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CoMargin

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Abstract: We present CoMargin, a new methodology to estimate collateral requirements in derivatives central counterparties (CCPs). CoMargin depends on both the tail risk of a given market participant and its interdependence with other participants. Our approach internalizes trading externalities and enhances the stability of CCPs, thus, reducing systemic risk concerns. We assess our methodology using proprietary data from the Canadian Derivatives Clearing Corporation that include daily observations of the actual trading positions of all of its members from 2003 to 2011. We show that CoMargin outperforms existing margining systems by stabilizing the probability and minimizing the shortfall of simultaneous margin-exceeding losses.

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1. Introduction

In an effort to enhance market stability after the recent financial crisis, G20 member countries mandated the central clearing of standardized derivatives (US Department of Treasury, 2009; European Union, 2012). As a consequence, a growing fraction of the $700 trillion derivatives market, in notional terms (BIS, 2014), is now being channeled through central counterparties (CCPs). The resulting concentration of default risk in CCPs has increased their systemic importance (Acharya et al., 2009; Pirrong, 2011; Duffie, Li, and Lubke, 2010; Duffie, 2013a; Menkveld, 2013), making it necessary for regulators and market participants to assess and potentially re-evaluate the risk management practices of these institutions.[1]

Precautionary measures used to protect CCPs against default include margin (collateral) requirements, minimum capital levels, and contributions to risk mutualization arrangements, such as a default fund. In addition, CCP members are required to segregate their firm (i.e., proprietary) and client accounts (Jordan and Morgan, 1990), and enter into private insurance agreements (Jones and Pérignon, 2013). Among all of these clearinghouse safeguards, however, margin requirements are the most widely used, the most important and, arguably, the most effective.[2]

In this paper, we propose a new margining methodology, called CoMargin, which accounts for the variability and interdependence of the profits and losses (P&Ls) of clearing members. Indeed, other margining systems commonly used in CCPs, such as the Standard Portfolio Analysis

[1] In a derivatives exchange, the clearing house confirms, matches, and settles all trades. Clearing houses operate with a small number of clearing members who are allowed to submit proprietary and client trades for clearing. The process of novation allows the clearing house to become the sole counterparty to every trade cleared by its members (Bliss and Steigerwald, 2006; Pirrong, 2011). Thus, the clearing house concentrates credit risk but remains market risk neutral by matching/netting long and short positions.

[2] Aggregate margins exceed 90% of total default protection (i.e., margin + default fund + default insurance + other guarantees) for most leading CCPs (source: CCP websites). Margins in CCPs primarily consist of Treasuries (>90%) and other high-quality liquid assets. Cash accounts for less than 1% of all collected margin.

of Risk (SPAN) or Value-at-Risk (VaR) approaches, estimate collateral requirements based on a coverage level of potential losses for an individual contract or portfolio of contracts (Figlewski 1984; Kupiec 1994; Booth et al., 1997; Cotter, 2001; Day and Lewis, 2004; Chicago Mercantile Exchange (CME), 2012). However, by focusing primarily on individual exposures, these alternative systems ignore the fact that the P&Ls of clearing members are often highly interdependent, and can leave the CCP exposed to simultaneous extreme losses.

To highlight the importance of P&L interdependence, consider the role of collateral in a CCP. Margin requirements are designed to protect the CCP against losses in clearing member portfolios. However, there are times when these losses exceed the value of pledged collateral, resulting in negative margin account balances. Clearing members facing these margin "exceedances" may delay payment or even default on their obligations; thus, generating a shortfall in the market and forcing the CCP to compensate all winning counterparties using the default fund.[3] Usually, financing the shortfall of a single clearing member over a short period does not impose a hefty financial burden on the CCP. On the other hand, when two or more large clearing members experience simultaneous exceedances due to their P&L interdependence, the resources of the CCP could be eroded to the point of financial distress or even failure. While rare, CCP failures have occurred in the past and proven to be economically significant. Examples include Paris in 1973, Kuala Lumpur in 1983 and Hong Kong in 1987 (Bernanke, 1990; Knott and Mills, 2002; Duffie, 2013b).[4]

[3] The typical default waterfall system in a CPP works as follows: (1) if firm i incurs losses that it cannot cover within a predefined period, then the CCP liquidates the assets in its margin account to cover these losses. (2) If the proceeds are insufficient, the CCP liquidates firm i's contribution to the default fund. (3) Remaining losses are then mutualized by liquidating other firms' contributions to the default fund. (4) If unfunded losses remain, they can be covered with CCP equity or (5) calls for additional capped or uncapped clearing member contributions to previously unfunded default facilities (see Duffie, Li and Lubke, 2010, and Ghamami, 2014).

[4] Default of clearing members is of course, more frequent. Recent examples in the CME include Refco in 2005, Lehman in 2008, and MF Global in 2011 (see Jones and Pérignon, 2013).

P&L interdependence increases (1) with trade crowdedness and (2) underlying asset comovement. Trade crowdedness occurs when clearing members hold similar portfolio positions (i.e., portfolio weights). Hirshleifer, Subrahmanyam and Titman (1994) argue that this situation arises among large market participants with common informational advantages that lead them to pursue similar directional trades, arbitrage opportunities or hedging strategies.[5] Underlying asset comovement, on the other hand, occurs when asset returns move in the same direction. This is a common phenomenon during economic slowdowns or periods of high volatility.[6]

Given the potentially severe ramifications from two or more clearing members defaulting simultaneously, this paper formally incorporates P&L interdependence in the calculation of margin requirements. By doing so, we contribute to a global effort to design macro-prudential regulations for systemically important institutions and market infrastructures. To the best of our knowledge, CoMargin is the first margining methodology that explicitly provides "financial resources sufficient to cover a wide range of potential stress scenarios that should include, but not be limited to the default of the two participants and their affiliates that would potentially cause the largest aggregate credit exposure to the CCP" (CPSS-IOSCO, 2012).[7]

[5] Much of the proprietary trading activity on derivatives exchanges consists of arbitraging futures and OTC or cash markets (e.g. cash-futures arbitrage of the S&P 500 index, Eurodollar-interest rate swap arbitrage, etc.).

[6] Menkveld (2014) proposes a factor-based measure of trade crowdedness. By construction, however, his Margin(A) measure bundles both components of loss dependence and cannot differentiate between the portion that arises from similar portfolio positions and that which arises from increasing asset comovement. Differentiating between these two sources is critical for regulatory and risk management purposes, as the first can be linked to individual trading decisions and the second to aggregate market conditions. Gemmill (1994), for example, highlights the importance of asset comovement, noting the diversification benefit clearing houses obtain from combining contracts on low correlated assets. Extreme dependence and contagion are discussed in Longin and Solnik (2001), Bae, Karolyi and Stulz (2003), Longstaff (2004), Poon, Rockinger and Tawn (2004), Boyson, Stahel and Stulz (2010), Harris and Stahel (2011), and Christoffersen et al. (2012) among others. Billio et al. (2012) connect extreme financial equity comovements with systemic risk.

[7] The Principles of Financial Market Infrastructures have been widely advocated and are in the process of being adopted by G20 member countries. For instance, Mark Carney, Governor of the Bank of England explicitly states that "It is extremely important that CCPs organize themselves to make sure they can provide the necessary resilience plan … [to] cover the failure of one or two major institutions" (Risk Magazine, November 28, 2013).

We define CoMargin as the P&L VaR of a clearing member's portfolio, *conditional* on one or several other members being in financial distress. In our context, a clearing member is said to be in financial distress if its losses exceed the VaR of its P&L distribution. In this light, CoMargin builds on existing margin methods and reduces clearinghouse risk. We show that this intuitive adjustment to existing margining systems is sufficient to capture the interdependence of clearing member P&Ls. CoMargin exhibits a wide range of desirable features, including systematically increasing with P&L dependence, stabilizing the probability of exceedance events given financial distress, and reducing the probability that the clearing house might exhaust its funds.

Moreover, CoMargin is simple to estimate. The process starts by taking the trading positions of all clearing members at the end of the trading day as given. Then, in the spirit of stress testing, a series of one-day-ahead scenarios, based on projected price and volatility changes in the underlying assets, are used to estimate hypothetical P&Ls; that is, potential changes in the value of member portfolios. Using these hypothetical P&Ls we compute margin requirements that target a pre-specified probability of margin exceedances conditional on the financial distress of other members.[8]

Through Monte Carlo simulations, we first show that CoMargin indeed enhances the stability of the CCP by adjusting collateral allocations such that the probability of margin exceedances, conditional on the financial distress of other firms, remains constant. Our simulations also show that CoMargin improves the resilience of the CCP relative to other methods by

[8] CoMargin adopts a conditional VaR setting similar to that in Adrian and Brunnermeier's (2014) CoVaR. However, it is important to note that the objective and estimation of these methodologies are radically different. CoMargin is a methodology for computing robust margin requirements. Its estimation is based on a simulation or stress testing approach. CoVaR, on the other hand, is a methodology to assess the contribution and exposure of financial institutions to market-wide tail risks and its estimation is based on quantile regressions.

minimizing the average shortfall when simultaneous exceedances occur. Most importantly, CoMargin's stability and resiliency are robust to a variety of distributional assumptions.

In addition to simulations, we also employ actual trader position data from the Canadian Derivatives Clearing Corporation (CDCC) to empirically assess the performance of CoMargin relative to existing margining systems.[9] Our dataset, the first of its kind to be used in an academic study, includes actual daily positions for all forty-eight CDCC members from 2003 to 2011. Each trading day, we observe the number of long and short closing positions held in the firm, client, and omnibus account of each member for the three most actively traded TMX Montreal derivatives contracts; namely, the three-month Canadian Bankers' Acceptance Futures, the ten-year Government of Canada Bond Futures, and the S&P/TSX 60 Index Standard Futures.

Confirming the results from our simulations, this real world data shows that CoMargin leads to fewer exceedances and lower expected shortfalls than SPAN and VaR margins. Moreover, CoMargin exceedances are less clustered in time and are more likely to emanate from small, less active clearing members--those less likely to pose a systemic threat to the clearinghouse. This real world data shows that the relative performance of CoMargin improves when when the level of P&L dependence across market participants increases, as was the case during the recent financial crisis. Therefore, our evidence demonstrably shows that CoMargin provides more protection to CCPs when needed the most.[10]

This paper makes two important contributions to the literature. Our first contribution is methodological. We develop a novel margining system that internalizes market interdependencies

[9] The CDCC is the CCP of the TMX Montreal Exchange.

[10] These results are robust to equalizing aggregate margin collections across CoMargin, VaR and SPAN. When the P&Ls of clearing members are independent, CoMargin collects the same collateral as existing margining systems. When P&Ls are not independent, the additional CoMargin collected depends on the coverage level selected by the CCP. In these cases, it is possible to select CoMargin coverage rates to the *same* aggregate margin as SPAN. We compare systems using this "budget neutral" approach below.

and enhances the stability and resiliency of CCPs. Our approach aims at addressing current concerns associated with CCPs, particularly in terms of systemic risk, and offers a general framework to think about collateral allocation in a broad range of financial markets. Our second contribution is empirical. We conduct an extensive analysis of the margining systems widely used in CCPs. To the best of our knowledge, this is the first study that uses actual trading positions of clearing members. Our proprietary data set allows us to conduct our analysis with actual trader positions and without the need to simulate or assume the trading behavior of market participants.

