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Beta Risk in the Cross-Section of Stocks and Options*

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Abstract

In order to study beta risk, we develop a multivariate stochastic volatility model in which individual equity and market returns co-vary stochastically. In the model, the stochastic covariance matrix of an individual equity return with the market follows a bivariate Wishart process and the associated beta risk is priced. We estimate the model on returns and options jointly for a large cross-section of stocks. When analyzing the model's empirical performance, we find that it outperforms the standard rolling regression approach that is prevalent in empirical asset pricing.

JEL Classification: G10; G12; G13.
Keywords: Factor models; stochastic beta; option-implied beta; Wishart processes.

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1 Introduction

An extensive literature documents pronounced time-variation in equity betas.[1] When modelling equity return dynamic, not accounting for the dynamics in market exposures might be relatively harmless during normal market conditions. However, missing sudden variation in beta during market downturns can be costly. Equity stock and option portfolios' optimal allocation, hedging, and performance evaluation require appropriate measures of the joint dynamics of the equity and market index returns. Capturing the dependence of equity on the market index is also of paramount importance for cross-sectional asset pricing. Yet, little is known about the risks associated with time-varying betas.

We study the dynamics of individual equity beta with the market index. To capture time variation in equity risks, we build on the work of Gouriéroux and Sufana (2003), Gouriéroux (2006), and Buraschi, Porchia, and Trojani (2010). We develop a stochastic volatility-covolatility model in which individual equity and market returns covary dynamically. In the model, the variance-covariance matrix of index and equity returns follows a bivariate Wishart process which can be viewed as a multivariate extension of Heston (1993). Our structural model extends the static Capital Asset Pricing Model (CAPM) by allowing beta to vary over time. This makes the model appropriate to study the cross-sectional implications of equity risk dynamics.[2]

The model provides several new insights about beta risk. First, it shows that part of equity premium can be attributed to a premium for time-varying betas. This prediction is consistent with the empirical findings in Ang, Chen, and Xing (2006) and Petkova and Zhang (2005). Moreover, the model predicts that stock betas and firm-specific innovations should comove negatively in the presence of a negative leverage effect. Finally, the model suggests that a substantial part of co-movements in betas is driven by market return and variance risks.

Several studies argue that better capturing changes in equity beta helps decrease CAPM pricing anomalies (see, among others, Jagannathan and Wang, 1996; and Petkova and Zhang, 2003). When estimating the CAPM conditionally, the common practice is to adopt an ad-hoc recursive estimation method to capture fluctuations in betas using ordinary least square regressions (OLS). An inherent problem with this methodology is that all observations in a given window impact equally the estimation of beta. The longer the estimation window the more backward looking the OLS beta becomes. This is in stark contrast with our model which, in addition to account for the dynamics

[1] See Jagannathan and Wang, (1996), Lettau and Ludvigson (2001), Lustig and Van Nieuwerburgh (2005), and Ang, Chen, and Xing (2006) for evidence on time-varying betas.

[2] See for example Da Fonseca, Grasselli, and Tebaldi (2007), Da Fonseca and Grasselli (2011), and Gruber, Tebaldi, and Trojani (2015) for other studies that use Wishart processes for modelling purposes. Note that none of these papers study the joint dynamic of market index and individual equity returns.

of beta, can be used to filter spot betas from one return observation only conditional on a set of structural model parameters.

Another key attribute of the model is that it yields analytical solutions for equity and index option prices. A large literature shows that option prices are highly informative about underlying risk dynamics (see, among others, Pan, 2002; Carr and Wu, 2009; Bollerslev, Todorov, and Xu, 2015). Motivated by these studies, we exploit the model's semi-closed form solution for option prices and develop a joint estimation methodology. We fit the model by maximizing the return and option likelihoods of a large cross-section of stocks over sixteen years. Our sample includes the Tech bubble and the recent financial and sovereign debt crises. This allows us to study fluctuations in betas over various economic-cycles. Combining options with return information is key to reliably estimate the dynamics of equity beta in the model.

Overall, the model fits the return and option data well. Unconditionally, the model's betas are closely related to OLS betas. Interestingly, the largest discrepancy between models' estimates is observed for firms with high total volatility such as Financials and IT firms. When inferring the market premium from the cross-section of average stock returns and betas, the premium implied by our model is much closer to the sample average of market index returns. Conditionally, our model's betas substantially outperform OLS betas when predicting future realized returns. Unlike OLS betas, the model forecasts for integrated beta obtain positive loadings that are highly statistically significant and economically meaningful. The model integrated betas help predict the cross-section of equity returns up to six months ahead.

The recent financial crisis has highlighted the dramatic impact of systematic risk on stock returns during periods of high uncertainty. Because the model captures the dynamics of beta, it should be informative about the future systematic exposure of stocks. We use the model and various OLS beta measures to forecast systematic equity risk. During the turbulent 2007-2009 period, the model forecast error decreases by more than fifteen percent relative to OLS betas.

Consistent with the model predictions, we find that betas are negatively impacted by firm-specific news. We also find that a substantial part of co-movements in betas are driven by aggregate uncertainty captured by macro-economic news and the VIX index. Together, these factors explain up to 79% of the co-movements in betas.

Our paper is related to the option-implied beta literature. For instance, Chang, Christoffersen, Jacobs, and Vainberg (2011), or Buss and Vilkov (2012) exploit the information in equity and index option prices to construct daily model-free estimates of beta. Overall, these studies conclude that option-implied betas help reduce CAPM anomalies. We complement these papers by modelling the dynamics of beta and by conducting a joint estimation on returns and options.

Comprehensive evidence exists on equity and market variance risk premia (see, among others,

Bollerslev, Tauchen, and Zhou, 2009; Carr and Wu, 2009; Todorov, 2010). However, the study of equity beta premium has been largely unexplored. We document substantial cross-sectional variations of beta risk premium.

Our work is related to the literature extending CAPM to account for market or equity risk dynamics. Campbell, Giglio, Polk, and Turley (2016) study the pricing of volatility risk using an intertemporal CAPM with stochastic volatility. Engle (2016) extends the standard CAPM to allow for time-varying regression coefficients. We complement these papers by developing a model that accounts for market and equity stochastic volatility and that allows for time-varying beta.

Also related to our work is the literature extending traditional factor models to allow for the pricing of options. This is done in Mo and Wu (2007), Christoffersen, Fournier, and Jacobs (2013), and Gourier (2016) among others. We complement these papers by relaxing the time-invariant beta assumption on which these studies rely.

The remainder of the paper is organized as follows. Sections 2 presents the model. Section 3 discusses model properties. Section 4 implements the model using return and option data. Section 5 assesses the model performance relative to OLS betas. In Section 6, we analyze the dependence of beta on news. Finally, Section 7 concludes.

2 A Capital Asset Pricing Model with Stochastic Beta

We develop the model in a continuous-time economy in which the physical dynamic of market index return is

$$\frac{dI_t}{I_t} = \mu_{I,t}dt + \sigma_{I,t}dZ_{I,t}, \tag{2.1}$$

where $\mu_{I,t}$ is the market premium, $\sigma_{I,t}$ denotes the market variance, and $Z_{I,t}$ is a Brownian motion capturing aggregate market return risk.[3] The investment opportunity set is further composed of J individual equities. In the spirit of CAPM, the law of motion for the return of individual equity j under the P-measure is

$$\frac{dS_t^j}{S_t^j} - rdt = \beta_{j,t}\left(\frac{dI_t}{I_t} - rdt\right) + \left(\sqrt{\sigma_{j,t}^2 - \beta_{j,t}^2\sigma_{I,t}^2}\right)d\varepsilon_{j,t}, \tag{2.2}$$

where r is the risk-free rate, $\beta_{j,t} \equiv \sigma_{jI,t}/\sigma_{I,t}^2$ is the market beta, $\sigma_{jI,t}$ is the equity and market index covariance, and $\sigma_{j,t}^2$ denotes stock j's total variance. In addition to the market innovations, the equity return is driven by an idiosyncratic shock $d\varepsilon_{j,t}$ independent of $dZ_{I,t}$.

[3]The information available to agents consists of the trajectories generated by all the Brownian motions driving the uncertainty in this economy (the Brownian filtration $\mathcal{F}$). The underlying probability space is $(\Omega, \mathcal{F}, P)$, where P is the physical probability measure.

Recall that we are interested in modelling the joint dynamic of market index and individual equity risks. To uncover the form of the market index and individual equity returns variance-covariance matrix, we can express the P-dynamics (2.1) and (2.2) as

$$\begin{bmatrix} \frac{dI_t}{I_t} \\ \frac{dS_t^j}{S_t^j} \end{bmatrix} = \begin{bmatrix} \mu_{I,t} \\ \mu_{j,t} \end{bmatrix} dt + \sqrt{\Sigma_t} \begin{bmatrix} dZ_{I,t} \\ d\varepsilon_{j,t} \end{bmatrix}, \tag{2.3}$$

where $\mu_{j,t} = r + \beta_{j,t} \cdot \big(\mu_{I,t} - r\big)$. In the previous equation, $\sqrt{\Sigma_t}$ is the square-root of the variance-covariance matrix of the market index and individual equity returns which satisfies

$$\sqrt{\Sigma_t} \equiv \begin{bmatrix} \sigma_{I,t} & 0 \\ \beta_{j,t}\sigma_{I,t} & \sqrt{\sigma_{j,t}^2 - \beta_{j,t}^2\sigma_{I,t}^2} \end{bmatrix} \Longrightarrow \Sigma_t = \sqrt{\Sigma_t}\sqrt{\Sigma_t}^{'} = \begin{bmatrix} \sigma_{I,t}^2 & \sigma_{Ij,t} \\ \sigma_{jI,t} & \sigma_{j,t}^2 \end{bmatrix}, \tag{2.4}$$

where we have used the fact that $\beta_{j,t}\sigma_{I,t}^2 = \sigma_{jI,t}$. We model the return variance-covariance matrix Σ_t under the physical measure as a bivariate Wishart process

$$d\Sigma_t = \left(\gamma Q^{'}Q - K\Sigma_t - \Sigma_t K'\right) dt + \sqrt{\Sigma_t} dW_t Q + \left(\sqrt{\Sigma_t} dW_t Q\right)^{'}, \tag{2.5}$$

for which all components are 2×2 matrices excepted γ which is a positive scalar such that $\gamma \geqslant 3$ in our bivariate setting.[4] Because the long-term mean matrix θ of Σ_t solves the system of equations $K\theta + \theta K' = \gamma Q^{'}Q$, Σ_t is a mean-reverting process since $\gamma Q^{'}Q - K\Sigma_t - \Sigma_t K'$ can be replaced by $K\left(\theta - \Sigma_t\right) + \left(\theta - \Sigma_t\right) K'$ in (2.5). In the model, K is thus a positive definite matrix which captures the mean-reversion speed of Σ_t and Q is the volatility of volatility matrix.

In (2.5), W_t is a matrix of independent Brownian innovations such that

$$W_t = \begin{bmatrix} W_{I,t}^{(1,1)} & W_{I,t}^{(1,2)} \\ W_{j,t}^{(2,1)} & W_{j,t}^{(2,2)} \end{bmatrix}, \tag{2.6}$$

where $W_{I,t}^{(1,1)}$ and $W_{I,t}^{(1,2)}$ are the aggregate market variance shocks while $W_{j,t}^{(1,1)}$ and $W_{j,t}^{(1,2)}$ are firm-specific innovations.[5] We refer to Appendix A for more details about the dynamics of $\sigma_{I,t}^2$, $\sigma_{jI,t}$, and $\sigma_{j,t}^2$ consistent with (2.4), (2.5), and (2.6).

