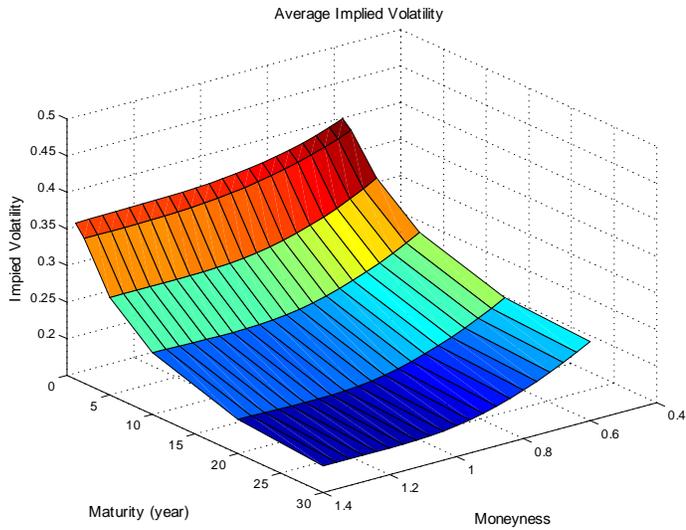


Figure 2: Time Series of TAIL(2) and TAIL(10)

This figure plots the monthly time series of TAILS at 2- and 10-year tenors, in the top and bottom panels, respectively. The sample period is June 1993–January 2013. The tail events labeled include the Fed rate hike in February 1995, the Asian financial crisis starting in November 1997, the LTCM crisis in October 1998, the dot-com bubble burst in March 2000, the 9/11 terrorist attack in September 2001, the stock market downturn starting in September 2002, the GM/Ford downgrade in May 2005, the quant crisis in August 2007, the Bear Stearns collapse in March 2008, the Lehman default in September 2008, the QE1 in November 2008, the flash crash in May 2010, the QE2 in November 2010, the Operation Twist in September 2011, and the QE 3 in September 2012.