Our paper complements the nascent literature on derivatives clearing. Duffie and Zhu (2011) show that the optimal number of CCPs needed to minimize counterparty default exposure is one. Duffie, Scheicher and Vuillemey (2014) provide an empirical analysis of the collateral requirements induced by broader application of central clearing. Ghamami (2014) theoretically studies the default waterfall resources of a CCP and derives optimal levels for the capital of clearing members and the CCP. Menkveld (2013) develops a model for centralized clearing focused on systemic liquidation risk due to the default of market participants.[11] Loon and Zhong (2014) empirically show that centrally clearing credit default swap (CDS) contracts reduces counterparty risk, hence increasing the value of credit protection and improving liquidity (see also Slive, Witmer and Woodman, 2012, Arora, Gandhi and Longstaff, 2012, and Bernstein, Hughson and Weidenmier, 2014). Following Gârleanu and Pedersen (2011), Hedegaard (2013) tests the effect of margins on futures prices using data on 597 margin changes on CME futures contracts. More directly relevant to our study is Jones and Pérignon (2013) who show that the most severe losses

[11] Other theoretical studies on derivatives clearing include Biais, Heider and Hoerova (2012, 2014), Koeppl, Monnet and Temzelides (2012), Fontaine, Pérez Saiz and Slive (2013), and Acharya and Bisin (2014).

of the largest clearing members of the CME tend to occur on the same days. These events are precisely what our CoMargin framework aims to protect against by internalizing P&L dependence.

In a similar vein, Menkveld (2014) proposes an alternative margining system called Margin(A) that accounts for similarities in trading positions across clearing members. Following Duffie and Zhu (2011), he considers the aggregate exposure of the CCP as the sum of all potential losses in clearing members' portfolios. He then derives the total amount of margin that the CCP should collect from all of its members under the assumption that losses are normally distributed and assigns margin requirements for each member using an allocation rule (see Brunnermeier and Cheridito, 2014). Margin(A), unlike CoMargin, does not reflect tail dependencies in the P&Ls of clearing members that often occur with derivatives.

CoMargin has other significant practical and theoretical advantages over Margin(A). First, because it is scenario-based, CoMargin is robust to a wide variety of distributions and loss dependence structures, while Margin(A) crucially relies on the normality assumption of clearing member losses and therefore is constrained to linear dependence. CoMargin's ability to capture different dependence structures makes it appropriate and practical for many derivative products with non-linear payoffs, including CDS and options. This is a crucial advantage, given the push across the globe in the wake of the recent financial crisis to require central clearing of these products. Second, by construction, CoMargin allows risk managers to target an actual coverage probability that can be backtested according to regulatory principles, something that cannot be properly targeted in Margin(A).

The remainder of the paper is organized as follows. In Section 2, we describe how margin requirements are currently estimated under the SPAN and VaR margining systems. In Section 3, we

present the theoretical foundations of the CoMargin system and highlight some of its properties with simulations. We discuss the implementation of CoMargin in Section 4. In Section 5, we demonstrate its empirical effectiveness using data from the Canadian Derivatives Clearing Corporation and provide concluding remarks in Section 6.

2. Standard Margining Systems

2.1. Derivatives Market

Consider a derivatives exchange with N clearing members and D derivatives securities (futures, options, CDS, etc.) written on U underlying assets. In practice, each clearing member usually has two accounts with the CCP: one to manage client trades, called a client account, and one to manage proprietary trades, called a firm account. We turn our focus to firm accounts as they are the most important source of risk within the CCP (Jones and Pérignon, 2013). Nevertheless, our methodology can be generalized to include all accounts.

Let $w_{d,i,t}$ be the number of d contracts in the derivatives portfolio of the firm account of clearing member i, for $d = 1, \dots, D$ and $i = 1, \dots, N$, at the end of day t, such that:

$$w_{i,t} = \begin{bmatrix} w_{1,i,t} \\ \vdots \\ w_{D,i,t} \end{bmatrix} \tag{1}$$

Initial margins are collected every day for each clearing member's account to guarantee the performance of their obligations and to guard the clearing house against default over a period of one day. Unlike margins collected for client accounts, those for firm accounts are computed at the portfolio level and allow for netting across positions. Let $B_{i,t}$ be the initial firm margin collected by

the CCP from clearing member i at the end of day t. This margin is a function of the outstanding firm trading positions $w_{i,t}$ of member i.

We assume that current exposures are settled daily. Thus, a variation margin amount, $V_{i,t}$, equivalent to the profits or losses in the portfolio of clearing member i on day t, is deposited or withdrawn, respectively, from its margin account at the end of the trading day. By construction, the sum of the variation margins across all accounts (*N* firm accounts plus *N* client accounts = 2*N* accounts), is equal to zero as for every buyer there is a seller. However, the sum of the variation margins across the *N* firm accounts can be different from zero.

Our interest focuses on cases when trading losses exceed margin requirements; that is, when $V_{i,t} \leq -B_{i,t-1}$. In these cases, we say that firm i has experienced an exceedance. Identifying firms in this state is important because they have an incentive to default on their positions or to delay payment on their obligations, which generates a shortfall in the market that needs to be covered by the CCP. Given the limited funds available to the CCP, simultaneous exceedance events can threaten its stability and survival.

2.2. SPAN Margin

The CME, the world's largest derivatives exchange, introduced the SPAN margining methodology in 1988. It has since become the most widely used margining system in derivatives exchanges around the world. Every day following the market close, clearing houses such as the CDCC, the CME, Eurex, LCH.Clearnet, Nymex and the Options Clearing Corporation (OCC), among others, use the SPAN system to determine the margin requirements of their members.

SPAN is a scenario-based methodology that is used to assess potential changes in the value of the derivatives held by each clearing member. However, SPAN does not take a portfolio-wide approach. Instead, it divides each portfolio into contract families, defined as groups of contracts that share the same underlying asset, and estimates a charge for each family independently. Thus, for a portfolio with $d \leq D$ derivatives written on $u \leq U$ underlying assets, the SPAN system computes initial margin requirements for each of the u contract families.

To compute margin requirements for a family of derivatives, the SPAN system simulates one-day-ahead changes in the value of each contract by using sixteen scenarios that vary the price and the volatility of the underlying asset, as well as the time to expiration of the contract (see Table 1). The range of the potential price changes of the underlying asset usually covers 99% of its daily price movements over a historical calibration window. A similar approach is adopted for the volatility. Extreme price changes are used to assess potential changes in deep out of the money options. The scenario analysis yields a risk array for each contract that contains sixteen one-day-ahead potential value changes (i.e., each maturity and each strike price has its own array).[12] The scenario with the worst potential loss for the entire contract family is identified and that loss becomes the first part of the contract family charge.

The second part of the contract family charge consists of a discretionary adjustment that is needed because contracts with different expiration months are assumed to be equivalent in the scenario analysis. In other words, long and short positions written on the same underlying asset but with different expiration months offset each other. Therefore, risk managers are required to add an *intra-commodity spread charge* to the worst case scenario loss to account for time-spread

[12] The projected price changes of non-linear contracts, such as options, are obtained by using numerical valuation methods or option pricing models.

trading. A third component of the contract family charge consists of another discretionary adjustment to account for commodity-spread trading (i.e., simultaneous long and short positions in contracts with the same expiration months but written on different but correlated underlying assets). These adjustments are known as *inter-commodity spread charges.*

It is important to note that both intra- and inter-commodity spread charges involve the discretion of risk managers. Thus, these adjustments are rarely consistent across commodities, market conditions or clearing houses. This situation, coupled with the fact that the SPAN system targets underlying price and volatility ranges, instead of the probability of portfolio-wide margin-exceeding losses, make the SPAN system inconsistent (in terms of its P&L coverage) across time and markets.

2.3. VaR Margin

VaR is defined as a lower quantile of a P&L distribution. It is the standard measure used to assess the aggregate risk exposure of banks (Berkowitz and O'Brien, 2002; Berkowitz, Christoffersen and Pelletier, 2011), as well as their regulatory capital requirements (Adrian and Shin, 2014). VaR can also be used to set margins on a derivatives exchange. In this case, the margin requirement corresponds to a given quantile of a clearing member's one-day-ahead P&L distribution.

Definition 1: The VaR margin of firm i, B_i, corresponds to the α quantile of its P&L distribution:

$$\Pr\left(V_{i,t+1} \leq -B_{i,t}\right) = \alpha \qquad (2)$$

The coverage level, determined by α, depends on the CCP's risk aversion and additional available financial resources (e.g. default fund) that could be used if a member defaults.

Like the SPAN system, the VaR margin method is applied on a firm-by-firm basis using a scenario analysis. However, the scenarios are applied to the entire portfolio. More specifically, we consider S scenarios derived from simulated one-day-ahead changes in the value of the price and the volatility of the underlying assets and use them to evaluate each clearing member's entire portfolio. The hypothetical P&L or variation margin of each clearing member is computed by *marking-to-model* its positions in each scenario. Thus, for each clearing member and date t, we obtain a simulated sample of daily P&Ls denoted $\left\{v_{i,t+1}^{s}\right\}_{s=1}^{S}$ that can be used to estimate the VaR margin requirement as follows:

$$\widehat{B}_{i,t} = -quantile\left(\left\{v_{i,t+1}^{s}\right\}_{s=1}^{S}, \alpha\right) \tag{3}$$

3. CoMargin

The VaR and SPAN collateral systems only focus on firm specific risk; that is, the unconditional probability of an individual clearing member experiencing a margin-exceeding loss. By adopting either method, the clearing house guards itself from unique or independent exceedances, but it leaves itself exposed to simultaneous exceedance events across firms. These events, however, tend to be more economically significant because they place a more substantial burden on the resources of the clearing house.

3.1. Methodology

Consider the firm accounts of two of the *N* firms, denoted firms i and j. The probability of simultaneous exceedances is given by:

$$\Pr\left[\left(V_{i,t+1} \leq -B_{i,t}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right] \tag{4}$$

$$= \Pr\left(V_{i,t+1} \leq -B_{i,t} | V_{j,t+1} \leq -B_{j,t}\right) \times \Pr\left(V_{j,t+1} \leq -B_{j,t}\right)$$

Equation 4 shows that simultaneous exceedance events tend to happen more frequently not only when firm specific risk increases (i.e., when $\Pr\left(V_{j,t+1} \leq -B_{j,t}\right)$ increases), but also when P&L dependence increases (i.e., when $\Pr\left(V_{i,t+1} \leq -B_{i,t} | V_{j,t+1} \leq -B_{j,t}\right)$ increases). In the first case, firms are more likely to experience losses that exceed their collateral levels in all states of the world. In the second case, firms are more likely to experience these losses at the same time as other firms, either because they hold similar positions (i.e., trade crowdedness is high) or because underlying assets have a tendency to move together (i.e., underlying asset comovement is high). However, VaR and SPAN margins completely disregard P&L dependence and its potential effect on the stability of the CCP. In the case of the VaR system, risk managers only target unconditional exceedance probabilities by setting a coverage level $1 - \alpha$ for each clearing member individually. In the case of the SPAN system, risk managers do not have direct control over the unconditional exceedance probabilities, so the clearing house is potentially left even more vulnerable to simultaneous exceedance events.

Now, consider a hypothetical situation in which two of the *N* firms have orthogonal risk exposures and their exceedances are independent.[13] Under the VaR system this means:

$$\Pr\left(V_{i,t+1} \leq -B_{i,t} | V_{j,t+1} \leq -B_{j,t}\right) = \Pr\left(V_{i,t+1} \leq -B_{i,t}\right) = \alpha \tag{5}$$

and

$$\Pr\left[\left(V_{i,t+1} \leq -B_{i,t}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right] = \alpha^2 \tag{6}$$

[13] Obviously, *all 2N accounts* (i.e., all firm and client accounts) cannot exhibit orthogonal risk exposures as it would violate the zero-sum property for variation margins.