A large empirical literature documents the presence of a negative contemporaneous correlation between return and variance innovations. This is commonly referred to as the leverage effect (see

[4]The parameter restriction $\gamma \geqslant N+1$ in a N-dimensional set-up ensures that the Wishart process admits a unique strong solution in the set of positive-definite matrices. Given our bivariate setting, it implies $\gamma \geqslant 3$.

[5]Throughout the paper $A^{(l,m)}$ refers to the element located on l^{th} row of column m of the matrix A.

Black, 1976; and Christie, 1982). We account for this effect by linking the Brownian motions in returns and variances according to

$$\begin{array}{l} dZ_{I,t} = \sqrt{1-\rho_{lev}^2}dB_{I,t} + \rho_{lev}dW_{I,t}^{(1,1)} \\ d\varepsilon_{j,t} = \sqrt{1-\rho_{lev}^2}dB_{j,t} + \rho_{lev}dW_{j,t}^{(2,1)} \end{array}, \tag{2.7}$$

where ρ_{lev} is the leverage parameter, and $B_{I,t}$ and $B_{j,t}$ are two Brownian motions independent of each other and of the shocks in W_t. From (2.7), we see that only the first column of W_t is correlated with the Brownians in returns. As we discuss in more details later, this specification facilitates the filtering of the spot variance-covariance matrices from returns. From (2.7) and the dynamics of $\sigma_{I,t}^2$ and $\sigma_{j,t}^2$, we can show that the market and equity leverage effects satisfy

$$\begin{array}{l} Corr_t(\frac{dI_t}{I_t}, d\sigma_{I,t}^2) = \rho_{lev}\frac{Q^{(1,1)}}{\sqrt{\left(Q^{(1,1)}\right)^2+\left(Q^{(2,1)}\right)^2}} \\ Corr_t(\frac{dS_t^j}{S_t^j}, d\sigma_{j,t}^2) = \rho_{lev}\frac{Q^{(1,2)}}{\sqrt{\left(Q^{(1,2)}\right)^2+\left(Q^{(2,2)}\right)^2}}. \end{array} \tag{2.8}$$

Thus, not only do the market and individual equity leverage effects depend on ρ_{lev} but they also depend on volatility of volatility. While the specification (2.7) relies on one single correlation parameter ρ_{lev}, the model can generate a wide range of leverage effects for the market and individual equity.

One of the main objectives of this paper is to combine the information in returns and options to extract the dynamics of equity risk. As options are priced under the risk-neutral measure, a mapping from physical dynamics to risk-neutral ones is needed. To this end, we assume that the stochastic discount factor (SDF) is linear in aggregate risks and follows

$$\frac{d\zeta_t}{\zeta_t} = -rdt - \sigma_{I,t}\left(\lambda^{R_I}dB_{I,t} + \lambda_1^{\sigma_I}dW_{I,t}^{(1,1)} + \lambda_2^{\sigma_I}dW_{I,t}^{(1,2)}\right), \tag{2.9}$$

where $\sigma_{I,t}\lambda^{R_I}$ is the price of market return-specific risk $B_{I,t}$, and $\sigma_{I,t}\lambda_1^{\sigma_I}$ and $\sigma_{I,t}\lambda_2^{\sigma_I}$ are the prices of the market variance risks $W_{I,t}^{(1,1)}$ and $W_{I,t}^{(1,2)}$.

We model the pairwise dynamics of the market index and equity j. Strikly speaking, the structural parameters, W_t, and the spot variance-covariance Σ_t in (2.5) should all be indexed by jI as they describe the joint evolution of the market index and individual equity j. For ease of notation, we omit these subscripts but elaborate on the dependence of the parameters on the equity j considered when necessary.

Without any constraint on the Wishart dynamic (2.5), the marginal distribution of $\sigma_{I,t}$ depends on individual equity variance and covariance. Given (2.5) and (2.9), setting $K^{(1,2)} = 0$ ensures that

the market variance is conditionally independent of individual equity risks under both the physical and risk-neutral measures. To obtain consistent market variance dynamic across various pairs of the market index and individual equities, we thus restrict $K^{(1,2)}$ to be 0 in the model.

3 Model Properties

We now explore the main implications of the model. First we investigate the joint dynamics of market index and individual equity variances and covariance. We then discuss model implications for the time variation in beta. Finally, we study the risk premia and the risk-neutral dynamics, and we introduce the model closed-form solution for the option price.

3.1 Spillover of Market Variance in Equity Variance and Covariance

Standard factor models focus on decomposing total equity variance into systematic and idiosyncratic components. Because these models rely on a constant beta assumption, they are not informative about the way market index and individual equity variances and covariance interact with each other. This is in stark contrast with our Wishart factor model.

We derive the conditional expectation of the market index and equity variances and covariance given the dynamics $d\sigma^2_{I,t}$, $d\sigma^2_{j,t}$, and $d\sigma_{jI,t}$ (see Appendix A). This allows us to study the way market and equity risks interact. Conditioning on time t, the expected change in market index variance, individual equity variance, and covariance satisfy

$$\frac{E^P_t\left[d\sigma^2_{I,t}\right]}{dt} = 2K^{(1,1)}\cdot(\theta_I-\sigma^2_{I,t}) \tag{3.1}$$

$$\frac{E^P_t\left[d\sigma^2_{j,t}\right]}{dt} = 2K^{(2,2)}\cdot(\theta_j-\sigma^2_{j,t})+2K^{(2,1)}\cdot(\theta_{Ij}-\sigma_{jI,t}) \tag{3.2}$$

$$\frac{E^P_t\left[d\sigma_{jI,t}\right]}{dt} = \left(K^{(1,1)}+K^{(2,2)}\right)\cdot(\theta_{Ij}-\sigma_{jI,t})+K^{(2,1)}\cdot(\theta_I-\sigma^2_{I,t}), \tag{3.3}$$

where the expectations are taken under the physical measure and θ_I, θ_j, and θ_{Ij} denote the variances and the covariance long-term means which solve the system of equations $K\theta+\theta K'=\gamma Q'Q$. As in standard volatility models, the dynamics of $\sigma^2_{I,t}$, $\sigma^2_{j,t}$, and $\sigma_{jI,t}$ are mean reverting. From equations (3.1), (3.2), and (3.3), we see that $2K^{(1,1)}$, $2K^{(2,2)}$, and $K^{(1,1)}+K^{(2,2)}$ capture the mean reversion speeds of market and individual equity variances and covariance, respectively.

The second term in equations (3.2) and (3.3) reveals an interesting property of the Wishart model. Whenever $K^{(2,1)}\neq 0$, the covariance impacts stock variance in expectation and market variance influences the covariance dynamic. Thus, fluctuations in market variance spillover directly

to covariance and indirectly to the stock variance through its impact on the covariance. Overall, the interactions of $\sigma_{I,t}^2$ with $\sigma_{jI,t}$ and $\sigma_{j,t}^2$ in the Wishart factor model can generate substantial equity co-movements and tail dependence with the market index. This has important implications for the dynamics of beta.

3.2 Beta Risk Dynamics

A key feature of our Wishart factor model is that it allows for stochastic market beta. In this regard, Proposition 1 is informative about the physical dynamics of equity risk.

Proposition 1 *Given (2.5), the physical dynamics of market beta, $\beta_{j,t} \equiv \sigma_{jI,t}/\sigma_{I,t}^2$, is such that*

$$d\beta_{j,t} = \beta_{j,t}\left(\frac{(d\sigma_{jI,t} - \Phi_1 dt)}{\sigma_{jI,t}} - \frac{\left(d\sigma_{I,t}^2 - \Phi_2 dt\right)}{\sigma_{I,t}^2}\right), \tag{3.4}$$

where $\Phi_1 \equiv 2\left(Q^{(1,1)}Q^{(1,2)} + Q^{(2,1)}Q^{(2,2)}\right)$ *and* $\Phi_2 \equiv -2\left(\left(Q^{(1,1)}\right)^2 + \left(Q^{(2,1)}\right)^2\right)$.

Proof. See Appendix B. ■

Proposition 1 provides several insights about beta risk. As readily apparent in (3.4), $\beta_{j,t}$ follows a two factor dynamics. By definition, the market beta is proportional to equity covariance with the market index and is inversely proportional to the market index variance. Accordingly, the first factor, $(d\sigma_{jI,t} - B_1 dt)/\sigma_{jI,t}$, corresponds to the relative change in covariance which positively impacts the change in beta. The second factor, $\left(d\sigma_{I,t}^2 - B_2 dt\right)/\sigma_{I,t}^2$, captures the relative change in the market index variance and is negatively related to $d\beta_{j,t}$. In the model, beta increases whenever the change in the covariance factor is greater than the relative change in market variance.

Because $d\beta_{j,t}$ is equal to beta times the factors, the sensitivity of beta to these factors is non-linear. Thus, the model predicts that beta will be more influenced by covariance and market variance innovations at high level of beta compared to times when beta is relatively low. The model suggests that accounting for time variation in beta is critically important during market downturns when equity risk is usually large.

The model has a prediction about the impact of firm-specific return shocks $d\varepsilon_{j,t}$ on beta. Beta co-moves with firm idiosyncratic risk according to

$$cov_t\left(d\beta_{j,t}; d\varepsilon_{j,t}\right) = \rho_{lev} Q^{(1,1)}\left(\frac{\sqrt{\sigma_{j,t}^2 - \beta_{j,t}^2\sigma_{I,t}^2}}{\sigma_{I,t}}\right). \tag{3.5}$$

The previous equation has important implications. For the empirically relevant case $\rho_{lev} < 0$ (i.e. negative leverage effect) and $Q^{(1,1)} > 0$, beta co-moves negatively with firm idiosyncratic shocks. When $d\varepsilon_{j,t}$ is large and positive, beta decreases. Note the way covariance depends on the ratio of idiosyncratic to market volatility. The covariance of beta with firm-specific innovation will be more negative when firm idiosyncratic volatility is high relative to market volatility.

Beta is also influenced by aggregate return risk $Z_{I,t}$. In the model, the instantaneous covariance of $\beta_{j,t}$ and $Z_{I,t}$ satisfies

$$cov_t\left(d\beta_{j,t}; dZ_{I,t}\right) = \rho_{lev}\left(\frac{Q^{(1,2)} - Q^{(1,1)}\beta_{j,t}}{\sigma_{I,t}}\right). \tag{3.6}$$

Given that $Q^{(1,2)}$ and $Q^{(1,1)}$ should be positive in the model, the covariance of beta with aggregate shock depends on the relative level of $\beta_{j,t}$. More precisely, $cov_t\left(d\beta_{j,t}; dZ_{I,t}\right) < 0$ whenever $Q^{(1,2)}/Q^{(1,1)} > \beta_{j,t}$ when $\rho_{lev} < 0$. Empirically, we find that $Q^{(1,2)}/Q^{(1,1)}$ is close to 0.86. Thus, the beta of firms with relatively low beta tends to covary negatively with aggregate shocks $Z_{I,t}$. In contrast, the beta of firms with relatively large beta (i.e. $Q^{(1,2)}/Q^{(1,1)} < \beta_{j,t}$) tends to co-move positively with $Z_{I,t}$.

In our model, firm's beta risk is priced. More precisely, the instantaneous beta premium is given by

$$\begin{aligned} & E_t^P\left[d\beta_{j,t}\right] - E_t^Q\left[d\beta_{j,t}\right] \\ = \; & -cov_t\left(d\beta_{j,t}; \frac{d\zeta_t}{\zeta_t}\right) \\ = \; & -\left(\lambda_1^{\sigma_I} Q^{(1,1)} + \lambda_2^{\sigma_I} Q^{(2,1)}\right)\beta_{j,t} + \left(\lambda_1^{\sigma_I} Q^{(1,2)} + \lambda_2^{\sigma_I} Q^{(2,2)}\right). \end{aligned} \tag{3.7}$$

Thus, we see that the model predicts that the instantaneous beta premium has two components and is a linear function of spot beta. The sign and the magnitude of the premium depend on beta but also on the structural parameter. Arguably, the model is sufficiently rich to deliver a wide range of premium. As we discuss in more details in the next section, beta risk has important pricing implications for the cross-section of stock returns.