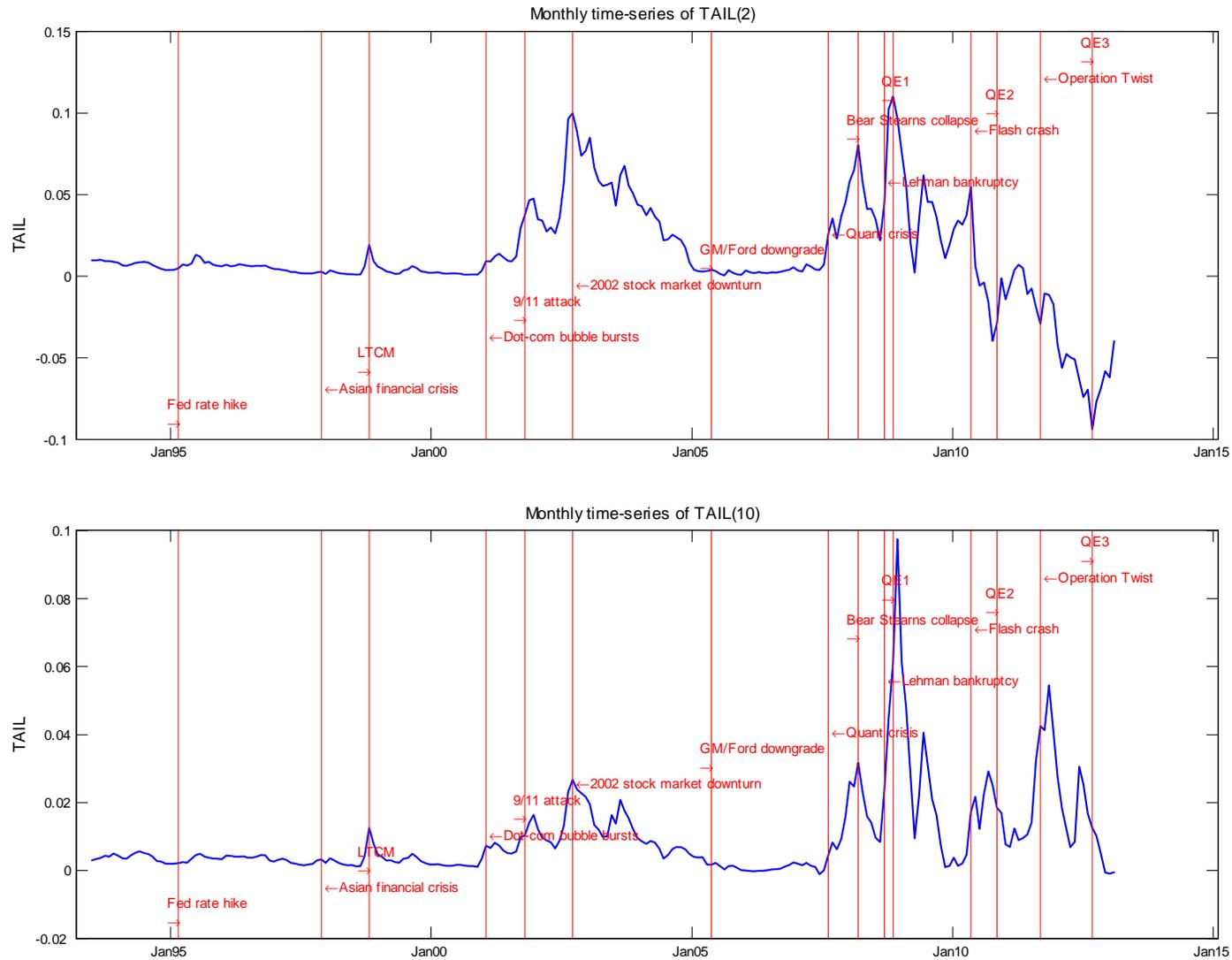


Figure 3: Time Series of TAIL

This figure plots the monthly time series of the term structure of TAIL, with 1-, 2-, 5-, 10-, 20-, and 30-year swap tenors. The sample period is from June 1993 to January 2013, with monthly time series of the TAIL with 2-year and 10-year tenors (middle panel), and slope of the TAIL term structure (bottom panel) defined as the difference between TAIL of 10- and 2-year tenors. The sample period is from June 1993 through January 2013.