Equation 5 has an important implication for market stability, which we define as constancy in the probability of exceedance events for i, regardless of the situation of j. Moreover, equation 6 shows that, once the risk manager selects the target α, the probabilities of simultaneous exceedance events are also fixed (i.e., α^2 for two events, α^3 for three events and so on). This property holds true regardless of the collateral system being adopted by the clearing house. Therefore, a setting with fully orthogonal exposures provides a conceptual benchmark to assess the stability of a margining system.

With this in mind, we propose a new collateral system, called CoMargin, which enhances financial stability by taking into account the dependence of clearing members' P&Ls.

Definition 2: The CoMargin of firm i, $B_t^{i|j\neq i}$, corresponds to the α quantile of its P&L distribution conditional on an event affecting a set of firms that exclude firm i:

$$\Pr\left(V_{i,t+1} \leq -B_t^{i|j\neq i} | \mathbf{C}\left(V_{j\neq i,t+1}\right)\right) = \alpha \quad (7)$$

Notice that CoMargin requires us to define an event for a conditioning set of firms that exclude firm i. We define this event as the financial distress (i.e., a loss in excess of $\alpha\%$ VaR) of at least one of the firms in the conditioning set.

In the most general case, the conditioning set of firms could include all of the remaining *N-1* firms in the CCP, excluding firm i, such that: [14]

[14] The conditioning event presented in equation 8 is different from that used by Adrian and Brunnermeier (2014) to define CoVaR. They consider either the distress of a single institution or the market. In contrast, our conditioning event is multidimensional and requires that at least one firm in the conditioning set is in financial distress. Furthermore, similar to Girardi and Ergun (2013), we use a less-than-or-equal sign in the conditioning event. This reflects the fact that we define financial distress as a range, as opposed to a point, in the P&L distribution. Moreover, the conditioning event in equation 8 could be based on CoMargin instead of VaR, $B_{j,t}$. In this case, all CoMargins would be jointly defined by solving a system of equations.

$$\mathbf{C}\left(V_{j\neq i,t+1}\right) = \bigcup_{j=1,j\neq i}^{N} \left(V_{j,t+1} \leq -B_{j,t}\right) \tag{8}$$

Alternatively, we can narrow the number of firms contained in the conditioning set based on their size or risk. For example, the Principles for Financial Market Infrastructures (CPSS-IOSCO, 2012) suggest selecting the two firms with the largest expected shortfall. If we denote these firms by F1 and F2, this means that the CoMargins of all other firms depend on F1 and F2. However, as shown in equation 7, the CoMargin of F1 does not depend on F1, but instead on F2 and F3, the firm with the third largest expected shortfall. As a result, being a conditioning firm does not increase margin.

A nice feature of CoMargin is that it achieves exceedance probabilities that are consistent with those of a fully orthogonal exposure case. For instance, for two of the N firms, i and j, we know through Bayes' theorem that:

$$\Pr\left(V_{i,t+1} \leq -B_t^{i|j} | V_{j,t+1} \leq -B_{j,t}\right) = \frac{\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right]}{\Pr\left(V_{j,t+1} \leq -B_{j,t}\right)} \tag{9}$$

where $B_t^{i|j}$ denotes the CoMargin of firm i, conditional on the financial distress of firm j. Thus, the numerator represents the joint probability of i exceeding its CoMargin and j experiencing financial distress. From definitions 1 and 2, we can see that:

$$\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right] = \alpha^2 \tag{10}$$

This alternative expression shows that with CoMargin, the probability of a firm exceeding its margin when another firm is in financial distress corresponds to that obtained with fully orthogonal exposures (α^2). Put simply, CoMargin "orthogonalizes" the risk exposures of the CCP; thus,

enhancing its stability.[15] Furthermore, comparing equations 6 and 10 shows that when the exposures are orthogonal, CoMargin and VaR margin requirements are equivalent. Thus, any differences between the collateral requirements of these two systems are attributed to P&L dependence.

Lastly, notice that there are other techniques that we could have used to capture P&L dependence in a margining system. These range from simple methods, such as linear correlation or quadrant dependence, to more sophisticated extreme dependence measures discussed in Poon, Rockinger and Tawn (2004), Bae, Karolyi and Stulz (2003) and Christoffersen et al. (2013) or systemic risk measures surveyed in Bisias et al. (2012). However, in our case, dependence measures need to be transformed to dollar values to compute margins.[16] The quantile-based dependence measure embedded in CoMargin offers the distinctive advantage of directly corresponding to the dollar value that the CCP should collect as margin.

3.2. Monte Carlo Simulations

In order to illustrate the performance of the CoMargin system, we consider the case of four clearing members, where two of them, members 1 and 2, have correlated P&Ls, such that $V \sim N(0, \Sigma)$ where:

$$V = (V_1, V_2, V_3, V_4)' \text{ and } \Sigma = \begin{pmatrix} 1 & \boldsymbol{\rho} & 0 & 0 \\ \boldsymbol{\rho} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

[15] This property also holds when the number of firms is greater than two.

[16] For instance, one could compute the SES/MES proposed by Acharya et al. (2010), the SRISK of Brownlees and Engle (2012), or any other dependence measure for all clearing members. Then, a bucketing procedure or another transformation would be required to translate risk estimates into dollar-values for margin requirements.

We allow the correlation between the P&Ls of firms 1 and 2, ρ, to increase from 0 to 0.8. As explained above, the rising correlation between the P&Ls of these firms can reflect an increase in the similarity of their firm trading positions or an increase in the comovement of the underlying assets. Panel A of Figure 1 shows the margin requirements for each clearing member under both the VaR and CoMargin systems for the levels of correlation being considered. To estimate the CoMargin of a given clearing member, we define the conditioning event as at least one of the other three firms being in financial distress.

Panel A of Figure 1 shows that VaR margin remains constant for all firms regardless of their correlation level because this method does not take into account P&L dependence. Consistent with the explanation in the previous section, CoMargin and VaR margin are equivalent when $\rho = 0$; that is, when there is no P&L dependence. However, notice that CoMargin is greater than VaR margin when $\rho > 0$; that is, when there is P&L dependence. In addition, CoMargin increases as ρ increases; that is, as P&L dependence increases.

It is important to note that there are two possible reasons why CoMargin could be more effective than VaR margin at reducing conditional margin exceedances. The first reason is that there is an "allocation effect"; that is, CoMargin provides a better allocation of collateral. An alternative reason is that there is a "level effect". This means that CoMargin simply collects more aggregate collateral and, as a consequence, has a higher coverage level. We show that allocations and not aggregate margin collections drive the relative effectiveness of CoMargin by reporting a Budget-Neutral VaR (BNVaR) margin. This margining scheme collects the same amount of aggregate collateral as the CoMargin system but it does so by distributing additional margin requirements evenly across all clearing members (rather than as a function of their P&L

dependence).[17] Thus, this process effectively cancels out the level effect and allows us to assess the merits of each margining system based on their allocations.

The BNVaR margin of firm i at time t, $B_{i,t}^{0}$, is defined as:

$$B_{i,t}^{0} = B_{i,t} + \frac{\sum_{i=1}^{N} B_{t}^{i|j\neq i} - \sum_{i=1}^{N} B_{i,t}}{N} \tag{11}$$

Panel B of Figure 1 shows the theoretical performance of the VaR, BNVaR and CoMargin systems at the clearing member level. The horizontal line in these charts highlights the values prevalent when $\rho = 0$; that is, when the risk exposures of members are orthogonal. Panel B shows the probability of a given clearing member exceeding its margin conditional on at least one other clearing member being in financial distress. When $\rho = 0$, all three margining systems provide the same level of coverage. However, as ρ increases, the VaR and BNVaR margins provide less coverage when at least one clearing member is in financial distress. On the other hand, only CoMargin keeps the coverage level constant.

Table 2 reports the theoretical performance of the different margining systems at the CCP level. The table reports the unconditional probability of having at least one exceedance, the probability of having additional exceedances given that one has occurred, and the expected shortfalls associated with these events. Panel C of Figure 1 extends these last set of results to up to four exceedance events. Our findings show once again that when $\rho = 0$, all three margining systems provide the same coverage to the CCP, but as ρ increases, CoMargin provides the best overall coverage.

[17] An alternative budget-neutral margin scheme would be to adjust the α parameter for CoMargin such that its aggregate margin corresponds to that under an alternative system (i.e., VaR or SPAN).

The unconditional probabilities in Table 2 suggest that BNVaR margin provides the best unconditional coverage as correlation increases. Nevertheless, this result is expected. In our example all four firms are identical except for their correlation level. Since BNVaR collects more aggregate funds than VaR margin and it does so evenly across all clearing members, it is equivalent to a VaR margin with a lower α. This higher coverage level embedded in BNVaR reduces the unconditional probability of individual margin exceedances. However, as shown in Table 2, this does not reduce the conditional probability of experiencing additional (i.e. simultaneous) exceedance events, particularly as P&L dependence increases.

Lastly, Panel C of Figure 1 shows the shortfall that the CCP is expected to cover given a minimum number of margin exceedances. Notice that both CoMargin and BNVaR margin provide similar results that outperform VaR for $\rho > 0$.

We repeat the previous exercise but for P&Ls that are jointly Student t distributed with degrees of freedom υ, $V \sim t_{\upsilon}(0, \Sigma)$ and present the results of this exercise in Table 2. Changing the distributional assumption of the previous exercise from a normal to a Student t multivariate distribution allows us to simulate scenarios where all clearing members have some level of tail dependence in their P&Ls, which is consistent with empirical evidence. The variance-covariance structure, Σ, is the same as that considered in the Gaussian distribution example, however, in this case, we set $\rho = 0.4$ and let the degrees of freedom decrease from 30 to 5.[18] Thus, the resulting P&L distributions have progressively fatter tails.

As shown in Table 2 these results are consistent with those obtained under the Gaussian assumption, but they highlight an important finding: CoMargin is able to capture P&L dependence

[18] We conducted a similar experiment using $\rho = 0$ which leads to the same conclusions. The results are available upon request by contacting the authors.

structures that go beyond simple correlations. Recall that the entire P&L dependence structure is fully characterized by correlation only if P&Ls are normally distributed. However, asset prices and P&Ls, particularly those of non-linear portfolios, rarely follow normal distributions, making CoMargin uniquely capable of incorporating this fact.

4. Implementing CoMargin

4.1. Scenario Generation

One common feature of all margining methods is that they are scenario based. As a consequence, generating meaningful scenarios is a crucial stage when setting margin requirements. Unlike the SPAN margining system, VaR margin and CoMargin use a portfolio-wide approach. This allows us to take into account the asset comovement within the portfolio of each clearing member without the need for ad-hoc netting adjustments (i.e., SPAN's inter- and intra-commodity spreads explained above). In order to assess the potential P&L of the entire portfolio of each clearing member, we need to jointly simulate *S* vectors of one-day-ahead changes in the underlying asset prices, $\{r_{u,t+1}^{s}\}_{s=1}^{S}$. While there are different ways to estimate multivariate probability distribution functions, we suggest using a copula to link the marginal probability distribution functions, say $F_1(r_1)$, $F_2(r_2)$,…, $F_U(r_U)$, to form a multivariate probability distribution function, $F(r_1, r_2, \dots, r_U)$, where r_x is the standardized return of underlying asset x and U is the number of assets underlying the derivatives cleared by the CCP.