3.3 Return Premia

Expressions for return risk premia can be derived in the model. The following proposition is informative about market index and individual equity expected returns.

Proposition 2 *Given (2.9), (2.3), and (2.5), the h-day integrated market return premium at time*

t, which we denote $RP^I_{t,h}$*, satisfies*

$$RP^I_{t,h} \equiv E^P_t\left[\int_t^{t+h/252} \frac{dI_s}{I_s}\right] - E^Q_t\left[\int_t^{t+h/252} \frac{dI_s}{I_s}\right] = \Lambda^I \sigma^2_{I,t,h}, \tag{3.8}$$

where $\Lambda^I = \left(\sqrt{1-\rho^2_{lev}}\right)\lambda^{R_I} + \rho_{lev}\lambda_1^{\sigma_I}$ *and* $\sigma^2_{I,t,h} \equiv \left(\int_t^{t+h/252} E^P_t\left[\sigma^2_{I,s}\right] ds\right)$ *is the h-day expected integrated market variance under the physical measure. Moreover, equity j's h-day integrated expected return,* $RP^j_{t,h}$*, is given by*

$$\begin{aligned} RP^j_{t,h} &\equiv E^P_t\left[\int_t^{t+h/252} \frac{dS^j_s}{S^j_s}\right] - E^Q_t\left[\int_t^{t+h/252} \frac{dS^j_s}{S^j_s}\right] = \Lambda^I \sigma_{Ij,t,h} &(3.9)\\ &= \int_t^{t+h/252} E^P_t\left[\beta_{j,s}\frac{dI_s}{I_s}\right] - E^Q_t\left[\beta_{j,s}\frac{dI_s}{I_s}\right] - r\left(\beta_{j,t,h} - \tilde{\beta}_{j,t,h}\right). &(3.10)\end{aligned}$$

where $\sigma_{Ij,t,h} \equiv \int_t^{t+h/252} E^P_t\left[\sigma_{Ij,s}\right] ds$*,* $\beta_{j,t,h} \equiv \int_t^{t+h/252} E^P_t\left[\beta_{j,s}\right] ds$*, and* $\tilde{\beta}_{j,t,h} \equiv \int_t^{t+h/252} E^Q_t\left[\beta_{j,s}\right] ds$ *are the expected integrated physical covariance, and physical and risk-neutral betas, respectively.*

Proof. See Appendix C & D. ■

The previous proposition delivers important insights. As in univariate stochastic volatility models, the market premium is the product of the integrated market variance times Λ^I which reflects aggregate preference for return specific and variance risks. Similarly, we see from (3.9) that equity premium is Λ^I times the integrated covariance. Note that we discuss in details model's predictions for expected integrated market index and equity variances, covariance, and beta in section (3.5).

Proposition 2 provides an interesting decomposition of the equity return premium. From (3.10), we see that equity j's total return premium is composed of two parts. The first component corresponds to the difference between equity j's systematic risk exposures under P and Q. In addition to systematic risk premium, we see that stock j's beta risk premium impacts expected return. When beta is time-invariant, (3.10) simplifies to the standard unconditional CAPM equation $RP^j_{t,h} = \beta_j \times RP^I_{t,h}$.

To build intuition for why the beta risk premium matters, let us consider two firms, firm 1 and firm 2. Both firms have similar levels of physical beta on average but $\beta_{1,t}$ co-moves more positively with the SDF and negatively with market returns than $\beta_{2,t}$. Consequently, the beta of firm 1 is higher than firm 2's beta when market returns are low and the SDF is high. Because of the positive co-variance of its beta with the SDF, the beta premium for stock 1 is more negative than that of stock 2. As (3.7) suggests, we have $E^P_t\left[\beta_{1,s}\right] - E^Q_t\left[\beta_{1,s}\right] < E^P_t\left[\beta_{2,s}\right] - E^Q_t\left[\beta_{2,s}\right]$ for any $s > t$

whenever $cov_t\left(d\beta_{1,t};\frac{d\zeta_t}{\zeta_t}\right) > cov_t\left(d\beta_{2,t};\frac{d\zeta_t}{\zeta_t}\right)$ on average, which implies after integration that

$$-r\left(\beta_{1,t,h}-\tilde{\beta}_{1,t,h}\right) > -r\left(\beta_{2,t,h}-\tilde{\beta}_{2,t,h}\right).$$

Thus, the model predicts that the return premium of firms for which beta substantially increases during market turmoil (i.e. positively comoves with the SDF and negatively comoves with market returns) should incorporate a large and positive compensation for beta risk.

Ang, Chen, and Xing (2006) document that stocks that comove more with the market index when market index returns are low earn an additional positive premium relative to the standard CAPM. This is consistent with our model prediction. Other evidence in the literature suggests that beta risk matters to explain the cross-section of stock returns. For instance, Petkova and Zhang (2005) show that time variation in betas helps explain the value premium puzzle.

3.4 Option Prices

One key objective of this study is to combine the information in returns and options when estimating equity risk dynamics and premia. This is motivated by the findings in Chang, Christoffersen, Jacobs, and Vainberg (2011), or Buss and Vilkov (2012) who show that beta constructed from risk-neutral moments perform well in the cross-section of stock returns. Because options are priced under the risk-neutral measure, all processes must be risk-neutralized. The following proposition presents the risk-neutral dynamics of market index and equity returns and of the Wishart process.

Proposition 3 *Given (2.3), (2.5), and (2.9), the risk-neutral dynamics of the market index and stock return, and the variance-covariance matrix are*

$$\frac{dI_t}{I_t} = rdt+\sigma_{I,t}d\tilde{Z}_{I,t} \tag{3.11}$$

$$\frac{dS_t^j}{S_t^j} = rdt+\beta_{j,t}\left(\frac{dI_t}{I_t}-rdt\right)+\left(\sqrt{\sigma_{j,t}^2-\beta_{j,t}^2\sigma_{I,t}^2}\right)d\tilde{\varepsilon}_{j,t} \tag{3.12}$$

$$d\Sigma_t = \left(\gamma Q'Q-\tilde{K}\Sigma_t-\Sigma_t\tilde{K}'\right)dt+\sqrt{\Sigma_t}d\tilde{W}_tQ+\left(\sqrt{\Sigma_t}d\tilde{W}_tQ\right)', \tag{3.13}$$

where $d\tilde{Z}_{I,t}$, $d\tilde{\varepsilon}_{j,t}$, and $d\tilde{W}_t$ are risk-neutral Brownian motions, and $\tilde{K}$ is the risk-neutral mean reversion speed of Σ_t which is $\tilde{K}=K+Q'L_V'$ with $L_V\equiv\begin{bmatrix}\lambda_1^{\sigma_I} & \lambda_2^{\sigma_I}\\ 0 & 0\end{bmatrix}$.

Proof. See Appendix C. ■

As apparent from (3.13), the affine structure of the Wishart dynamics is preserved. This is especially attractive given that affine models allow for option pricing formulas.

Several studies argue that option prices are important in order to reliably estimate risk dynamics (see, among others, Pan 2002; Carr and Wu, 2009; Bollerslev, Todorov, and Xu, 2015). We now derive the model price for index and equity options. Given (2.3), (2.9), and (3.13), the price of a call written on the market index is

$$C_t^I(I_t, X, \tau) = I_t \left(\frac{1}{2} - \Pi_t^I \right), \tag{3.14}$$

and the price of a call written on the individual equity is

$$C_t^j(S_t^j, X, \tau) = S_t^j \left(\frac{1}{2} - \Pi_t^j \right), \tag{3.15}$$

where $\tau = T - t$ is the option time-to-maturity and X is the strike price. In the previous equations, the risk-neutral probabilities Π_t^I and Π_t^j are derived in Appendix E. Armed with a panel of index and equity options, these equations can be used to infer the dynamics of market index and individual equity risks.

3.5 Forecasting Market and Equity Risks

In the aftermath of the recent financial crisis, forecasting individual equity and market risks has become of particular interest to market participants. Furthermore, Proposition 2 also shows that measures of integrated beta is important for equity return premia. However, standard reduced-form factor models do not deliver forecast of beta. In contrast, our Wishart factor model can be used to obtain beta as well as equity and market risks predictions. In this regard, Proposition 4 provides some key results.

Proposition 4 *Conditional on time t, the h-day ahead expected integrated variance-covariance matrix is*

$$\Sigma_{t,h} = E_t^P \left[\int_t^{t+h/252} \Sigma_{t,s} ds \right] = \int_t^{t+h/252} \left(e^{-K(s-t)} \Sigma_t e^{-K'(s-t)} + \Gamma_{t,s} \right) ds, \tag{3.16}$$

where the expression for $\Gamma_{t,s}$ is given in Appendix F and is function of γ, Q, and K. Moreover, the annualized h-day ahead expected integrated beta under the physical measure is approximately

$$\beta_{j,t,h} = E_t^P \left[\int_t^{t+h/252} \beta_{j,s} ds \right] \approx \frac{\sigma_{jI,t,h}}{\theta_I} - \frac{\left(\sigma_{I,t,h}^2 - \theta_I \frac{h}{252} \right) \theta_{jI}}{\left(\theta_I \right)^2}, \tag{3.17}$$

where $\sigma^2_{I,t,h}$ *and* $\sigma_{jI,t,h}$ *denote the h-day ahead expected integrated market variance and covariance under the physical measure, respectively. They are given by* $\sigma^2_{I,t,h} = (\Sigma_{t,h})^{(1,1)}$ *and* $\sigma_{Ij,t,h} = (\Sigma_{t,h})^{(2,1)}$. *Based on these results, the annualized h-day ahead integrated variance-covariance matrix and beta are*

$$\Sigma^{ann.}_{t,h} = \Sigma_{t,h}\frac{252}{h} \quad \textit{and} \quad \beta^{ann.}_{j,t,h} = \beta_{j,t,h}\frac{252}{h}, \tag{3.18}$$

respectively.

Proof. See Appendix F. ■

Equation (3.16) can be used in order to obtain model's forecast about market and equity risks. Note that the results in Proposition 3 and 4 can be combined to calculate the model-based risk-neutral expectation of integrated variance-covariance matrix, $\tilde{\Sigma}_{t,h}$, by replacing K by $\tilde{K}$ in (3.16). Based on model physical and risk-neutral expectations, variance-covariance risk premia satisfy $\Sigma_{t,h} - \tilde{\Sigma}_{t,h}$.

Using (3.17) and (3.18), one can obtain model's forecast of future realized beta. By construction, beta is a non-linear function of the state variables. Thus, no exact analytical expression exists for its conditional expectation. Equation (3.17) approximates expected integrated future beta based on a first order Taylor expansion around the long-term variance-covariance means. While this expression is not exact, it closely approximates expected integrated future beta.[6] As suggested by Proposition 2, the difference between expected physical and risk-neutral beta (i.e. beta risk premium) influences the expected return on equity. Using risk-neutral expected integrated market variance and covariance measures combined with risk-neutral long-term means $\tilde{\theta}_I$ and $\tilde{\theta}_{jI}$ in (3.17), model's forecast for integrated beta under the risk-neutral measure can be obtained.