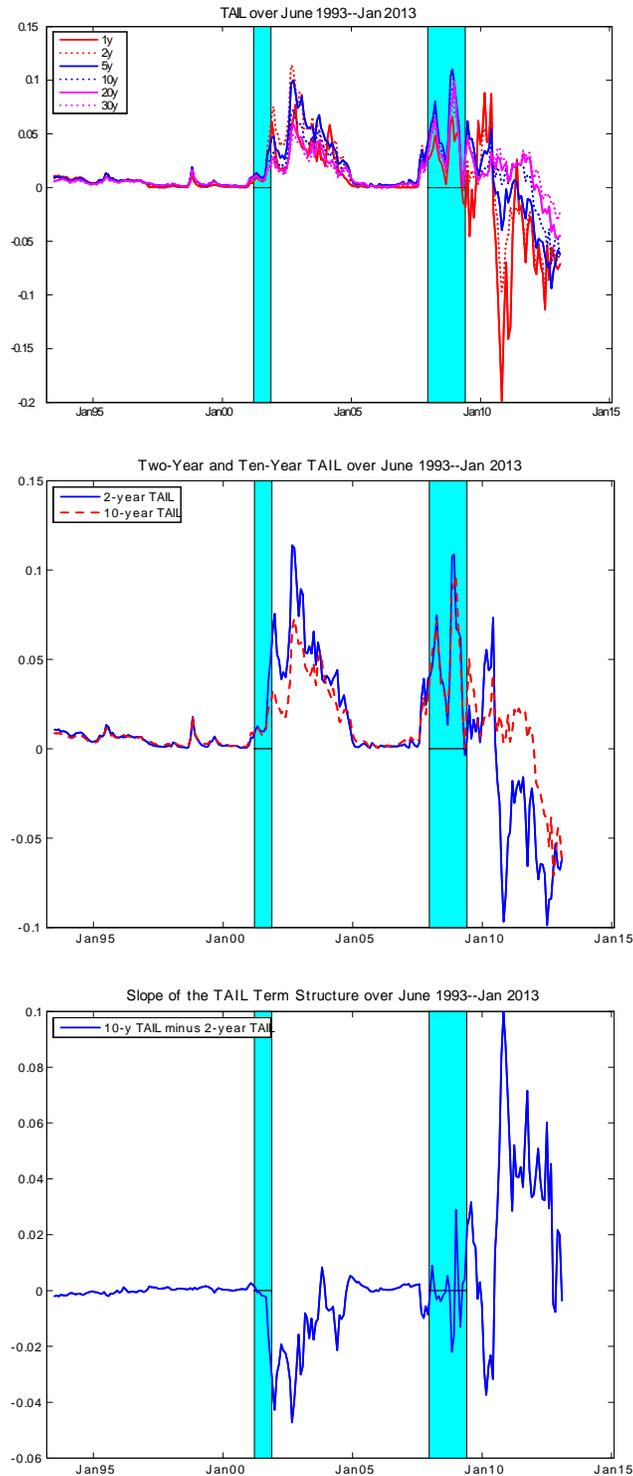
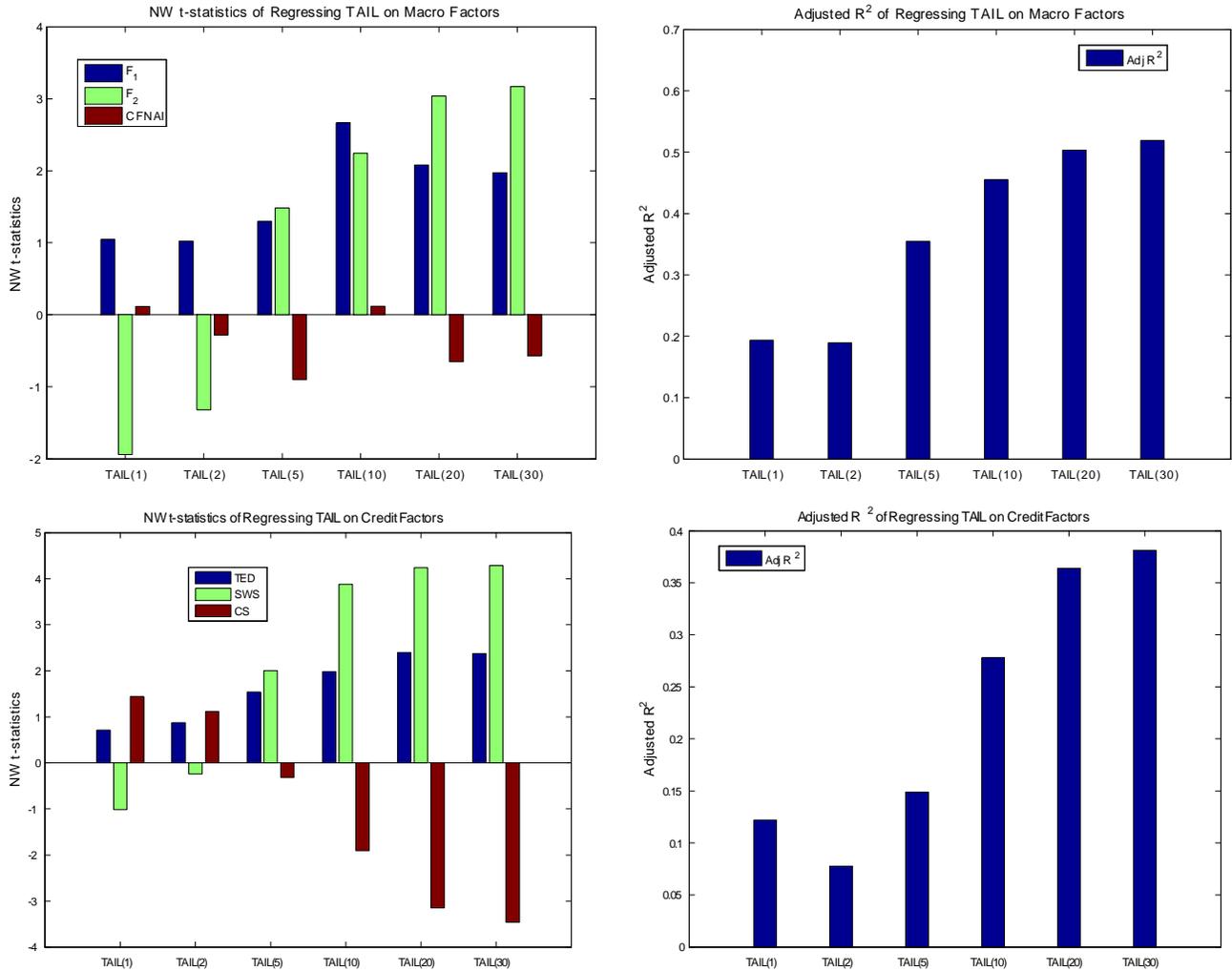