Using copulas to model the multivariate structure of underlying asset returns has advantages in this context. First, marginal distributions do not need to be similar to each other to be linked together with a copula structure. Second, the choice of the copula or multivariate structure is not constrained by the choice of the marginal distributions. Third, copulas can be used with U marginal

distributions to cover all of the underlying assets cleared by the CCP (see Oh and Patton, 2012). Lastly, the use of copulas allows us to model the tails of the marginal distributions and the tail dependence across underlying assets separately. This feature is particularly important in our case because the likelihood of an extreme underlying asset return might increase either because of fatter tails in their marginal distributions or because of fatter tails in their multivariate distribution. We use Student t copulas in our modeling because, unlike their Gaussian counterparts, they more closely resemble some of the stylized features of asset returns, such as fat tails in the marginal distributions and multivariate tail dependence.[19]

We implement a two-stage semi-parametric approach to estimate a U-dimensional copula for the underlying asset returns. The first stage consists of estimating the empirical marginal distributions of the returns of each underlying asset. The second stage consists of estimating the t-copula parameters, R (correlation matrix) and υ (degrees of freedom), through maximum likelihood. This approach is commonly known as the canonical maximum likelihood estimation method (Genest, Ghoudi and Rivest, 1995). Once the copula parameters are estimated, we use the implied multivariate structure to simulate potential changes in the price of the underlying assets.[20]

We use a fixed-length estimation window that is rolled daily to simulate new scenarios every day. On each day, once the S potential changes in the price of the underlying assets have been simulated, we mark-to-model all of the derivatives in the portfolio of each clearing member. We then use this simulated sample $\left\{v_{i,t+1}^{s}\right\}_{s=1}^{S}$ to estimate VaR margin as described in Section 2.3. To

[19] A Student t copula corresponds to the dependence structure implied by a multivariate Student t distribution. It is fully characterized by the variance-covariance matrix of standardized returns and the degrees of freedom, υ. The degrees of freedom define the probability mass assigned to simultaneous extreme returns (both positive and negative); the lower the degrees of freedom, the higher the probability of experiencing simultaneous extreme returns relative to the Gaussian copula. However, as $\upsilon \rightarrow \infty$ the Student t copula converges to its Gaussian counterpart.

[20] These simulations can also be obtained for different correlation matrices. For instance, a pre-defined range can be obtained from 95% confidence intervals used to forecast R through the Dynamic Conditional Correlation method proposed by Engle (2002).

estimate CoMargin, we use the simulated samples for all clearing members, $\left\{v_{1,t+1}^{s}, v_{2,t+1}^{s}, \ldots, v_{n,t+1}^{s}\right\}_{s=1}^{S}$, and follow the procedure described in the following section.

4.2. Estimation

In this section, we show how to estimate CoMargin requirements from simulated P&Ls for the case of two clearing members. The Appendix explains the estimation procedure for the more general case when N>2.

Given the simulated sample $\left\{v_{i,t+1}^{s}, v_{j,t+1}^{s}\right\}_{s=1}^{S}$, conditional on $B_t^{i|j}$, a simple estimate of the joint probability $\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right]$, denoted $P_t^{i,j}$, is given by:

$$\hat{P}_t^{i,j} = \frac{1}{S}\sum_{s=1}^{S} \mathbf{I}\left(v_{i,t+1}^{s} \leq -B_t^{i|j}\right) \times \mathbf{I}\left(v_{j,t+1}^{s} \leq -B_{j,t}\right) \tag{12}$$

where $v_{i,t+1}^{s}$ and $v_{j,t+1}^{s}$ correspond to the s^{th} simulated P&L of firms i and j, respectively. Given this result, we can now estimate $B_t^{i|j}$. For each time t and for each firm i, we look for the value $B_t^{i|j}$, such that the distance $\hat{P}_t^{i,j} - \alpha^2$ is minimized:

$$\hat{B}_t^{i|j} = \arg\min_{\left\{B_t^{i|j}\right\}} \left(\hat{P}_t^{i,j} - \alpha^2\right)^2 \tag{13}$$

Thus, for each firm i, we end up with a time series of CoMargin requirements $\left\{\hat{B}_t^{i|j}\right\}_{t=1}^{T}$ for which confidence bounds can be bootstrapped.

4.3. Backtesting

One way to assess the effectiveness of a margining system is by backtesting it (Hurlin and Pérignon, 2012). Backtesting aims to identify misspecified models that lead to either excessive or

insufficient coverage for the CCP relative to a target. Therefore, if a margining system cannot be backtested using formal statistical methods, we cannot identify its potential shortcomings and fine tune it to meet its objectives.

VaR margins can be easily backtested because they are defined by the quantile of a P&L distribution. As it is the case with VaR margin, CoMargin allows us to test the null hypothesis of an individual member exceeding its margin requirement. More importantly, however, is the fact that we can also test the probability of exceedances conditional on the financial distress of other firms, as defined by the CoMargin of firm i, $B_t^{i|j}$. The null hypothesis in this case becomes:

$$H_0: \Pr\left(V_{i,t+1} \leq -B_t^{i|j} | V_{j,t+1} \leq -B_{j,t}\right) = \alpha \tag{14}$$

Since the null implies that $E\left[\mathbf{I}\left(V_{i,t+1} \leq -B_t^{i|j}\right) \times \mathbf{I}\left(V_{j,t+1} \leq -B_{j,t}\right)\right] = \alpha$, a simple likelihood-ratio (*LR*) test can be used for the testing procedure (Christoffersen, 2009).

To assess the conditional probability of exceedances, we use the historical paths of the P&Ls for both members i and j; i.e., $\left\{v_{i,t+1}\right\}_{t=1}^{T}$ and $\left\{v_{j,t+1}\right\}_{t=1}^{T}$. The corresponding LR test statistic, denoted $LR_{i|j}$, takes the form:

$$LR_{i|j} = -2\ln[(1-\alpha)^{T-N_{i|j}}\alpha^{N_{i|j}}] + 2\ln\left[\left(1-\frac{N_{i|j}}{T}\right)^{T-N_{i|j}}\frac{N_{i|j}}{T}^{N_{i|j}}\right] \tag{15}$$

where $N_{i|j}$ denotes the total number of joint past violations observed for both members i and j; that is, $N_{i|j} = \sum_{t=1}^{T}\mathbf{I}\left(v_{i,t+1} \leq -B_t^{i|j}\right) \times \mathbf{I}\left(v_{j,t+1} \leq -B_{j,t}\right)$.

5. Empirical Analysis

5.1. Data and Descriptive Statistics

In this section we compare the empirical performance of the SPAN, VaR and CoMargin systems by using proprietary data from the Canadian Derivatives Clearing Corporation (CDCC). The CDCC is the clearing house of the TMX Montreal Exchange. The dataset includes the daily trading positions at market close for each clearing member on the three-month Canadian Bankers' Acceptance Futures (BAX), the ten-year Government of Canada Bond Futures (CGB), and the S&P/TSX 60 Index Standard Futures (SXF) for the forty-eight clearing members active in the CDCC between January 2, 2003 and March 31, 2011. To the best of our knowledge no other academic study has ever used actual clearing member positions.

Table 3 presents a short description of the data. In a derivatives exchange, on any given day, there are many delivery dates available on each underlying asset. Over the sample period there were 45 different delivery dates available for BAX contracts and 34 for CGB and SXF contracts. Thus, the sample includes a total of 113 futures contracts. Table 4 summarizes the specifications of these contracts. The contracts in our sample do not constitute the full set of derivatives cleared by the CDCC. However, they represent around 75% its clearing activity. The documentation provided by CDCC states that the BAX, CGB and SXF are among the most actively traded derivatives in Canada. Furthermore, BAX and CGB are the most actively traded cleared interest rate contracts in the country (Campbell and Chung, 2003 and TMX Montreal Exchange, 2013a,b,c).

Table 5 shows the summary statistics for the contracts in the sample. Panel A shows the aggregate statistics for all 113 contracts and Panels B to D report the summary statistics by underlying asset. On a typical day, there were approximately 20 active contracts, 12 on the BAX,

four on the CGB and four on the SXF. On average, contracts remained active for 363 trading days. However, there is a significant dispersion across underlying assets. BAX contracts remained active for 551 days, whereas CGB and SXF contracts remained active for 239 and 237 days, respectively. BAX contracts were the most actively traded, with an average daily gross open interest of 275,000. The corresponding open interest for CGB and SXF contracts were 131,000 and 111,000, respectively.

CDCC members have access to three accounts to submit trades for clearing: a firm, a client and an omnibus account. The firm account is used by clearing members to submit their own trades (i.e., conduct proprietary trading). The client account is used to submit trades on behalf of clearing members' clients. The omnibus account is used for all other clearing activities and is the least active account across all clearing members.

Our dataset includes 21 firm, 23 client, and 16 omnibus accounts that were active on at least one day of the sample period. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Due to disclosure restrictions, we are unable to report the owners of each active account by type. However, Table 6 provides the full list of clearing members that had at least one active client, firm or omnibus account during the sample period. Notice that this list includes more clearing members than those currently affiliated with the CDCC because some members entered and exited the market during this period.

In our tests, we only consider firm accounts and we do so for three main reasons. First, house trading has been identified as the main source of risk on CCPs (Jones and Pérignon, 2013). Second, considering only a subset of accounts allows us to apply the CoMargin methodology with any

number of firms in the conditioning set because the sum of the variation margins is not zero. Third, unlike firm margins, client margins are not set at the portfolio level but, instead, on a client-by-client basis.

In Figure 2 we plot the daily settlement prices (obtained from Bloomberg) for the underlying assets and the futures contracts in the sample. Panel A shows the time series of underlying asset prices, Panel B shows the underlying asset returns and Panel C shows the settlement futures prices for all delivery dates. Lines in different colors represent different delivery dates. It is evident from Panel B that the volatility of the underlying assets increased dramatically after the onset of the financial crisis in mid-2007. In addition, Panel C shows an increase in the spread of futures prices during the same period, particularly for BAX contracts.

Figure 3 plots the daily stacked P&L values implied from the positions in active firm accounts. For each date t, $n_t^a \in N$ observations are plotted, which correspond to the P&L of the n_t^a clearing members that had an active account on that day. Notice how the volatility of the P&Ls increased dramatically at the beginning of the financial crisis as well, consistent with the patterns in the underlying and futures prices in Figure 2. Therefore, we consider two sub-periods in our analysis: the pre-crisis period, from January 2, 2003 to July 31, 2007, and the crisis period, from August 1, 2007 to March 31, 2011.

Table 7 presents the summary statistics for the firm accounts in the sample. Panel A reports the values for the full sample period and Panels B and C present the values for the pre-crisis and crisis periods, respectively. On a typical day, there were approximately 12 clearing members with active firm accounts. This number remained relatively stable across sub-periods. The average account was active for 56% of the days in the full sample (1,145 out of 2,066 days), with 75%

during the pre-crisis period and 56% during the crisis period. The relatively lower activity during the crisis period was partially influenced by clearing members exiting the market. The P&L numbers reported in the table focus exclusively on active accounts. The typical active account reported an implied daily loss of $60,000 on the futures contracts listed in the sample. During the pre-crisis period, the average account reported daily losses of $164,000. However, during the crisis period, the average account reported a daily profit of $65,000. These profits were mostly derived from long positions in BAX contracts. Over the entire sample period, the typical account made an implied loss of $38,000, with corresponding losses of $119,000 and profits of $39,000 for the pre-crisis and crisis periods, respectively.[21]

5.2. Empirical Performance

Using the daily trading positions in each firm account, we compute the initial margin collected from each clearing member under the SPAN, VaR and CoMargin systems. We set the risk level α to 2% for the VaR and CoMargin systems and use a rolling estimation window of 500 trading days in all cases. As mentioned in Section 2.2, the SPAN system is estimated using the sixteen scenarios in Table 1. In this study, as we are only dealing with futures contracts, we ignore scenarios based on volatility changes or extreme scenarios, which are targeted to deep out of the money options.[22] For the VaR and CoMargin systems we consider $S = 100{,}000$ scenarios that are obtained using the methodology described in Section 4.1. Consistent with our earlier discussion,

[21] Notice that the P&L values reported in this paper are those implied by the positions held by the clearing members in their firm accounts on the contracts included in the sample. The actual accounts of these clearing members include positions in other contracts cleared by the CDCC. In addition, our P&L values do not include trading revenues from other sources, such as non-cleared OTC transactions.