4 Estimation and Fit

Several approaches for estimating stochastic volatility models have been proposed in the literature. Jacquier, Polson, and Rossi (1994) use Markov Chain Monte Carlo to estimate a discrete-time stochastic volatility model. Pan (2002) uses GMM to estimate the objective and risk neutral parameters from returns and option prices. Christoffersen, Jacobs, and Mimouni (2010) adopt a particle filtering approach to estimate stochastic volatility models using returns and panels of option prices. Another approach treats the latent volatility states as parameters to be estimated and thus avoids filtering the latent volatility factor. This strategy has been adopted by Bates (2000), Santa-Clara and Yan (2010), and Christoffersen, Fournier, and Jacobs (2013) among others.

[6] We have verified by Monte-Carlo simulations that the forecasting error of formula (3.17) is negligible.

We follow a joint estimation approach in which market and equity variances and covariance dynamics are estimated jointly by maximizing the sum of returns and options likelihoods. In the estimation, the spot variances and covariance required to compute the likelihoods are filtered from index and equities returns only.

In this section we describe the data used and discuss the estimation methodology. Subsequently, we present parameter estimates.

4.1 Data

Our empirical analysis relies on two main datasets. We obtain daily return data from CRSP and end-of-day implied-volatility surfaces from OptionMetrics. The sample period starts on January 1, 1996 and ends on December 30, 2011. To assess model performance in the cross-section, it is important to include a sufficiently large number of firms. To this end, we obtain all the constituents of the S&P 500 index as of January 1, 1996 with option data available for the full sample. We filter out firms with major corporate event during the sample period. This ensures a reliable match between the return and option data.[7] Of the stocks remaining, we consider the first ninety three firms with the biggest market capitalization.

Due to the computational intensity when estimating the model, we apply a series of filters to reduce the size of the option sample. It is well-known that S&P500 index put options are more liquid than S&P500 calls. In contrast, calls are the most actively traded options for equities. Accordingly, we restrict our focus on S&P500 puts and equity calls when estimating the model. Arguably, the term structure and smile of implied-volatilities are informative about the underlying risks dynamics spanned by index and equities options. It is thus important to retain contracts that are informative about the term structure and smile when estimating the model. To capture the term structure, we retain at-the-money S&P500 index puts (i.e. -0.5 delta) and equity calls (i.e. 0.5 delta) with 30, 91, and 182 days-to-maturity. To obtain a reliable representation of the smile, we further include -0.25 and -0.75 delta puts for the index and 0.25 and 0.75 delta calls for equities with 91 days to maturity. Based on these filters, on any given day, five options spanning implied-volatilities across maturity and moneyness are retained for each asset. When estimating the model, we use a weekly discretization of the states from Wednesday close to Tuesday Close. Consequently, we restrain our analysis to Wednesday option data to avoid Friday maturity biases in implied-volatilities.

We obtain data on cash dividends from CRSP. In all our calculations, we assume that dividend payments are known during the life of the option. We thus subtract the present value of future

[7] Note that corporate events such as M&As or change of company name among others result in change of company identifier which makes the matching of CRSP with OptionMetrics data cumbersome.

dividends from the stock price when calculating model prices. Finally, interest rates are calculated by linear interpolation of the zero coupon yield curve provided by OptionMetrics.

The cross-section of equities that constitute our sample are split into ten industries. For convenience, we report on data and model based results by industry in the core paper, and relegate stock level results in the Online Appendix.

Table 1 presents some important descriptive statistics of the return and option data for the index and for each industry separately. The first four columns report the mean, variance, OLS beta, systematic risk, and correlation. To describe the option sample, we present the average of implied-volatilities across all contracts as well as for in- and out-of-the-money options. In the three last columns, we report on the average vega across all contracts, and the average moneyness (S/K) for $S/K > 1$ and $S/K < 1$ that correspond to absolute delta of 0.25 and 0.75. Note that OLS betas are obtained by regressing daily excess equity returns on daily excess S&P 500 returns over the full sample. Moreover, all statistics are annualized. Industry averages are obtained by first calculating each statistic at the firm level and then taking the average across all firms in a given industry. In the cross-section of industries, the betas vary from 0.57 for Utilities to 1.47 for Financials. Because our sample includes the financial crisis, Financials also have the largest sample variance. Systematic risk also varies widely across industries with a minimum of 20.16% for Utilities and a maximum of 42.51% for Financials.

On average, risk-neutral index skewness is usually more negative than that of individual equities. This is true in our sample as well. This is readily apparent by comparing the average implied volatility of out-of-the-money ($S/K > 1$) and in-the-money ($S/K < 1$) index puts. The implied volatility of out-of-the-money index puts exceeds the implied volatility of in-the-money puts by 3.42% which implies a negative skewness of market index risk-neutral distribution. Interestingly, equity implied-volatilities also display a smirk but somewhat less pronounced than that of the index. Across all industries, the implied volatility of in-the-money equity calls ($S/K > 1$) exceeds the implied volatility of out-of-the-money calls ($S/K < 1$) by 2.39% on average. The Telecommunication industry has the least pronounced smile while Financials have the most pronounced smile on average. For transparency, we report on firm-level statistics in Tables A.1.A to A.1.D and in Tables A.2.A to A.2.D in the Online Appendix.

4.2 Estimation Methodology

We adopt a pairwise estimation strategy in which we estimate return and variance-covariance dynamics of the index and individual equity j together. Thus, we need to estimate the set of structural parameters $\Theta_{jI} \equiv \{\gamma, K, Q, \rho_{lev}, \lambda_1^{\sigma_I}, \lambda_2^{\sigma_I}\}_{jI}$ along with the paths of the spot market variance

$\{\sigma^2_{I,t}(\Theta_{jI})\}$, spot equity variance $\{\sigma^2_{j,t}(\Theta_{jI})\}$, and the spot covariance $\{\sigma^2_{jI,t}(\Theta_{jI})\}$ for every pair $\{j, I\}_{j\in 1,...,93}$. Our estimation methodology builds on a weekly discretization of model dynamics. This substantially reduces the computational burden as it decreases the number of latent variables to filter by a factor of five compared with a daily discretization of the states. We compute Wednesday close to Tuesday close weekly log- returns from the daily observations and restrict our analysis to Wednesday options data.

When estimating underlying risks dynamics and their premia, it is important to combine returns with options data to obtain internally consistent P- and Q-dynamics. We thus adopt a joint estimation strategy.

The estimation procedure involves maximizing the sum of index and individual equity returns and options log-likelihoods. We denote these log-likelihoods by $\mathcal{L}^R(\cdot)$ and $\mathcal{L}^O(\cdot)$, where $\mathcal{L}^R$ is the bivariate index and equity return log-likelihood, and $\mathcal{L}^O$ corresponds to the sum of the index and equity options log-likelihoods.

For each pair of index and equity considered, the set of optimal structural parameters $\hat{\Theta}_{jI}$ solves the following maximization problem

$$\hat{\Theta}_{jI} = argmax(\mathcal{L}^R(\Theta^P_{jI}) + \mathcal{L}^O(\Theta^Q_{jI})), \tag{4.1}$$

where $\Theta^P_{jI} \equiv \{\gamma, K, Q, \rho_{lev}\}_{jI}$, $\Theta^Q_{jI} \equiv \{\gamma, \tilde{K}, Q, \rho_{lev}\}_{jI}$, and $\Theta_{jI} \equiv \{\Theta^P_{jI}, \Theta^Q_{jI}\}$

Arguably, one of the key challenges when estimating the model jointly is the filtering of the unobserved state variables which takes place within the optimization. We tackle this challenge by developing a bivariate Particle Filter (PF) algorithm that allows us to extract the spot Wishart variance-covariance matrix from weekly returns.

Specific details on the construction of $\mathcal{L}^R(\cdot)$ and $\mathcal{L}^O(\cdot)$ and on the PF algorithm can be found in Appendix G.

4.3 Parameter Estimates and Fit

This section presents estimation results for the market index and the 93 firms for the 1996-2011 period. As discussed in the previous section, we estimate the model pairwise for the market index and one firm at a time. This is not computationally efficient but it is required to ensure the positive definiteness of the filtered spot variance-covariance matrices. For ease of notation, we omit the subscript jI when referring to the structural parameters γ, K, Q, ρ_{lev}, $\lambda_1^{\sigma_I}$, and $\lambda_2^{\sigma_I}$ in what follows.[8]

[8]Recall that the set of parameters γ, K, Q, ρ_{lev}, $\lambda_1^{\sigma_I}$, and $\lambda_2^{\sigma_I}$ have been estimated 93 times in total (i.e. one set for each pair $\{j, I\}_{j\in 1,...,93}$).

In Panel A of Table 2 we report on the estimated structural parameters. For convenience, we report on the results by industry in this table, but relegate stock level results to Tables A.3.A to A.3.D in the Online Appendix. The estimated γ across industries display a low dispersion and ranges from 4.847 for Financials to 5.005 for Consumer Staples. In the model, $K^{(1,1)}$ captures the speed of mean reversion of the market variance while $Q^{(1,1)}$ and $Q^{(2,1)}$ are market variance volatility of volatility parameters. Note the way the estimates for $K^{(1,1)}$, $Q^{(1,1)}$, and $Q^{(2,1)}$ are very stable across industries. $K^{(1,1)}$ is 2.38 on average which corresponds to a mean reversion speed of 4.75 for the market index variance (i.e. 2×2.38). From the estimated $Q^{(1,1)}$, $Q^{(2,1)}$ and ρ_{lev}, we can infer the model implied market leverage, see (2.8) for more details. The market index leverage effect is -0.81 on average and ranges from -0.78 to -0.83. From the coefficients reported, we can also infer market volatility of volatility which is $2\sqrt{\left(Q^{(1,1)}\right)^2 + \left(Q^{(2,1)}\right)^2}$. It is 0.361 on average and ranges from 0.357 to 0.367. Arguably, the parameter restriction $K^{(1,2)} = 0$, which ensures that the market variance is conditionally independent of individual equity risks, allows us to obtain relatively stable market parameters across the 93 estimations.

Relative to the market parameter, the equity speed of mean reversion parameter is more disperse across industries. $K^{(2,2)}$ ranges from 1.40 for Financials to 1.79 for Utilities. These estimates translate to speeds of mean reversion of 2.8 and 3.58, respectively. This suggests that the individual equity variances are less persistent than market variance during the 1996-2011 period. Interestingly, $K^{(2,1)}$ varies substantially from one industry to another. The highest estimate of 0.03 is obtained for Industrials and the lowest estimate of -0.62 is obtained for Telecommunication firms. $K^{(2,1)}$ is -0.27 on average across all industries. Recall that this parameter captures the sensitivity of equity covariance to market variance. The negative estimate of -0.27 implies that the equity covariance tends to increase with market variance. This is interesting as very little is known on the way equity covariance tends to co-move with aggregate uncertainty. From Panel A, we can also infer the equity leverage and volatility of volatility. Equity leverage varies from -0.45 for Consumer Staples to -0.56 for Materials. This is substantially less in absolute value than the market leverage effect of -0.81. This is consistent with the stylized fact that the market index skewness is more negative than the skewness of individual equities on average. Equity volatility of volatility, which satisfies $2\sqrt{\left(Q^{(1,2)}\right)^2 + \left(Q^{(2,2)}\right)^2}$, ranges from 0.386 for Utilities to 0.604 for Financials. It is 0.48 on average which is much bigger than the market index volatility of volatility.