Figure 4: TAIL and Contemporaneous Economic Fundamentals

This figure reports the t-statistics (left panel) and adjusted R^2 s (right panel) of regressing TAIL (of 1-, 2-, 5-, 10-, 20-, and 30-year tenors) on contemporaneous factors of economic fundamentals. The macro factors \hat{F}_1 and \hat{F}_2 are estimated (following Ludvigson and Ng (2009, 2010)) as the first two principal components from a dataset of 104 macro variables updated until July 2012, and the $CFNAI$ is downloaded from the Chicago Fed web page. The three credit variables, TED as the difference between the three-month LIBOR and 3-month Treasury bill rate, SWS as the difference between ten-year swap rate and the corresponding Treasury yield, and CS as the difference between the Moody's AAA and BAA corporate bond yields, are computed using data from the FRED and H.15 statistical release of the Fed. The mortgage factors DUR and CV are obtained from Merrill Lynch, while the Mortgage Bankers Association (MBA) Refinancing Index REF is from J.P. Morgan. The funding liquidity factor ILQ^{FG} of Fontaine and Garcia (2011) for fixed-income market is obtained from Jean-Sebastien Fontaine's webpage, whereas the market liquidity factor ILQ^{HPW} of Hu, Pan, and Wang (2012) is from Jun Pan's webpage. The collateral-based liquidity factor ILQ^{COL} is the dollar amount of repo failures involving Treasury bonds as collateral, which we obtain from the New York Fed. Regressions are standardized, meaning all variables are de-meaned and divided by their respective standard deviation. The t-statistics based on the Newey and West (1987) standard errors. All data are monthly, and the sample period is June 1993–January 2013, with variations according to the availability of each specific fundamental factor.