[22] The shocks in extreme scenarios 15 and 16 are defined by the risk managers as a percentage increase over the price range (e.g., 20% over the price range). Using an ad hoc extreme change would prevent us from keeping the same 99% confidence interval for all underlying assets. Adding an ad-hoc extreme change only collects more money (i.e., increases the SPAN coverage) but does not change the results.

we set the financial distress threshold for CoMargin at the VaR margin level of the conditioning firms. The conditioning firms are the two clearing members with the highest one-day-ahead Expected Shortfall, $ES_{i,t+1} = E\left(V_{i,t+1} | V_{i,t+1} \leq -B_{i,t}\right)$.

For consistency across time periods, we ignore the ad hoc inter- and intra-commodity spreads used in the SPAN system and impose a minimum margin of $10,000 on all active accounts under all systems. This amount allows us to avoid cases when clearing members are not required to post any collateral because they have matched long and short positions. These cases are likely to result in small exceedances as P&Ls in different contracts do not always offset each other. Thus, imposing a minimum margin amount prevents an upward bias in the number of SPAN exceedances. This amount, however, does not influence the rest of our results as it represents a constant that accounts for less than 0.2% of the average individual daily margin required under all systems.[23]

We find that accounting for P&L dependence increases margin significantly. Table 8 reports the summary statistics for the daily margin collected over the full sample period under different margining methods (Panels A and B). The table also reports budget-neutral margins for both SPAN (BNSPAN) and VaR (BNVaR) systems using the same approach as that described in equation 11. The average aggregate daily margin collected across all clearing members is $112, $101 and $161 million for the SPAN, VaR and CoMargin systems, respectively. The fact that CoMargin collects an extra 44% compared to SPAN margin might appear relatively costly at first sight. However, the CCP has the freedom to choose a higher risk level α to collect less margin, still benefiting from the CoMargin allocation of the collateral. Using budget-neutral margins, we show below that the

[23] We computed our results under different minimum collateral amounts ranging from $0 to $100,000. The results are consistent in all cases. With a minimum collateral of $0, however, the SPAN system yields a high number of small exceedances.

better performance of CoMargin relative to other systems is not simply the result of collecting higher margins, but rather the outcome of a better collateral allocation process across clearing members.

Panels C and D report the summary statistics for the daily margin collected over the pre-crisis and crisis periods, respectively. As expected given the increased volatility during the financial crisis, both aggregate and individual collateral levels are higher during the crisis period. However, the ranking of margin collections is consistent throughout the full sample period and the two sub-periods being considered. VaR margin consistently collects the least and CoMargin consistently collects the most collateral. Similarly, VaR margin consistently shows the least dispersion and CoMargin consistently shows the most dispersion of collected margin as measured by the standard deviation. This situation arises because CoMargin takes into account the variation of more factors than the other two margining methods (i.e., the factors driving P&L dependence).

Panel A of Figure 4 shows the daily stacked initial margin requirements under the SPAN, VaR and CoMargin systems. The stacking process is the same as that used in the previous section for the P&L values of Figure 3. Notice that all three approaches produce margin requirements that are highly correlated. The average cross-sectional correlation for the full sample period is 0.99 between SPAN and VaR, 0.90 between SPAN and CoMargin, and 0.90 between VaR and CoMargin. The high correlation and low dispersion between the SPAN and VaR systems coupled with the average collection values shown in Table 8, indicate that at the individual clearing member level, SPAN margins behave much like VaR margins but at a higher coverage probability. However, notice that CoMargin is the least correlated of the three systems and shows the widest dispersion. This dispersion is more pronounced during the crisis-period, when P&L dependence is higher. As

mentioned in Section 3.2, this can be explained by the fact that CoMargin converges to VaR margin as P&L dependence decreases and diverges as P&L dependence increases.

Panel B of Figure 4 plots the daily P&L of each active clearing member against its initial margin requirement for the entire time series. The 45 and -45 degree lines are indicated in red. Observations falling below the -45 degree line denote margin-exceeding losses. Notice that of the three margining systems, CoMargin shows the least number of margin exceedances. In addition, unlike the other systems, CoMargin tends to concentrate exceedances in low initial margin, low P&L points. These points represent clearing members with the smallest or least active portfolios; that is, those that are the least likely to pose a systemic threat to the CCP.

Panel C of Figure 4 shows the daily stacked values of the relative variation margin, which is defined as the ratio of P&L to posted initial margin ($V_{i,t}/B_{i,t-1}$). Once again, the stacking process is the same as that used in Figure 3. Observations with a relative variation margin below -1, the level depicted with a red line, represent margin exceedances. Consistent with the study of the CME margins by Jones and Pérignon (2013), SPAN margins exceedances tend to cluster in time. In contrast, the CoMargin system exhibits the lowest number of simultaneous margin exceedances.

Panels A and B of Table 9 summarize the performance of different margining systems over the full sample period. Panels C and D show the corresponding values for the pre-crisis and crisis periods, respectively. The left part of the panels measures unconditional performance in terms of the probability of experiencing at least one exceedance and the expected shortfall if at least one exceedance occurs. The right part of the panels reports the same measures but conditional on at least one member exceeding its margin.

Consistent with the simulation results presented in Section 3, our empirical results show that the CoMargin system outperforms the SPAN and VaR systems in all dimensions (lower frequency of exceedances and expected shortfall), whether these are estimated unconditionally or conditionally. In addition, the relative performance of CoMargin increases when we condition on at least one exceedance event. This finding shows that CoMargin does a better job at protecting the CCP from simultaneous exceedances, even after a clearing member has surpassed its margin. Furthermore, Panels C and D indicate that the relative performance of CoMargin also improves during the crisis period, when P&L dependence is more persistent. This indicates that CoMargin tends to provide more protection to the CCP when it is needed most; that is, when simultaneous exceedance events are more likely to occur.

The clearing member-level results in Panels B to D show the probability that a typical clearing member surpasses its margin on any given day and the expected shortfall associated with this event. Once again, the conditional frequency and expected shortfalls are lower for CoMargin than for SPAN and VaR margins. In addition, notice how both the conditional and unconditional frequencies increase for the SPAN and VaR systems during the crisis period relative to the pre-crisis period. These results imply that when P&L dependence increases, SPAN and VaR exceedances are more likely than under CoMargin.

As mentioned earlier, CoMargin might outperform other systems because of a “level effect” (i.e., CoMargin imposes higher margins on average) or an “allocation effect” (i.e., CoMargin collects relatively more margin from interconnected firms). We dissect these two effects by contrasting the performance of CoMargin with that of budget-neutral SPAN and VaR systems (BNSPAN and BNVaR,

respectively).[24] By design, the aggregate margin collected by these systems is the same as that collected by CoMargin. Therefore, the level effect is neutralized and any differences in relative performance must be due to allocation effects. More precisely, if CoMargin is truly providing a better allocation mechanism, then budget-neutral methods would effectively transfer margin requirements from firms with high P&L dependence to those with low P&L dependence; thus, leaving the CCP exposed to higher shortfalls. Nevertheless, budget-neutral margins remain risk-based as they collect VaR or SPAN margin plus the difference in aggregate margins between them and CoMargin (see equation 11).

The results displayed in Table 9 show that the allocation effect of CoMargin significantly reduces average shortfall, and especially conditionally on at least one exceedance. Switching from CoMargin to BNSPAN or BNVaR increases the average conditional shortfall by a factor of five (from 1.85 to 10.15 or 9.45). Since expected shortfalls ultimately determine the impact on the funds available to the CCP, our findings show that CoMargin allocations enhance the resilience of clearing houses by minimizing the economic impact of adverse events. Furthermore, the allocation effect of CoMargin reduces the conditional frequency of exceedances at the CCP level (from 0.38 to 0.28 for BNSPAN and from 0.36 to 0.28 for BNVaR) but increases the unconditional frequency of exceedances, which mainly stem from isolated and moderate losses.

[24] Similar to BNVaR, BNSPAN is defined by equation 11 in which $B_{i,t}$ is set to the SPAN margin. Note that when there is large dispersion in the size of the clearing members, as it is the case in our sample of real world positions, the budget-neutral scheme can impose unrealistically large margins on small clearing members when the largest clearing members have correlated P&Ls.

6. Conclusion

In this paper, we present a new methodology, called CoMargin, to estimate margin requirements in derivatives CCPs. Our innovative approach explicitly takes into account both the individual risk and the interdependence of the P&Ls of market participants. As a result, CoMargin produces collateral allocations that enhance the stability and resilience of CCPs, thus reducing their systemic risk.

Using both simulated and real world derivatives position data we show that CoMargin outperforms the widely popular SPAN and VaR margining approaches. CoMargin performs particularly well relative to these alternatives when the level of P&L dependence across market participants increases, as was the case during the recent financial crisis. Therefore, our evidence suggests that CoMargin provides more protection to the CCP when it needs it most.

While CoMargin significantly increases aggregate collateral levels during certain periods, using budget-neutral method comparisons we show that the relative performance of CoMargin is driven by its allocation process and not the additional funds collected. Thus, increasing collateral requirements under other systems does not necessarily protect the CCP against simultaneous margin exceedance events.

As a cautionary note, our results also highlight that implementing CoMargin may lead to unintended consequences since individual clearing member position changes have consequences for other members' margins. For instance, by mimicking a smaller rival's trades (thus generating P&L dependence), a large, deep-pocket member might attempt to drive a smaller rival out of business. Furthermore, with CoMargin, actions such as taking trades off the CCP or merging accounts for firms with highly correlated positions, may lead to lower netting efficiency.

Nevertheless, our analysis fully reveals these possibilities which can be carefully analyzed by CCPs before implementation and monitored accordingly thereafter.

Overall, we consider CoMargin as a viable alternative to existing margin systems that might serve as the basis for further enhancements to margin systems. Potential improvements to the current CoMargin framework include reducing the pro-cyclicality of collateral requirements and enriching the scenario generating process. For instance, scenarios could account for endogenous risk or feedback effects. This feature would consider shocks that not only reflect the past evolution of the underlying variables but also the reaction of clearing firms to specific shocks.