Panel B of Table 2 reports the unconditional second moments, their respective risk premiums, and measures of model fit. The market index long term P-variance, θ_I, is very stable across industries and is equal to 0.034 on average. Similarly, the unconditional market variance risk premium, $VRP_I \equiv \theta_I - \tilde{\theta}_I$, does not vary much across sectors and is -1.4% on average. This estimate compares well to other measures of unconditional market variance risk premium reported

in the literature. The long term variance of equities, θ_j, ranges from 0.052 for the Utilities to 0.179 for the Financials. Equity variance risk premiums, $VRP_j \equiv \theta_j - \tilde{\theta}_j$, are negative and their magnitudes vary widely. Interestingly, Financials obtain the most negative VRP_j while Consumer Staples and Utilities have variance risk premium close to zero. The third and fourth columns are informative about the dispersion of covariance and beta across industries. Comparing column four of Panel B in Table 2 with column three of Table 1, we see that the Wishart unconditional betas are similar in magnitude to the OLS sample betas. Very little is known about covariance and beta risk premia. In that regard, Panel B is informative about the size of these premia and their distribution across industries. $CRP_{Ij} \equiv \theta_{Ij} - \tilde{\theta}_{Ij}$ is negative for all industries and varies from -0.8% for Consumer Staples to -2.9% for Financials. Interestingly, beta risk premium, $BRP_{Ij} \equiv \theta_{Ij}/\theta_I - \tilde{\theta}_{Ij}/\tilde{\theta}_I$, has substantial cross-sectional variation and takes on positive and negative values. It is largest for Utilities and lowest for Financials. Several studies use option implied information to estimate stock betas.[9] When estimating stock betas from option prices, an important issue should be noted. As evident from our results, the beta premium can be substantial for certain industries such as Financial. Thus, one needs to be careful when interpreting the cross-sectional performance of betas constructed from risk-neutral moments.

Interestingly, firms with high unconditional variance are also the ones with large unconditional betas. As a result, the industry averages of θ_j and β_j in Panel B have a correlation coefficient of 94%.

Figure 1 presents the daily average, minimum, and maximum spot variances filtered by industry. We also plot the minimum, maximum, and average of the market variance paths filtered for the 93 estimations. While the estimation is done at a weekly frequency, we filters market and equity variances from daily returns using the structural parameters estimated. The impact of the tech bubble and of the recent financial crisis on spot variances is readily apparent. Comparing the minimum and maximum of equity spot variance, we see that equity variance displays substantial dispersion across firms within each industry. In contrast, the market variance paths are consistent across the 93 estimations as suggested by the overlap of the lines representing the minimum, maximum, and average.

Three important conclusions can be drawn from the results discussed in this section. First, market parameters and market variance paths are stable across the 93 estimations. Second, model

[9]French, Groth, and Kolari (1983) compute betas based on historical correlations and implied volatilities. Siegel (1995) proposes the creation of a derivative from which implied betas can be estimated. Chang et al. (2011) show that in a one-factor model, and under the assumption of zero skewness for the idiosyncratic returns, the implied betas can be estimated as the product of the ratio of equity to market implied skewness, and the ratio of equity to market implied volatilities. Buss and Vilkov (2012) use implied volatilities along with a parametric model for implied correlations to estimate betas.

implied leverage and volatility of volatility are consistent with well-known stylized facts related to the relative pricing of market index and equity options. Finally, the model delivers unconditional beta estimates that compare well to OLS unconditional betas. We now investigate in detail the properties of the Wishart betas in the cross-section.

5 Time-Varying Betas and Cross-Sectional Pricing

In this section, we provide an in-depth analysis of the pricing performance of the Wishart betas for the cross-section of stock returns. As a natural candidate, we use OLS betas as benchmark in all tests. We start by analyzing the ability of the Wishart model to predict future realized OLS betas. Subsequently, we compare the in-sample market return average to model based market premium estimates. We then study whether Wishart and OLS betas can predict subsequent equity returns and future systematic risk.

5.1 A Comparison of Wishart and OLS Betas

In this section we analyze the way Wishart betas compare to OLS betas. Recall that our estimation frequency is weekly. To obtain daily estimates of Wishart betas, we apply the PF algorithm on daily market index and equity returns. Armed with the daily spot variance-covariance matrices, we apply the results (3.17) and (3.18) in Proposition 4 and construct annualized one-month horizon (i.e. 21 trading days) Wishart forecasts for future realized betas for each firm. We denote them $\beta^W_{j,t,21}$. We also construct daily estimates of OLS betas. The common practice in the literature is to use ex-ante betas to proxy for future realized betas. To obtain OLS beta predictions for the $h = 21$ day horizon of future betas on day t, we regress the last h day daily excess equity return against the market index excess returns

$$R^j_s = \alpha_j + \beta_j \times R^I_s + \varepsilon_{j,s} \text{ for } s \in \{t-h+1, ..., t\}. \tag{5.1}$$

Accordingly, we define OLS model forecast for the h day future realized beta on day t as $\beta^{OLS}_{j,t,h} = \hat{\beta}_j$. Setting $h = 21$, we run the regression above on every day and for each firm to obtain the time-series of one-month OLS betas over the sample period. For any horizon, the h day ex-post realized beta on day t satisfies $\beta^{Real}_{j,t,h} = \beta^{OLS}_{j,t+h,h}$. Based on the one-month OLS beta estimates, the 21 day ex-post realized beta on day t is $\beta^{Real}_{j,t,21} = \beta^{OLS}_{j,t+21,21}$.

In the top panel of Figure 2, we scatter plot the sample average of the daily one-month OLS beta against the average of one-month Wishart betas for each firm. In the bottom panel, we scatter

plot the industry averages of the one-month OLS and Wishart betas. In both panels, the solid line corresponds to the regression fit obtained from regressing OLS betas against Wishart betas. The top panel uncovers the close relationship between OLS and Wishart betas. Unconditionally, we see that the models' betas are close. The coefficient obtained when regressing OLS unconditional betas on Wishart average betas is 1.01 and the regression R-squared is 89%. No particular outliers are apparent. However, we see that the models' estimates diverge more for high beta firms than for firms with low beta. Interestingly, firms that have large beta are also the one with high return variance. When averaged by industry in the bottom panel, we see that OLS and Wishart betas are also close to each other. The coefficient obtained when regressing the industry average of OLS betas on Wishart betas is 1.03 and the regression R-squared is 95%. Note, however, that the divergence of betas for the two models is the greatest for Financials and IT which are the most volatile industries as suggested by the results documented in Table 1 and 2. We conclude that the unconditional OLS and Wishart betas are close on average but diverge more for firms and industries that are more volatile.

In Figure 3, we plot the time-series of daily one-month OLS betas (grey) and Wishart betas (black) averaged by industries. Overall, the patterns in the two beta time-series are similar across industries. For instance, note the way OLS and Wishart betas substantially increase for IT during the tech bubble, and for Financials during the recent financial crisis. This is encouraging because it demonstrates the ability of the Wishart model to adequately capture large variations in equity risks during periods of high uncertainty. Sporadically, however, OLS and Wishart betas diverge substantially. Note the way OLS betas take on more extreme values than Wishart betas during the last quarter of 2008 and the first quarter of 2009. The results documented in Figure 2 and 3 suggest that OLS and Wishart betas share common characteristics but can also diverge both in the cross-section and in the time-series.

To investigate the informational content of Wishart betas, we regress one-month ex-post realized betas against one-month expected integrated Wishart betas controlling for one-month OLS betas. Table 3 presents the regression coefficient estimates, t-statistics, and R-squared averaged by industries. Note that Wishart beta forecasts are statistically significant for predicting future realized betas for all industries expected Industrials. Interestingly, Wishart betas obtain large coefficients and t-statistics for Financials and IT. Together, the results documented in Table 3 suggest that Wishart betas, which are filtered from one single realization of market and equity daily return conditional on estimated structural parameters, contain relevant information about future realized betas that goes beyond the informational content of ex-ante OLS betas.

We now investigate in more detail the pricing performance of Wishart betas.

5.2 Estimating Market Premium

Reliably estimating the market return premium is critical in many applications. Estimating the market premium is important for cost-of-equity measurement, asset allocation, and to assess the performance of actively managed portfolios, among others.

Table 4 presents the market premium implied by various measures of Wishart beta in Panel A, and OLS beta in Panel B. For a given model, we regress the cross-section of average daily equity returns (annualized) against the model beta estimates. We report on the regression slope, its t-statistics, as well as the regression R-squared. For the Wishart model, we consider four measures of beta that include the unconditional beta estimates from Table 2 (i.e. $\beta_j = \theta_{Ij}/\theta_I$), and the sample average of the daily one-, three-, and six-month expected integrated betas. For the OLS model, the unconditional beta is obtained by regressing daily excess firm return on daily excess market return over the full sample. In addition, we also consider the sample average of the daily one-, three-, and six-month OLS betas. On any day t, the OLS beta for a given horizon of h-day is obtained by regressing the last h-day excess equity returns against the market index excess returns. We refer to the previous section and equation (5.1) for more details. For all OLS estimations and Wishart beta calculations, we assume 21 trading days per month.

Comparing the slopes to the sample average of daily market return annualized in Panel A, we see that the Wishart model delivers consistent estimates of the market premium across the various measures of beta. The slopes range from 0.069 to 0.075 which is close to the sample average of market return of 0.0674. Strikingly, the market premium implied by Wishart unconditional betas is almost equal to the average of market returns.

Unlike Wishart betas, the ability of OLS betas to adequately capture information about the market premium varies across measures. The most accurate OLS estimate of market premium is obtained for the regression using unconditional OLS betas. However, the premium implied by unconditional OLS betas is not as close to the average annualized market daily returns as unconditional Wishart betas. When using average one-, three-, and six-month OLS betas, the estimated market premiums display a large downward bias.

The evidence presented in this section demonstrates that the cross-section of Wishart betas contain relevant information about the market premium. In addition, the dispersion of premia estimated for various Wishart beta measures is low compared to OLS-implied market premiums. Overall, the performance of the Wishart model is impressive.

5.3 Wishart Betas and Future Realized Equity Returns

Predicting stock returns constitutes the foundation of security picking and asset allocation. We now assess the performance of the Wishart betas to predict the cross-section of equity returns.

Based on daily excess stock returns, we construct daily measures of one-, three-, and six-month compounded realized returns. For consistency, we assume 21 trading days per month. We denote by $R^j_{t,h}$ the h-day ahead compounded excess return of equity j on day t. It is estimated using the daily excess returns R^j_t to R^j_{t+h-1}. To assess the models' predictive performance, we run cross-sectional Fama-MacBeth predictive regressions. On each day, we regress the cross-section of future realized excess equity returns on Wishart and OLS betas controlling for lagged returns,

$$R^j_{t+1,h} = b^0_{t,h} + b^W_{t,h} \times \beta^W_{j,t,h} + b^{OLS}_{t,h} \times \beta^{OLS}_{j,t,h} + Controls_t + \varepsilon_{t+1,h} \text{ for all } j, \tag{5.2}$$

to obtain daily estimates of the prices of risk. We consider three horizons of one-, three-, and six-month.

Table 5 presents the average of the coefficients, their t-statistics computed using the Newey-West approach, and the regression R-squared. We set the auto-correlation lags to the number of trading days considered for each horizon (i.e. 21, 63, and 126 lags, respectively). The R-squared obtained across horizons are high. They range from 18% to 22%. Comparing Wishart with OLS betas' coefficients reveal an interesting pattern. We see that the coefficients obtained for the Wishart betas have the expected positive sign while the coefficients obtained for OLS betas are small in magnitude, and take on positive and negative values. Note also that the statistical significance of the loadings on Wishart beta is much higher than the one of OLS beta coefficients. At three-month horizon, the coefficient of the Wishart beta is the most significant.

The Wishart coefficient estimates $b^W_{t,h}$ are economically meaningful. Multiplying the average of the $b^W_{t,h}$ by $252/h$ gives a model-based measure of the market return premium. The Wishart model market premium estimates are 6.61%, 7.14%, and 6.41%, respectively. These estimates compare well to the market return sample average of 6.74%.

5.4 Hedging Equity Systematic Risk

In light of the recent financial crisis, the measurement of financial assets' systematic risk has become critical to market participants. Portfolio, hedge fund, and risk managers need to appropriately forecast the systematic risk exposure of stocks. We now test the model ability to hedge equity systematic risk.