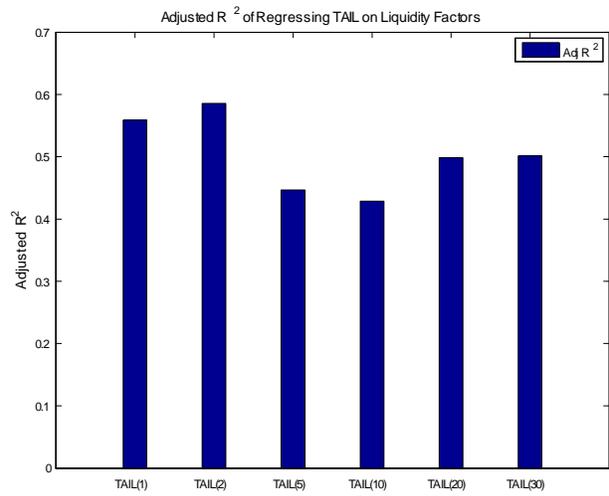
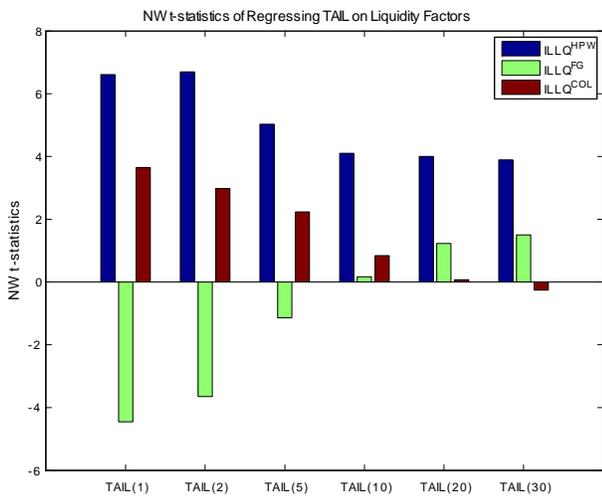
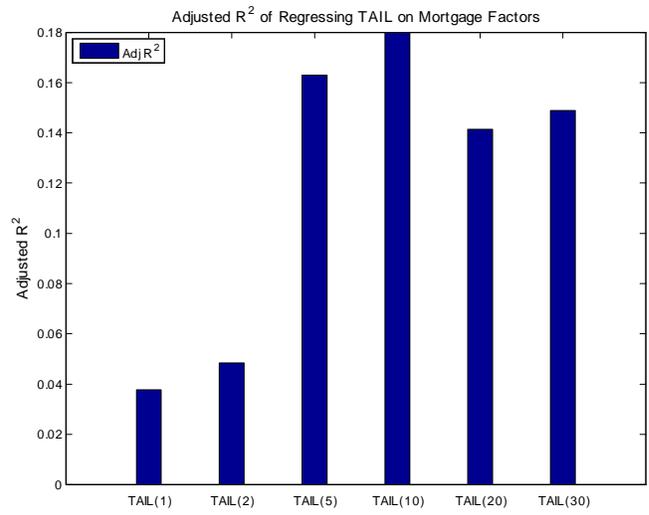
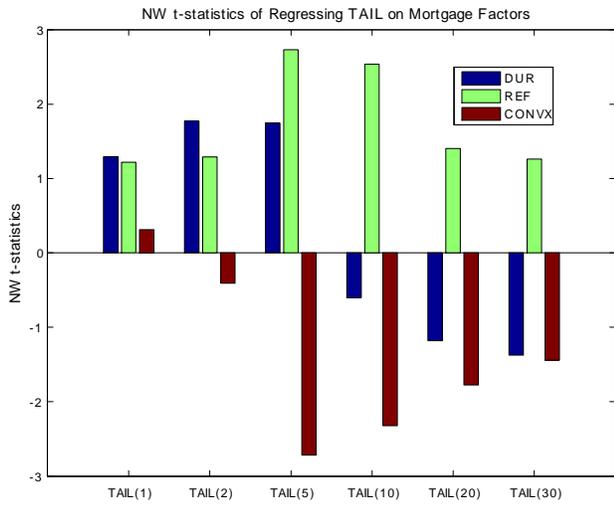


Figure 5: TAIL and Future Economic Fundamentals

This figure plots the values of the adjusted R^2 s from regressing values of fundamental factors, over 1-, 3-, 6-, 12-, and 18-month future horizons, on the current TAIL of 1-, 2-, 5-, 10-, 20-, and 30-year tenors jointly. The macro factors \hat{F}_1 and \hat{F}_2 are estimated (following Ludvigson and Ng (2009, 2010)) as the first two principal components from a dataset of 104 macro variables updated until July 2012, and the $CFNAI$ is downloaded from the Chicago Fed web page. The three credit variables, TED as the difference between the three-month LIBOR and 3-month Treasury bill rate, SWS as the difference between ten-year swap rate and the corresponding Treasury yield, and CS as the difference between the Moody's AAA and BAA corporate bond yields, are computed using data from the FRED and H.15 statistical release of the Fed. The mortgage factors DUR and CV are obtained from Merrill Lynch, while the Mortgage Bankers Association (MBA) Refinancing Index REF is from J.P. Morgan. The funding liquidity factor ILQ^{FG} of Fontaine and Garcia (2011) for fixed-income market is obtained from Jean-Sebastien Fontaine's webpage, whereas the market liquidity factor ILQ^{HPW} of Hu, Pan, and Wang (2012) is from Jun Pan's webpage. The collateral-based liquidity factor ILQ^{COL} is the dollar amount of repo failures involving Treasury bonds as collateral, which we obtain from the New York Fed. Regressions are standardized, meaning all variables are de-meaned and divided by their respective standard deviation. All data are monthly, and the sample period is June 1993–January 2013, with variations according to the availability of each specific factor.