Appendix: CoMargin with *N-1* Conditioning Firms

With $N-1$ conditioning firms, the CoMargin for firm i, $B_t^{i|j\neq i}$, is defined by:

$$\frac{\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j\neq i}\right) \cap \mathbf{C}\left(V_{j\neq i,t+1}\right)\right]}{\Pr\left[\mathbf{C}\left(V_{j\neq i,t+1}\right)\right]} = \alpha \tag{A1}$$

where the conditioning event is that at least one of the $N-1$ other firms is in financial distress:

$$\Pr\left[\mathbf{C}\left(V_{j\neq i,t+1}\right)\right] = \Pr\left[\bigcup_{j=1,j\neq i}^{N}\left(V_{j,t+1} \leq -B_{j,t}\right)\right] \tag{A2}$$

Note that only $N-1$ firms are considered in the conditioning event, since by definition, firm i is excluded. Then, the estimation method of the CoMargin is similar to the one used with two firms in Section 4.2. Given the simulated sample of P&Ls $\left\{v_{1i,t+1}^s, .., v_{N,t+1}^s\right\}_{s=1}^S$, a simple estimate of the joint probability given in equation A2, denoted $\hat{P}_t^{j\neq i}$, is given by:

$$\hat{P}_t^{j\neq i} = \frac{1}{S}\sum_{s=1}^{S}\mathbf{I}\left(\bigcup_{j=1,j\neq i}^{N}\left(v_{j,t+1}^s \leq -B_{j,t}\right)\right) \tag{A3}$$

where $v_{j,t+1}^s$correspond to the s^{th} simulated P&L of firms j. In a similar way, conditionally on $B_t^{i|j\neq i}$, a simple estimator of the numerator in equation A1, denoted $\hat{P}_t$, is defined as:

$$\hat{P}_t = \frac{1}{S}\sum_{s=1}^{S}\mathbf{I}\left(v_{i,t+1}^s \leq -B_t^{i|j\neq i}\right) \times \mathbf{I}\left(\bigcup_{j=1,j\neq i}^{N}\left(v_{j,t+1}^s \leq -B_{j,t}\right)\right) \tag{A4}$$

For each time t and for each firm i, we look for the value $B_t^{i|j\neq i}$ that minimizes the distance between the ratio $\hat{P}_t/\hat{P}_t^{j\neq i}$ and α:

$$\hat{B}_t^{i|j\neq i} = \arg\min_{\left\{B_t^{i|j\neq i}\right\}}\left(\frac{\hat{P}_t}{\hat{P}_t^{j\neq i}} - \alpha\right)^2 \tag{A5}$$

Thus, for each firm i, we end up with a time series of CoMargins $\left\{\hat{B}_t^{i|j\neq i}\right\}_{t=1}^T$ for which confidence bounds can be bootstrapped.

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Table 1: Scenarios used in the SPAN system

Scenario	Underlying Asset Price Change	Volatility Change	Time to Expiration
1	0	+ volatility range	$-1/252$
2	0	- volatility range	$-1/252$
3	$+1/3$ x price range	+ volatility range	$-1/252$
4	$+1/3$ x price range	- volatility range	$-1/252$
5	$-1/3$ x price range	+ volatility range	$-1/252$
6	$-1/3$ x price range	- volatility range	$-1/252$
7	$+2/3$ x price range	+ volatility range	$-1/252$
8	$+2/3$ x price range	- volatility range	$-1/252$
9	$-2/3$ x price range	+ volatility range	$-1/252$
10	$-2/3$ x price range	- volatility range	$-1/252$
11	$+3/3$ x price range	+ volatility range	$-1/252$
12	$+3/3$ x price range	- volatility range	$-1/252$
13	$-3/3$ x price range	+ volatility range	$-1/252$
14	$-3/3$ x price range	- volatility range	$-1/252$
15	Positive extreme change	0	$-1/252$
16	Negative extreme change	0	$-1/252$

Note: The table shows the sixteen scenarios used to determine the contract family charge in the SPAN system. Price and volatility ranges usually cover 99% of the data points over a rolling historical estimation window. Positive and negative extreme changes are designed to assess the effect of deep out of the money options.

Table 2: Theoretical performance of VaR and CoMargin systems

	Jointly Normally Distributed P&Ls				Jointly Student t Distributed P&Ls			
	Unconditional		Conditional on One Exceedance		Unconditional		Conditional on One Exceedance	
	Prob. of at least one Exceedance	Shortfall	Prob. of Additional Exceedances	Shortfall	Prob. of at least one Exceedance	Shortfall	Prob. of at least one Exceedance	Shortfall
	ρ = 0				**ν = 30, ρ = 0.4**			
VaR	0.185	0.084	0.076	0.451	0.177	0.094	0.123	0.531
CoMargin	0.185	0.084	0.076	0.451	0.115	0.056	0.074	0.485
BNVaR	0.185	0.084	0.076	0.451	0.108	0.052	0.088	0.485
	ρ = 0.4				**ν = 10, ρ = 0.4**			
VaR	0.179	0.084	0.109	0.466	0.171	0.119	0.151	0.695
CoMargin	0.138	0.060	0.069	0.433	0.081	0.053	0.090	0.650
BNVaR	0.129	0.055	0.083	0.430	0.077	0.051	0.104	0.658
	ρ = 0.8				**ν = 5, ρ = 0.4**			
VaR	0.165	0.084	0.193	0.505	0.164	0.175	0.191	1.068
CoMargin	0.110	0.048	0.062	0.432	0.051	0.060	0.129	1.171
BNVaR	0.077	0.033	0.144	0.428	0.049	0.059	0.141	1.193

Note: This table presents the theoretical performance of the VaR (equation 2), CoMargin (equation 7), and Budget-neutral VaR (BNVaR, equation 11) systems, assuming four clearing members whose P&Ls are jointly normally or Student t distributed. The left panel presents the case where P&Ls are jointly normally distributed, such that $V \sim N(0, \Sigma)$, $V = (V_1, V_2, V_3, V_4)'$ and $\Sigma = \begin{pmatrix} 1 & \boldsymbol{\rho} & 0 & 0 \\ \boldsymbol{\rho} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ and reports the results for different levels of the correlation parameter, ρ, that range from 0 to 0.8. The right panel shows the case when P&Ls are Student t distributed with degrees of freedom υ, $V \sim t_\upsilon(0, \Sigma)$. The variance-covariance structure, Σ, is the same as that considered under the normal distribution assumption, but in this case, we set ρ=0.4 and let the degrees of freedom decrease from 30 to 5.

Table 3: Description of the data used in the empirical analysis

Item	Number	Comments
Clearing members	48	There is entry and exit in the sample, so the number of clearing members varies over time.
Trading Days	2,066	The sample period is from January 2, 2003 to March 31, 2011.
Underlying Assets	3	The three underlying assets are: 1. Yield on the three-month Canadian bankers' acceptance. 2. Yield on the ten-year Government of Canada Bond Futures 3. Level of the S&P/TSX 60 Index
Three-Month Canadian Bankers' Acceptance Futures Contracts (BAX)	45	Delivery dates range from January 2003 to December 2013.
Ten-Year Government of Canada Bond Futures Contracts (CGB)	34	Delivery dates range from March 2003 to June 2011.
S&P/TSX 60 Index Standard Futures Contracts (SXF)	34	Delivery dates range from March 2003 to June 2011.
Total futures contracts	113	These represent all the futures contracts (i.e., all delivery dates) written on the three underlying assets during the sample period.
Active firm accounts	21	We report results only for this type of account.
Active client accounts	23	
Active omnibus accounts	16	

Note: The table presents an overview of the dataset used in the empirical analysis, which was obtained from the Canadian Derivatives Clearing Corporation. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset.

Table 4: Specifications of the contracts included in the empirical analysis

	S&P/TSX 60 Index Standard Futures (SXF)	Three-Month Canadian Bankers' Acceptance Futures (BAX)	Ten-Year Government of Canada Bond Futures (CGB)
Underlying Interest	The S&P/TSX 60 Index C$200 times the S&P/TSX 60 index futures value	C$1,000,000 nominal value of Canadian bankers' acceptance with a three-month maturity.	C$100,000 nominal value of Government of Canada Bond with 6% notional coupon.
Expiration Months	March, June, September and December.	March, June, September and December plus two nearest non-quarterly months (serials).	March, June, September and December.
Price Quotation	Quoted in index points, expressed to two decimals.	Index : 100 minus the annualized yield of a three-month Canadian bankers' acceptance.	Par is on the basis of 100 points where 1 point equals C$1,000.
Price Fluctuation	0.10 index points for outright positions. 0.01 index points for calendar spreads	0.005 = C$12.50 per contract for the nearest three listed contract months, including serials. 0.01 = C$25.00 per contract for all other contract months.	0.01 = C$10
Price Limits	A trading halt will be invoked in conjunction with the triggering of "circuit breaker" in the underlying stocks.	None	None
Settlement	Cash settlement	Cash settlement	Physical delivery of eligible Government of Canada Bonds.
Trading Hours (EST)	Early session*: 6:00 a.m. to 9:15 a.m. Regular session: 9:30 a.m. to 4:15 p.m. * A trading range of -5% to +5% (based on previous day's settlement price) has been established only for this session.	Early session: 6:00 a.m. to 7:45 a.m. Regular session: 8:00 a.m. to 3:00 p.m. Extended session*: 3:09 p.m. to 4:00 p.m. * There is no extended session on the last trading day of the expiring contract month.	Early session: 6:00 a.m. to 8:05 a.m. Regular session: 8:20 a.m. to 3:00 p.m. Extended session*: 3:06 p.m. to 4:00 p.m. * There is no extended session on the last trading day of the expiring contract month.

Source: TMX Montreal Exchange (http://www.m-x.ca).

Table 5: Summary statistics of the contracts included in the empirical analysis

Variable	Average	Median	St.Dev.	Min	Max
Panel A: All Contracts					
Active contracts per day	19.81	20.00	0.9279	13.00	21.00
Trading days per contract	362.25	253.00	221.72	6.00	756.00
Panel B: BAX Contracts					
Active contracts per day	11.99	12.00	0.14	8.00	13.00
Trading days per contract	550.58	699.00	249.25	6.00	756.00
Open interest	137.81	131.32	49.21	48.35	310.97
Panel C: CGB Contracts					
Active contracts per day	3.93	4.00	0.49	1.00	5.00
Trading days per contract	238.91	253.00	42.73	55.00	255.00
Open interest	65.26	60.81	27.36	16.15	176.97
Panel D: SXF Contracts					
Active contracts per day	3.89	4.00	0.43	1.00	4.00
Trading days per contract	236.32	250.00	42.58	52.00	255.00
Open interest	55.28	55.09	14.01	24.40	98.49

Note: The table shows the summary statistics of the 113 futures contracts included in the empirical analysis. These contracts are divided according to their underlying assets as follows: Three-month Canadian Bankers' Acceptance (BAX), Ten-year Government of Canada Bond (CGB) and S&P/TSX 60 Index Standard (SXF). Over the sample period (January 2, 2003 to March 31, 2011), there were 45 different delivery dates available for BAX contracts and 34 delivery dates available for CGB and SXF contracts. Open interest values are reported in thousands.

Table 6: Clearing members included in the empirical analysis

Number	Name	Number	Name
1	Newedge Canada Inc.	25	Morgan Stanley Canada LTD.
2	RBC Dominion Securities Inc.	26	CFG Financial Group Inc.
3	Union Securities LTD.	27	MF Global Canada Co.
4	T.D. Securities Inc.	28	Haywood Securities Inc.
5	BMO Nesbitt Burns LTD.	29	Goldman Sachs Canada Inc.
6	Macquarie Private Wealth Inc.	30	Timber Hill Canada Co.
7	UBS Securities Canada Inc.	31	Credit Suisse Securities
8	Desjardins Securities Inc.	32	CIBC World Markets Inc.
9	Macquarie Capital Markets Inc.	33	NBCN Clearing Services Inc.
10	Name not reported	34	HSBC Securities (Canada) Inc.
11	Merrill Lynch Canada Inc.	35	Mackie Research Capital Corporation
12	Odlum Brown LTD.	36	Benson-Quinn GMS Inc.
13	Penson Financial Services Inc.	37	Scotia Capital Inc.
14	Dundee securities corporation	38	E*trade Canada Securities Corporation
15	Daex Commodities Inc.	39	Raymond Kames LTD.
16	Canaccord Capital Corporation	40	Lévesque Beaubien Geoffrion Inc.
17	Friedberg Mercantile Group LTD.	41	TD Waterhouse Canada Inc.
18	W.D. Latimer Co. LTD.	42	Citigroup Global Markets Canada Inc.
19	Canadian Imperial Bank of Commerce (CIBC)	43	National Bank of Canada
20	Jones, Gable & Co. LTD.	44	J.P. Morgan Securities Canada Inc.
21	Name not reported	45	Merrill Lynch Canada Inc.
22	Timber Hill Canada Company	46	Name not reported
23	Laurentian Bank Securities Inc.	47	Fidelity Clearing Canada ULC
24	Deutsche Bank Securities LTD.	48	Maple Securities Canada LTD.