For each firm, we regress daily market-hedged equity excess returns on market realized excess

returns:

$$R^j_{t+1,h} - \beta^k_{j,t,h} \times R^I_{t+1,h} = c^k_{j,h} + d^k_{j,h} \times R^I_{t+1,h} + \varepsilon^k_{j,t+1,h}, \text{ for } k \in \{W, OLS\}. \tag{5.3}$$

Arguably, if the models adequately forecast future realized betas then $R^j_{t+1,h} - \beta^k_{j,t,h} \times R^I_{t+1,h}$ should not depend on market returns, and thus, $abs(d^k_{j,h})$ should be small. The larger $abs(d^k_{j,h})$ is the lower the hedging performance of the model is. To further assess model hedging performance, we also calculate the model beta root mean squared errors (i.e. BRMSE) across all firms for a given horizon. It is defined as

$$BRMSE^k_h = \sqrt{\frac{1}{N}\sum_j \left(d^k_{j,h}\right)^2}. \tag{5.4}$$

We first estimate the $d^k_{j,h}$ for each firm and each model based on (5.3). We then calculate the models' BRMSE, and industry average of the $abs(d^k_{j,h})$ and of $d^k_{j,h}$' s t-statistics. We report the results in Table 6. As in Table 5, we consider three horizons and use the Newey-West approach for computing t-statistics using h auto-correlation lags for a given horizon. As expected, the hedging performance of the Whishart and OLS betas decreases as the horizon increases. The Wishart model BRMSE is on average 1.4% lower than the OLS model BRMSE across horizons. Given an average of 22.50% for the OLS model, this represents a 6.22% decrease in BRMSE.

To investigate the performance of the model to hedge systematic risk during the financial crisis, we repeat the previous analysis for the turbulent 2007-2010 sample period. We report on the results obtained in Table 7. As expected, the performance of both models is lower during the 2007-2010 sample period. However, the Wishart model clearly outperform the OLS model. The Wishart BRMSE is 35% on average which is 6.7% less than the OLS BRMSE. Overall, this represent a 16% decrease in hedging error.

We conclude that the Wishart model performs well in hedging future equity systematic risk. This is particularly true during the financial crisis period.

6 The Dependence of Beta on News

Little is known about the way firm-specific and macro-economic news influence the cross-section of betas. As we discuss in more details in the next sections, the model has clear implications for the relation between beta and news. To test the model implications, we obtain macro- and firm-level news data from RavenPack Analytics (RPA) DowJones and Press Release files.[10] The database starts on January 3^{rd}, 2000 and includes one file with equity news and one file with global macro news. Firm-specific news stories reflect headlines such as news about earnings announcements, an-

[10] We are grateful to Gunnar Grass for providing us with this data.

alyst estimates, changes in credit rating, management changes, and M&A announcements among others. The macro-news data covers announcements about economic output, consumption, employment, foreign exchange and interest rates, credit, the balance of payments, production, and housing, among others.

RPA converts unstructured news into an event sentiment score (ESS) which reflects whether the news is positive or negative. ESS values vary between 0 and 100, with values below 50, equal to 50 and above 50 corresponding to negative, neutral, and positive news, respectively. To avoid lagged repeated news, we restrict our analysis to novel news stories with an RPA novelty score equal to 100. Furthermore, we only use news stories which are highly relevant as indicated by an RPA relevance score of 100. Finally, we discard all ex-post news that are written about market movements, as opposed to fundamental ex-ante news that (may) cause market reactions. Accordingly, we disregard firm-level news about trading imbalances, insider trading, technical trading signals, and trading halts, amongst others.

On each day, we obtain a single value for ESS by averaging the ESS scores of all news stories observed between the previous trading day's market close and the current trading day's close. News published during non-trading days are thus assigned to the next trading day. We do this for macro and firm-specific news separately.

Using the time-series of macro and firm-level ESS, we investigate the way betas depend on news arrival.

6.1 The Impact of Firm-Specific News

Patton and Verardo (2012) investigate whether stock betas vary with earning announcements using daily firm-level betas estimated from intraday prices. Our model has clear implications on the impact of firm-specific news on beta. Recall that equation (3.5) suggests that beta and firm-specific return shocks $d\varepsilon_{j,t}$ should comove negatively. To the extent that firm-specific news are model-free proxies of $d\varepsilon_{j,t}$, the model predicts that firm-level news should be contemporaneously negatively related with beta controlling for lagged beta.

To test this prediction, we run Fama-MacBeth cross-sectional regressions. On each day, we regress the models' betas/covariances against firm-specific news controlling for lagged betas/covariances. Firm-specific news is proxied by ESS divided by 1000. We report the average of the coefficients and their corresponding Newey-West t-statistics in Table 8. We consider three horizons of 5, 21, and 63 days, respectively. For each horizon and variable considered, we report the time-series average of the R-square. In all panels, the t-statistics correspond to Newey-West t-statistics computed using h auto correlation lags.

Table 8 uncovers an interesting pattern. Note that on average a daily cross-section of 46 stocks have news releases. This represents about 50% of the total cross-section in our sample. Independently of the model and horizon considered, firm-specific news negatively predict next period betas. This is consistent with the model prediction. Comparing the loadings on news in Panel A, B, and C, we see that there is a term structure of the coefficient. As expected, firm-level news have a much bigger impact on short term betas/covariances relative to long term betas/covariances. Unlike OLS betas, Wishart betas loads significantly on news. Recall that on any day, Wishart betas are filtered from one daily realization of equity and market index returns once model structural parameters have been estimated. Thus, Wishart betas capture much better the information in returns which explains their significant dependence on firm-level news. OLS betas are also negatively impacted by news but do not load significantly on them. Arguably, OLS betas that are estimated from rolling-windows are unable to appropriately capture the impact firm-level news have on stock returns.

We conclude that the model's prediction for the impact of firm-specific news on betas is consistent the data. Overall, the results documented suggests that Wishart betas, which are filtered from one daily return realization conditional on model structural parameters, are better able to capture relevant information of firm level news in comparison to OLS betas.

6.2 The Impact of Aggregate Uncertainty

To the best of our knowledge, no existing studies have investigated how betas co-move in response to macro uncertainty. Proposition 1 shows that the dynamics of beta are influenced by changes in covariance and market variance. Since changes in market variance and firm covariance are functions of $W_{I,t}^{(1,1)}$ and $W_{I,t}^{(1,2)}$, the model suggests that part of the co-movements in the cross-section of betas should be driven by aggregate variance risks. Moreover, we know from (3.6) that betas co-move with market index return risk, $Z_{I,t}$. We investigate now whether aggregate return and variance risks explain a substantial part of the co-movement in betas.

For a given horizon of h days, we run a principal component analysis on the daily Wishart integrated betas and extract the first three principal components (PC) driving the cross-section of the daily betas for the 93 firms in our sample. Based on the first three PCs extracted, we construct a beta PC factor by summing up the time-series of each of the first three components times the average of the firm loadings for that component. This allows us to obtain one single time-series of the factor capturing the co-movements driving the cross-section of integrated betas. We repeat this exercise for three horizons of 5, 21, and 63 days, respectively. We also repeat the previous analysis of the model's forecast of integrated covariances. This allows us to analyze whether the PC factor

in betas and covariances are impacted differently by aggregate uncertainty.

Table 9 presents the results obtained from regressing the PC factor of betas and covariances against macro news (i.e. macro announcement ESS divided by 1000) and VIX index. For comparison purposes, we also report the results obtained when considering the PC factor constructed from OLS betas and covariances. Each panel reports on the results for a given maturity. In all panels, the t-statistics correspond to Newey-West t-statistics computed using h auto correlated lags. As expected, co-movements in Wishart betas and covariances are largely driven by macro uncertainty. Across horizons, the R-squares are 77% for Wishart betas and 72% for Wishart covariances. The PC factor of betas and covariances negatively depends on macro news but positively loads on VIX. Overall, this result is intuitive. During market downturns, the cross-section of Wishart betas co-movements increase. Across horizons, the coefficient loading of the PC for covariances on the VIX index is 0.20 on average. From equation (3.3), we see that $K^{(2,1)}$ defines the dependence of firm covariance on market variance. In Table 2, the industry average of $K^{(2,1)}$ is 0.27. This is close to the regression-based loading of the covariance PC on the VIX index. Unlike Wishart betas, the PC of OLS betas are almost completely unrelated to aggregate uncertainty. The R-squares range from 0.51% to 1.58% which is surprisingly low. Except at the 5 day horizon, the PC of OLS betas does not load significantly on macro news and VIX. In contrast, the R-squares obtained for the PC of OLS covariances is high. In addition, the magnitude of the coefficient estimates is comparable with the one obtained for the PC of Wishart covariances. This makes the result obtained for OLS betas striking. It is well-known that OLS betas are usually estimated with noise. Because beta is the ratio of covariance to market variance, it is likely that when covariance and market variance estimates from returns are noisy, the ratio becomes even noisier. This perhaps explains the discrepancy between the results obtained for OLS betas in comparison with OLS covariances.

Overall, the results documented in this section uncover an important link between Wishart betas and macro uncertainty. Arguably, the ability of Wishart betas to capture aggregate uncertainty explains its pricing performance.

7 Summary and Conclusions

We study the implications of beta dynamics for the cross-section of stock returns. Building on recent advancement in statistics, we develop a CAPM model with stochastic beta. In the model, individual equity and market returns covary dynamically and their variance-covariance matrix follows a bivariate Wishart process which can be viewed as a multivariate extension of Heston (1993).

One of the key attribute of the model is that it can be used to filter spot betas from one return observation only and it allows for closed-form option pricing formulas. Based on spot betas, the

model can be used to forecast future integrated beta. This is another key advantage of the model in comparison to standard OLS beta estimations.

The model makes a series of predictions related to the dependence of beta on firm-specific and market innovations. The model also shows that part of the equity premium can be attributed to a premium for time-varying betas. This is consistent with Ang, Chen, and Xing (2006) and Petkova and Zhang (2005).

We develop a joint estimation methodology and fit the model by maximizing the return and option likelihoods of a large cross-section of stocks over sixteen years. Overall, the model fits the return and option data well. Unconditionally, the market premium estimated from the cross-section of average stock returns and model's betas is much closer to the sample average of market index returns than the market premium implied by OLS betas. Conditionally, the model betas substantially outperform OLS betas when predicting future realized returns and hedging future systematic risk. During the turbulent 2007-2009 period, the model forecast error decreases by more than sixteen percent on average relative to OLS betas. Consistent with the model, we find that betas are negatively impacted by firm-specific news. We also find that a substantial part of co-movement in betas is driven by aggregate uncertainty captured by macro-economic news and the VIX index.

Several issues are left for future research. First, it may be useful to extend the model, for instance by allowing for jumps in the market price (Bates, 2008; Bollerslev and Todorov, 2011; Kelly, Lustig, and Van Nieuwerburgh, 2016). Second, combining option information with high-frequency returns when estimating the parameters in our model, following Andersen, Fusari, and Todorov (2015) may lead to better estimates of the spot beta (see also Patton and Verardo, 2012; Bollerslev, Li, and Todorov, 2016). Third, we have focused on analyzing the implications of beta dynamics for stock returns, but additionally analyzing the implications of the model for option returns is of great interest as well. Finally, comparing Wishart betas with option-implied betas would also be interesting.

Appendix

This appendix collects proofs of some key mathematical results we refer to in the text or in the propositions.

A. The Physical Dynamics of $\sigma_{I,t}^2$, $\sigma_{jI,t}$, and $\sigma_{j,t}^2$

In (2.5), $\gamma Q^{'} Q = K\theta + \theta K'$ where θ is the long-term mean of Σ_t. We use this result and impose $K^{(1,2)} = 0$ to express the dynamics of $\sigma_{I,t}^2$, $\sigma_{jI,t}$, and $\sigma_{j,t}^2$. Given the dynamics of $d\Sigma_t$, the fact that $\gamma Q^{'} Q = K\theta + \theta K'$, and the form of $\sqrt{\Sigma_t}$, the dynamics of $\sigma_{I,t}^2$ is

$$d\sigma_{I,t}^2 = 2K^{(1,1)}(\theta_I - \sigma_{I,t}^2)dt + 2\sigma_{I,t}\left(Q^{(1,1)}dW_{I,t}^{(1,1)} + Q^{(2,1)}dW_{I,t}^{(1,2)}\right), \tag{7.1}$$

while individual equity total variance follows

$$\begin{aligned} d\sigma_{j,t}^2 &= \left(2K^{(2,2)}(\theta_j - \sigma_{j,t}^2) + 2K^{(2,1)}(\theta_{Ij} - \sigma_{jI,t})\right)dt \\ &\quad +2\beta_{j,t}\sigma_{I,t}\left(Q^{(1,2)}dW_{I,t}^{(1,1)} + Q^{(2,2)}dW_{I,t}^{(1,2)}\right) \\ &\quad +2\sqrt{\sigma_{j,t}^2 - \beta_{j,t}^2\sigma_{I,t}^2}\left(Q^{(2,2)}dW_{j,t}^{(2,2)} + Q^{(1,2)}dW_{j,t}^{(2,1)}\right), \end{aligned} \tag{7.2}$$

and the dynamics of the covariance of market index and individual equity returns is

$$\begin{aligned} d\sigma_{jI,t} &= \left(K^{(2,1)}(\theta_I - \sigma_{I,t}^2) + \left(K^{(2,2)} + K^{(1,1)}\right)(\theta_{Ij} - \sigma_{jI,t})\right)dt \\ &\quad + \left(\sigma_{I,t}Q^{(1,2)} + \beta_{j,t}\sigma_{I,t}Q^{(1,1)}\right)dW_{I,t}^{(1,1)} + \left(\sigma_{I,t}Q^{(2,2)} + \beta_{j,t}\sigma_{I,t}Q^{(2,1)}\right)dW_{I,t}^{(1,2)} \\ &\quad +\sqrt{\sigma_{j,t}^2 - \beta_{j,t}^2\sigma_{I,t}^2}\left(Q^{(1,1)}dW_{j,t}^{(2,1)} + Q^{(2,1)}dW_{j,t}^{(2,2)}\right). \end{aligned} \tag{7.3}$$

B. The Physical Dynamics of Beta

We now derive the dynamics of equity risk under the Physical measure. By definition, the stock beta satisfies $\beta_{j,t} = \sigma_{jI,t}/\sigma_{I,t}^2$. A straightforward application of Itô's lemma implies that

$$d\beta_{j,t} = \frac{1}{\sigma_{I,t}^2}d\sigma_{jI,t} - \frac{\sigma_{jI,t}}{\left(\sigma_{I,t}^2\right)^2}d\sigma_{I,t}^2 + \frac{\sigma_{jI,t}}{\left(\sigma_{I,t}^2\right)^3}d\left\langle\sigma_{I,t}^2;\sigma_{I,t}^2\right\rangle - \frac{1}{\left(\sigma_{I,t}^2\right)^2}d\left\langle\sigma_{I,t}^2;\sigma_{jI,t}\right\rangle, \tag{7.4}$$

where $d\langle\cdot;\cdot\rangle$ denotes the instantaneous covariance operator. From (7.1) and (7.3), we see that the quadratic variations $d\left\langle\sigma_{I,t}^2;\sigma_{I,t}^2\right\rangle$ and $d\left\langle\sigma_{I,t}^2;\sigma_{jI,t}\right\rangle$ satisfy

$$\begin{aligned} \frac{d\left\langle\sigma_{I,t}^2;\sigma_{I,t}^2\right\rangle}{dt} &= \sigma_{I,t}^2 \cdot \left(4\left(\left(Q^{(1,1)}\right)^2 + \left(Q^{(2,1)}\right)^2\right)\right) \\ &= \sigma_{I,t}^2 \cdot A \end{aligned} \tag{7.5}$$

where $A \equiv 4\left(\left(Q^{(1,1)}\right)^2 + \left(Q^{(2,1)}\right)^2\right)$, and

$$\begin{aligned}\frac{d\left\langle \sigma_{I,t}^2; \sigma_{jI,t}\right\rangle}{dt} &= \sigma_{I,t}^2 \cdot 2\left(Q^{(1,1)}Q^{(1,2)} + Q^{(2,1)}Q^{(2,2)}\right) + \sigma_{jI,t} \cdot 2\left(\left(Q^{(1,1)}\right)^2 + \left(Q^{(2,1)}\right)^2\right) \\ &= \sigma_{I,t}^2 \cdot B + \sigma_{jI,t} \cdot C \end{aligned} \tag{7.6}$$

where $B \equiv 2\left(Q^{(1,1)}Q^{(1,2)} + Q^{(2,1)}Q^{(2,2)}\right)$, and $C \equiv 2\left(\left(Q^{(1,1)}\right)^2 + \left(Q^{(2,1)}\right)^2\right)$, respectively. We can use these results to obtain

$$\begin{aligned} d\beta_{j,t} &= \frac{1}{\sigma_{I,t}^2}d\sigma_{jI,t} - \frac{\sigma_{jI,t}}{\left(\sigma_{I,t}^2\right)^2}d\sigma_{I,t}^2 + \frac{\sigma_{jI,t}}{\left(\sigma_{I,t}^2\right)^3}\sigma_{I,t}^2 A dt - \frac{1}{\left(\sigma_{I,t}^2\right)^2}\left(\sigma_{I,t}^2 \cdot B + \sigma_{jI,t} \cdot C\right)dt \\ &= \frac{1}{\sigma_{I,t}^2}d\sigma_{jI,t} - \frac{\sigma_{jI,t}}{\left(\sigma_{I,t}^2\right)^2}d\sigma_{I,t}^2 + \frac{\sigma_{jI,t}}{\left(\sigma_{I,t}^2\right)^2}A dt - \frac{1}{\sigma_{I,t}^2}B dt - \frac{\sigma_{jI,t}}{\left(\sigma_{I,t}^2\right)^2}C dt \\ &= \frac{1}{\sigma_{I,t}^2}\left(d\sigma_{jI,t} - B dt\right) - \frac{\sigma_{jI,t}}{\left(\sigma_{I,t}^2\right)^2}\left(d\sigma_{I,t}^2 - (C-A)\,dt\right) \\ &= \beta_{j,t}\left(\frac{\left(d\sigma_{jI,t} - \Phi_1 dt\right)}{\sigma_{jI,t}} - \frac{\left(d\sigma_{I,t}^2 - \Phi_2 dt\right)}{\sigma_{I,t}^2}\right), \end{aligned} \tag{7.7}$$

where $\Phi_1 \equiv 2\left(Q^{(1,1)}Q^{(1,2)} + Q^{(2,1)}Q^{(2,2)}\right)$ and $\Phi_2 \equiv -2\left(\left(Q^{(1,1)}\right)^2 + \left(Q^{(2,1)}\right)^2\right)$ which completes the proof.

C. Return and Variance-Covariance Risk-Neutral Dynamics

We now derive the return and variance-covariance risk-neutral dynamics. We make use of these results in Appendix D and Appendix E.

We obtain return and variance-covariance risk-neutral dynamics in two steps. First, we derive the model implication for the risk-neutralization of the Brownian motions driving the dynamics of the economy. Based on the risk-neutral shocks obtained, we subsequently risk-neutralize dI_t, dS_t^j, and $d\Sigma_t$.

We now derive the risk-neutralization of the Brownian motions $Z_{I,t}$, $\varepsilon_{j,t}$, and W_t consistent with the SDF ζ_t. To this end, let us define $L_R \equiv \begin{bmatrix} \lambda^{R_I} \\ 0 \end{bmatrix}$ and $L_V \equiv \begin{bmatrix} \lambda_1^{\sigma_I} & \lambda_2^{\sigma_I} \\ 0 & 0 \end{bmatrix}$. We can re-write the SDF using the following matrix notation

$$\frac{d\zeta_t}{\zeta_t} = -r dt - Tr\left[L_V' \sqrt{\Sigma_t} dW_t\right] - L_R' \sqrt{\Sigma_t} dB_t. \tag{7.8}$$

where $B_t \equiv [B_{I,t}\ B_{j,t}]'$ and $Tr[\cdot]$ is the trace operator. By application of the multivariate Girsanov theorem, we have

$$d\begin{bmatrix} \tilde{B}_{I,t} \\ \tilde{B}_{j,t} \end{bmatrix} = d\begin{bmatrix} B_{I,t} \\ B_{j,t} \end{bmatrix} + \sqrt{\Sigma_t}' L_R dt \tag{7.9}$$

$$d\begin{bmatrix} \tilde{W}_{I,t}^{(1,1)} & \tilde{W}_{I,t}^{(1,2)} \\ \tilde{W}_{j,t}^{(2,1)} & \tilde{W}_{j,t}^{(2,2)} \end{bmatrix} = d\begin{bmatrix} W_{I,t}^{(1,1)} & W_{I,t}^{(1,2)} \\ W_{j,t}^{(2,1)} & W_{j,t}^{(2,2)} \end{bmatrix} + \sqrt{\Sigma_t}' L_V dt, \tag{7.10}$$

where the tildes identify the risk-neutral Brownian motions. Note that the previous system is equivalent to

$$d\begin{bmatrix} \tilde{B}_{I,t} \\ \tilde{B}_{j,t} \end{bmatrix} = d\begin{bmatrix} B_{I,t} \\ B_{j,t} \end{bmatrix} + \sigma_{I,t}\begin{bmatrix} \lambda^{R_I} \\ 0 \end{bmatrix} dt$$

$$d\begin{bmatrix} \tilde{W}_{I,t}^{(1,1)} & \tilde{W}_{I,t}^{(1,2)} \\ \tilde{W}_{j,t}^{(2,1)} & \tilde{W}_{j,t}^{(2,2)} \end{bmatrix} = d\begin{bmatrix} W_{I,t}^{(1,1)} & W_{I,t}^{(1,2)} \\ W_{j,t}^{(2,1)} & W_{j,t}^{(2,2)} \end{bmatrix} + \sigma_{I,t}\begin{bmatrix} \lambda_1^{\sigma_I} & \lambda_2^{\sigma_I} \\ 0 & 0 \end{bmatrix} dt \tag{7.11}$$

Combining these results with the leverage effect decomposition of $dZ_{I,t}$, and $d\varepsilon_{j,t}$, we can infer the risk-neutral expression for the return shock dynamics

$$\begin{aligned} d\begin{bmatrix} \tilde{Z}_{I,t} \\ \tilde{\varepsilon}_{j,t} \end{bmatrix} &= \left(\sqrt{1-\rho_{lev}^2}\right) d\begin{bmatrix} \tilde{B}_{I,t} \\ \tilde{B}_{j,t} \end{bmatrix} + \rho_{lev} d\begin{bmatrix} \tilde{W}_{I,t}^{(1,1)} & \tilde{W}_{I,t}^{(1,2)} \\ \tilde{W}_{j,t}^{(2,1)} & \tilde{W}_{j,t}^{(2,2)} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &= \left(\sqrt{1-\rho_{lev}^2}\right) d\begin{bmatrix} B_{I,t} \\ B_{j,t} \end{bmatrix} + \left(\sqrt{1-\rho_{lev}^2}\right)\sqrt{\Sigma_t}' L_R dt \\ &\quad + \rho_{lev}\left(d\begin{bmatrix} W_{I,t}^{(1,1)} & W_{I,t}^{(1,2)} \\ W_{j,t}^{(2,1)} & W_{j,t}^{(2,2)} \end{bmatrix} + \sqrt{\Sigma_t}' L_V dt\right)\begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &= d\begin{bmatrix} Z_{I,t} \\ \varepsilon_{j,t} \end{bmatrix} + \sqrt{\Sigma_t}'\left(\left(\sqrt{1-\rho_{lev}^2}\right) L_R + \rho_{lev} L_V \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) dt \end{aligned} \tag{7.12}$$