Note: The table provides the full list of clearing members that had at least one active client, firm or omnibus account during the sample period (January 2, 2003 to March 31, 2011). An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Notice that this list includes more clearing members than those currently affiliated with the Canadian Derivatives Clearing Corporation (CDCC) because some of them entered and exited the market during the sample period.

Table 7: Summary statistics of the firm accounts included in the empirical analysis

Variable	Average	Median	St.Dev.	Min	Max
Panel A: Full Sample period					
Active accounts per day	11.64	12.00	1.09	8.00	15.00
Active days for an account	1145.19	1420.00	911.72	3.00	2066.00
Daily P&L across clearing members	-60.92	-97.80	2659.44	-15014.20	17502.52
P&L over time	-37.83	0.43	160.57	-455.52	237.50
Panel B: Pre-Crisis period					
Active accounts per day	11.96	12.00	0.95	9.00	15.00
Active days for an account	858.00	1148.00	431.12	3.00	1148.00
Daily P&L across clearing members	-163.50	-156.15	2027.37	-6813.50	10381.36
P&L over time	-119.12	-0.78	225.46	-671.62	39.80
Panel C: Crisis period					
Active accounts per day	11.24	11.00	1.13	8.00	15.00
Active days for an account	516.05	684.00	418.29	3.00	918.00
Daily P&L across clearing members	65.18	-57.43	3280.36	-15014.20	17502.52
P&L over time	39.83	-0.60	135.28	-110.76	484.42

Note: The table presents the summary statistics of the 21 active firm accounts used in the empirical analysis. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. The sample period is from January 2, 2003 to March 31, 2011. The pre-crisis period is from January 2, 2003 to July 31, 2007 and the crisis period is from August 1, 2007 to March 31, 2011. P&L values are reported in thousands of dollars and are estimated only for active firm accounts.

Table 8: Daily margin requirements under the SPAN, VaR and CoMargin systems

	Mean	Median	St.Dev.	Min	Max
			Panel A: Aggregate Market (CCP level)		
SPAN	112.04	105.71	38.49	49.83	328.77
VaR	101.40	95.80	36.03	42.90	301.97
CoMargin	161.31	156.20	64.93	56.89	475.27
BNSPAN	161.31	156.20	64.93	56.88	475.27
BNVaR	161.31	156.20	64.93	56.88	475.27
			Panel B: Per Clearing Member		
SPAN	9.76	9.16	3.54	3.83	29.86
VaR	8.83	8.26	3.28	3.30	27.45
CoMargin	14.14	13.41	6.08	4.38	43.01
BNSPAN	14.14	13.41	6.08	4.38	43.01
BNVaR	14.14	13.41	6.08	4.38	43.01
			Panel C: Per Clearing Member (Pre-Crisis)		
SPAN	8.49	8.32	2.38	3.83	14.98
VaR	7.72	7.47	2.15	3.30	13.79
CoMargin	11.94	11.62	4.45	4.38	25.98
BNSPAN	11.94	11.62	4.45	4.38	25.98
BNVaR	11.94	11.62	4.45	4.38	25.98
			Panel D: Per Clearing Member (Crisis)		
SPAN	11.36	10.85	4.07	4.88	29.86
VaR	10.21	9.51	3.87	4.46	27.45
CoMargin	16.89	16.21	6.69	5.64	43.01
BNSPAN	16.89	16.21	6.69	5.64	43.01
BNVaR	16.89	16.21	6.69	5.64	43.01

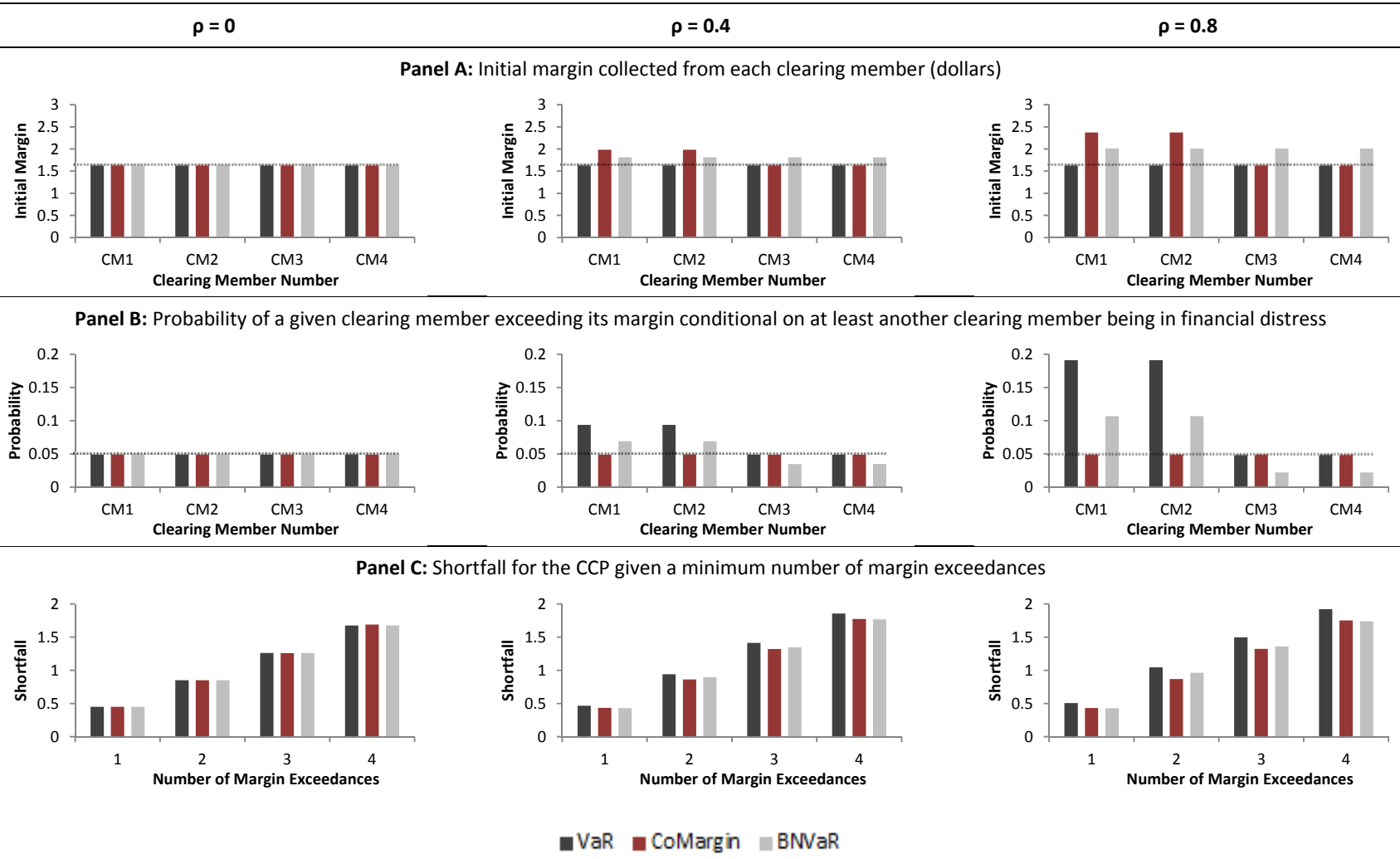
Note: The table presents the summary statistics of the daily margin requirements under the SPAN, VaR (equation 2) and CoMargin (equation 7) systems for the 21 active firm accounts during the full sample period, from January 2, 2003 to March 31, 2011 (Panels A and B), during the pre-crisis period, from January 2, 2003 to July 31, 2007 (Panel C), and during the crisis period, from August 1, 2007 to March 31, 2011 (Panel D). The budget-neutral versions of the SPAN (BNSPAN) and VaR (BNVaR) systems are also presented and were computed using equation 11. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Margin amounts are reported in millions of dollars.

Table 9: Performance of the SPAN, VaR and CoMargin systems

	Unconditional		Conditional on at least one exceedance	
	Frequency of Exceedances	**Avg. Shortfall (CAD Millions)**	**Frequency of Exceedances**	**Avg. Shortfall (CAD Millions)**
		Panel A: Aggregate Market (CCP level)		
SPAN	0.09	0.35	0.36	3.78
VaR	0.14	0.44	0.42	3.20
CoMargin	0.07	0.13	0.28	1.85
BNSPAN	0.02	0.16	0.38	10.15
BNVaR	0.02	0.16	0.36	9.45
		Panel B: Per Clearing Member		
SPAN	0.01	0.03	0.14	0.34
VaR	0.02	0.04	0.15	0.29
CoMargin	0.01	0.01	0.12	0.16
BNSPAN	0.00	0.01	0.12	0.90
BNVaR	0.00	0.01	0.12	0.84
		Panel C: Per Clearing Member (Pre-Crisis)		
SPAN	0.01	0.01	0.11	0.08
VaR	0.02	0.01	0.13	0.09
CoMargin	0.01	0.00	0.10	0.05
BNSPAN	0.00	0.00	0.10	0.17
BNVaR	0.00	0.00	0.10	0.18
		Panel D: Per Clearing Member (Crisis)		
SPAN	0.02	0.06	0.17	0.63
VaR	0.03	0.08	0.18	0.48
CoMargin	0.01	0.02	0.14	0.30
BNSPAN	0.00	0.03	0.14	1.40
BNVaR	0.00	0.03	0.14	1.21

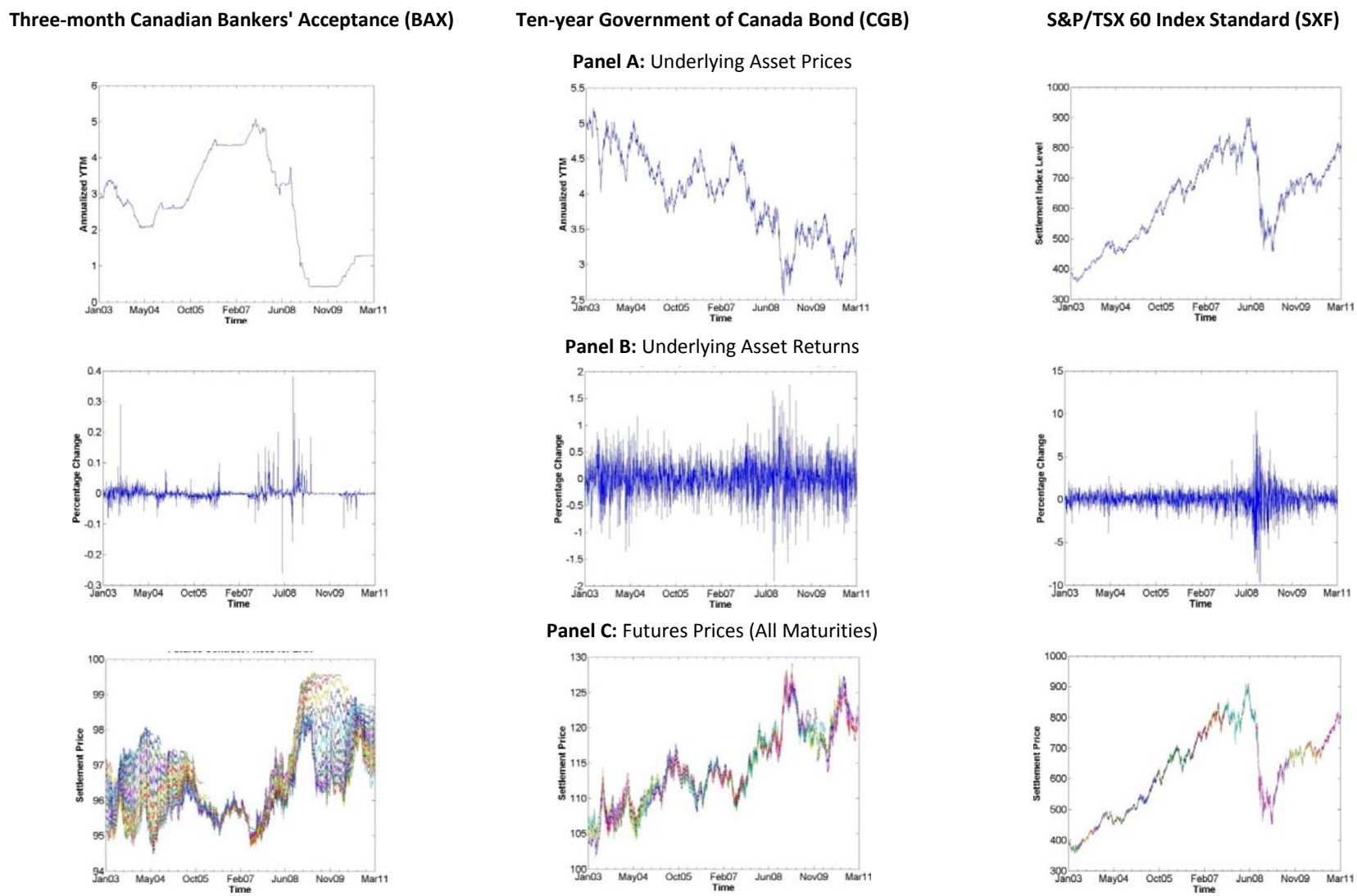
Note: The table compares the empirical performance of the SPAN, VaR (equation 2) and CoMargin (equation 10) systems computed for the 21 active firm accounts over the sample period, from January 2, 2003 to March 31, 2011 (Panels A and B), during the pre-crisis period, from January 2, 2003 to July 31, 2007 (Panel C), and during the crisis period, from August 1, 2007 to March 31, 2011 (Panel D). The left panel reports unconditional amounts and the right panel reports the same amounts conditional on at least one margin exceedance. An exceedance is defined as a loss that exceeds the margin posted by a clearing member at the end of the previous trading day. The budget-neutral versions of the SPAN (BNSPAN) and VaR (BNVaR) systems are also presented and were computed using equation 11. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Average shortfall values are reported in millions of dollars and correspond to the losses expected if margin exceedances occur.

Figure 1: Theoretical performance of VaR and CoMargin systems assuming jointly normally distributed P&Ls



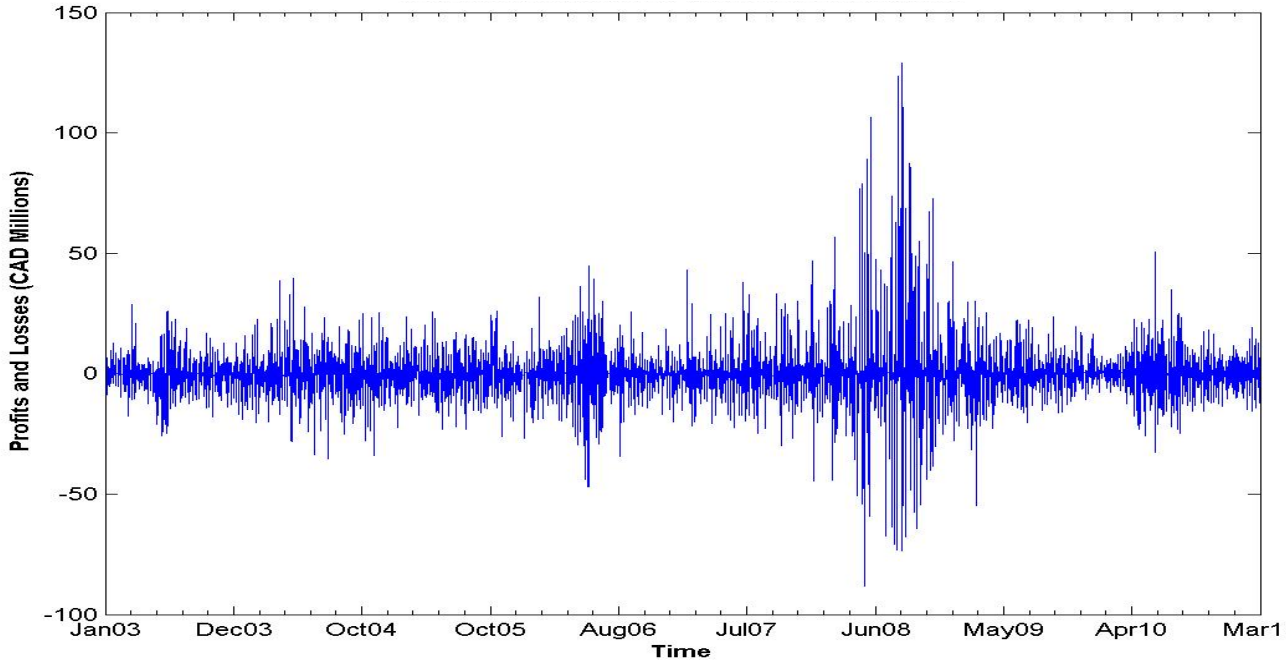
Note: This figure presents the theoretical performance of VaR margin (equation 2), CoMargin (equation 7), and BNVaR margin (equation 11). We consider four firms with jointly normally distributed P&Ls, such that $V \sim N(0, \Sigma)$, $V = (V_1, V_2, V_3, V_4)'$ and $\Sigma = \begin{pmatrix} 1 & \boldsymbol{\rho} & 0 & 0 \\ \boldsymbol{\rho} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$. We report our results for different levels of the correlation parameter, ρ, that range from 0 to 0.8.

Figure 2: Underlying assets and futures contracts used in the empirical analysis



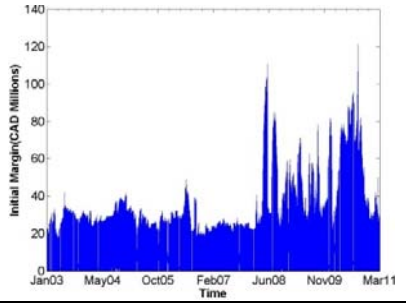
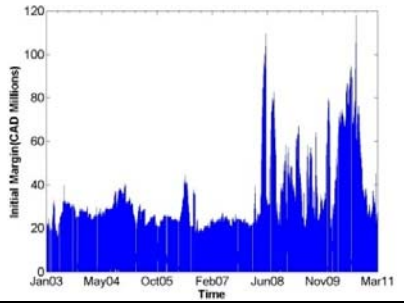
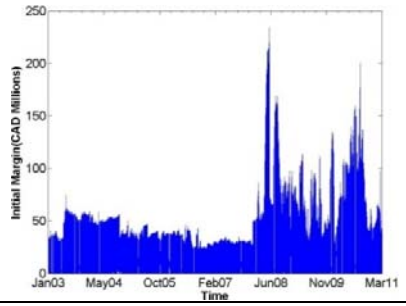
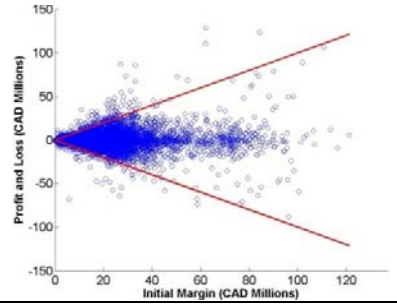
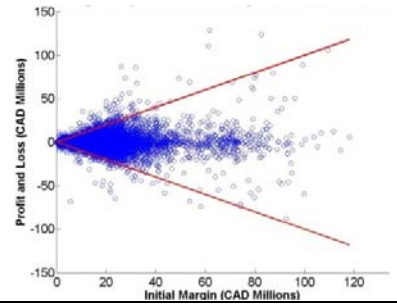
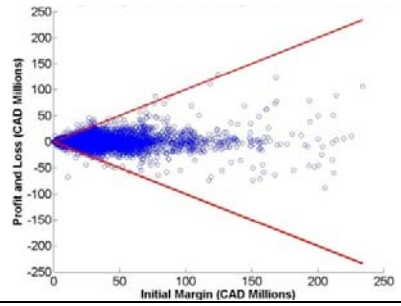
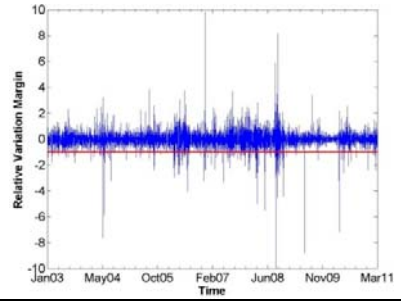
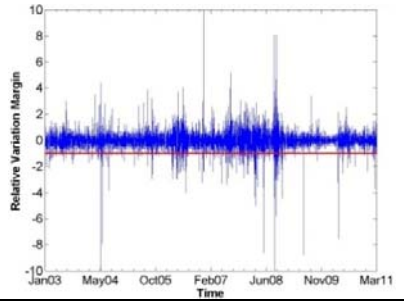
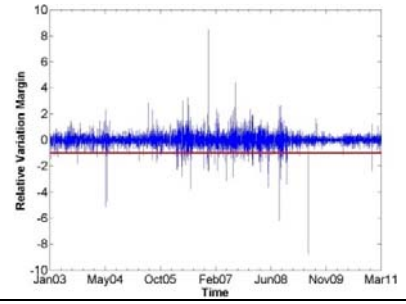
Note: Panel A presents the daily annualized settlement yield for the Three-month Canadian Bankers' Acceptance, the annualized yield on the Ten-year Government of Canada Bond and the settlement level of the S&P/TSX 60 Index. Panel B shows the daily returns (i.e., percentage changes) of the variables presented in Panel A. Panel C presents the daily settlement futures prices for the futures contracts (BAX, CGB, and SXF) written on the variables presented in Panel A and traded in the Montreal Exchange. Lines in different colours represent different delivery dates. The sample period is from January 2, 2003 to March 31, 2011. Source: Bloomberg.

Figure 3: P&L for active firm accounts



Note: The figure shows the daily stacked P&L implied from the positions of the 21 active firm accounts included in the sample; that is, accounts with an open interest (i.e., long or short position) in at least one underlying asset at the end of the trading day. For each date t, $n_t^a \in N$ observations are plotted, which correspond to the P&L of the n_t^a clearing members with an active account. The sample period is from January 2, 2003 to March 31, 2011.

Figure 4: SPAN, VaR and CoMargin collateral requirements over the full sample period

SPAN	VaR	CoMargin
Panel A: Initial Margin		
		
Panel B: Daily P&L and Initial Margins		
		
Panel C: Relative Variation Margin		
		

Note: The figure shows the implied margin requirements from the positions in the 21 active firm accounts under the SPAN, VaR and CoMargin systems. Panel A shows the daily stacked initial margin requirements. Panel B plots the daily implied P&L against its initial margin requirement. Panel C shows the daily stacked values of the relative variation margin, which is defined as the ratio of P&L to posted initial margin. The stacking method used in panels A and C is as follows: for each date t, $n_t^a \in N$ observations are plotted, which correspond to the observations of the n_t^a clearing members with an active account. The sample period is from January 2, 2003 to March 31, 2011.