Combining (7.11) and (7.12) implies that

$$\begin{aligned} d\tilde{Z}_{I,t} &= dZ_{I,t} + \sigma_{I,t}(\sqrt{1-\rho_{lev}^2}\lambda^{R_I} + \rho_{lev}\lambda_1^{\sigma_I}) \\ d\tilde{\varepsilon}_{j,t} &= d\varepsilon_{j,t} \\ d\tilde{W}_{I,t}^{(1,1)} &= dW_{I,t}^{(1,1)} + \sigma_{I,t}\lambda_1^{\sigma_I} dt \\ d\tilde{W}_{I,t}^{(1,2)} &= dW_{I,t}^{(1,2)} + \sigma_{I,t}\lambda_2^{\sigma_I} dt \\ d\tilde{W}_{j,t}^{(2,1)} &= dW_{j,t}^{(2,1)} \\ d\tilde{W}_{j,t}^{(2,2)} &= dW_{j,t}^{(2,2)}. \end{aligned} \tag{7.13}$$

Absence of arbitrage opportunities imposes for the market index that $I_t = E_t^P\left[\frac{\zeta_T}{\zeta_t} I_T\right]$, which implies

that the market index instantaneous return premium is

$$\mu_{I,t} - r = \sigma_{I,t}^2(\sqrt{1-\rho_{lev}^2}\lambda^{R_I} + \rho_{lev}\lambda_1^{\sigma_I}), \tag{7.14}$$

and thus the risk-neutral dynamics for market index return is given by

$$\frac{dI_t}{I_t} = rdt + \sigma_{I,t}d\tilde{Z}_{I,t}. \tag{7.15}$$

Given (2.2), (7.15), and the fact that $d\tilde{\varepsilon}_{j,t} = d\varepsilon_{j,t}$, the risk-neutral dynamics of individual equity follows

$$\frac{dS_t^j}{S_t^j} - rdt = \beta_{j,t}\left(\frac{dI_t}{I_t} - rdt\right) + \left(\sqrt{\sigma_{j,t}^2 - \beta_{j,t}^2\sigma_{I,t}^2}\right)d\tilde{\varepsilon}_{j,t}. \tag{7.16}$$

The Wishart dynamics is also influenced by the change of measure. Given the physical dynamics of the Wishart process

$$d\Sigma_t = \left(\gamma\Omega - K\Sigma_t - \Sigma_t K^{'}\right)dt + \sqrt{\Sigma_t}dW_tQ + Q'dW_t'\sqrt{\Sigma_t}^{'},$$

and the risk-neutralization (7.10) (i.e. $d\tilde{W}_t = dW_t + \sqrt{\Sigma_t}^{'}L_V$ with $L_V \equiv \begin{bmatrix} \lambda_1^{\sigma_I} & \lambda_2^{\sigma_I} \\ 0 & 0 \end{bmatrix}$), we have under the risk-neutral measure

$$\begin{aligned} d\Sigma_t &= \left(\gamma Q^{'}Q - K\Sigma_t - \Sigma_t K^{'}\right)dt + \sqrt{\Sigma_t}\left(d\tilde{W}_t - \sqrt{\Sigma_t}^{'}L_V dt\right)Q \\ &\quad + Q'\left(d\tilde{W}_t - \sqrt{\Sigma_t}^{'}L_V dt\right)^{'}\sqrt{\Sigma_t}^{'} \end{aligned}$$

$$\begin{aligned} \Leftrightarrow\ & d\Sigma_t = \left(\gamma Q^{'}Q - K\Sigma_t - \Sigma_t K^{'}\right)dt + \sqrt{\Sigma_t}d\tilde{W}_tQ - \Sigma_t L_V Qdt \\ & + Q'd\tilde{W}_t'\sqrt{\Sigma_t}^{'} - Q'L_V'\Sigma_t dt \end{aligned}$$

$$\Leftrightarrow d\Sigma_t = \left(\gamma Q^{'}Q - \tilde{K}\Sigma_t - \Sigma_t\tilde{K}'\right)dt + \sqrt{\Sigma_t}d\tilde{W}_tQ + Q'd\tilde{W}_t'\sqrt{\Sigma_t}^{'}, \tag{7.17}$$

where

$$\tilde{K} = K + Q'L_V' = K + \begin{bmatrix} \lambda_1^{\sigma_I}Q^{(1,1)} + \lambda_2^{\sigma_I}Q^{(2,1)} & 0 \\ \lambda_1^{\sigma_I}Q^{(1,2)} + \lambda_2^{\sigma_I}Q^{(2,2)} & 0 \end{bmatrix}.$$

Together, equations (7.15), (7.16), and (7.17) define the joint risk-neutral dynamics of market index and equity returns and variance-covariance.

D. Market Index and Individual Equity Return Premia

We now derive integrated return premia from t to $t + h/252$. From equations (2.1), (7.14), and (7.15), we have

$$\begin{aligned} E_t^P\left[\int_t^{t+h/252}\frac{dI_s}{I_s}\right] - E_t^Q\left[\int_t^{t+h/252}\frac{dI_s}{I_s}\right] &= E_t^P\left[\int_t^{t+h/252}\mu_{I,s}ds\right] - E_t^Q\left[\int_t^{t+h/252} rds\right] \\ &= E_t^P\left[\int_t^{t+h/252}\left(\mu_{I,s}-r\right)ds\right] \\ &= \Lambda^I\int_t^{t+h/252} E_t^P\left[\sigma_{I,s}^2\right]ds \\ &= \Lambda^I\sigma_{I,t,h}^2, \end{aligned} \tag{7.18}$$

where $\Lambda^I = \left(\sqrt{1-\rho_{lev}^2}\right)\lambda^{R_I} + \rho_{lev}\lambda_1^{\sigma_I}$ and $\sigma_{I,t,h}^2 \equiv \int_t^{t+h/252} E_t^P\left[\sigma_{I,s}^2\right]ds$ is the h-day expected integrated market variance under the P-measure. The factor structure of individual equity return (2.2) combined with (7.16) implies

$$\begin{aligned} E_t^P\left[\int_t^{t+h/252}\frac{dS_s^j}{S_s^j}\right] - E_t^Q\left[\int_t^{t+h/252}\frac{dS_s^j}{S_s^j}\right] &= E_t^P\left[\int_t^{t+h/252} rds + \beta_{j,s}\left(\frac{dI_s}{I_s} - rds\right)\right] \\ &\quad -E_t^Q\left[\int_t^{t+h/252} rds + \beta_{j,s}\left(\frac{dI_s}{I_s} - rds\right)\right] \\ &= E_t^P\left[\int_t^{t+h/252}\beta_{j,s}\frac{dI_s}{I_s}\right] - E_t^Q\left[\int_t^{t+h/252}\beta_{j,s}\frac{dI_s}{I_s}\right] \\ &\quad -r\int_t^{t+h/252}\left(E_t^P\left[\beta_{j,s}\right] - E_t^Q\left[\beta_{j,s}\right]\right)ds. \\ &= E_t^P\left[\int_t^{t+h/252}\beta_{j,s}\frac{dI_s}{I_s}\right] - E_t^Q\left[\int_t^{t+h/252}\beta_{j,s}\frac{dI_s}{I_s}\right] \\ &\quad -r\left(\beta_{j,t,h} - \tilde{\beta}_{j,t,h}\right). \end{aligned} \tag{7.19}$$

where $\beta_{j,t,h} \equiv \int_t^{t+h/252} E_t^P\left[\beta_{j,s}\right]ds$ and $\tilde{\beta}_{j,t,h} \equiv \int_t^{t+h/252} E_t^Q\left[\beta_{j,s}\right]ds$ are the h-day expected integrated physical and risk-neutral betas, respectively. Because the individual equity instantaneous

expected P-return is equal to $\mu_{j,s} = r + \beta_{j,s} \cdot \left(\mu_{I,s} - r\right)$, we must also have

$$\begin{aligned} E_t^P\left[\int_t^{t+h/252} \frac{dS_s^j}{S_s^j}\right] - E_t^Q\left[\int_t^{t+h/252} \frac{dS_s^j}{S_s^j}\right] &= E_t^P\left[\int_t^{t+h/252} \mu_{j,s} ds\right] - E_t^Q\left[\int_t^{t+h/252} r ds\right] \\ &= E_t^P\left[\int_t^{t+h/252} \beta_{j,s} \cdot \left(\mu_{I,s} - r\right) ds\right] \\ &= E_t^P\left[\int_t^{t+h/252} \beta_{j,s} \cdot \sigma_{I,s}^2(\sqrt{1-\rho_{lev}^2}\lambda^{R_I} + \rho_{lev}\lambda_1^{\sigma_I}) ds\right] \\ &= \Lambda^I \int_t^{t+h/252} E_t^P\left[\sigma_{jI,s}\right] ds \\ &= \Lambda^I \sigma_{jI,t,h}. \end{aligned} \tag{7.20}$$

where we used the fact that $\mu_{I,s} - r = \sigma_{I,s}^2(\sqrt{1-\rho_{lev}^2}\lambda^{R_I} + \rho_{lev}\lambda_1^{\sigma_I})$ and $\sigma_{jI,t,h} \equiv \int_t^{t+h/252} E_t^P\left[\sigma_{jI,s}\right] ds$ is the h-day expected integrated covariance under the P-measure.

E. Index and Individual Equity Option Prices

For ease of notation, we define the integrated Brownian $\tilde{Z}_{\Sigma,t,\tau} \equiv \int_t^{t+\tau} \sqrt{\Sigma_t} d \begin{bmatrix} \tilde{Z}_{I,u} \\ \tilde{\varepsilon}_{j,u} \end{bmatrix}$ and the integrated variance-covariance matrix $\Sigma_{t,\tau}^{Int} \equiv \int_t^{t+\tau} \Sigma_u du$ where $\tau = T - t$. Given the Q-dynamics in Appendix C for dI_t and dS_t^j, one can apply Itô's lemma to $\ln(P_t)$ where $P_t \equiv \begin{bmatrix} I_t & S_t^j \end{bmatrix}'$ and obtain (after integration) the following expression for log-returns

$$\ln\left(P_T\right) - \ln\left(P_t\right) = r\mathbf{1}\tau - \frac{1}{2} diag\left(\Sigma_{t,\tau}^{Int}\right) + \tilde{Z}_{\Sigma,t,\tau}, \tag{7.21}$$

where $\mathbf{1}$ is a 2×1 vector of ones. Therefore, the conditional characteristic function of the risk-neutral log-returns takes the form

$$\begin{aligned} \tilde{\phi}_t^{LR}\left(\tau, u_I, u_j\right) &= E_t^Q\left[\exp\left(iu'\left(\ln\left(P_T\right) - \ln\left(P_t\right)\right)\right)\right] \\ &= E_t^Q\left[\exp\left(iu'\left(r\mathbf{1}\tau - \frac{1}{2} diag\left(\Sigma_{t,\tau}^{Int}\right) + \tilde{Z}_{\Sigma,t,\tau}\right)\right)\right], \end{aligned} \tag{7.22}$$

where $u \equiv \begin{bmatrix} u_I & u_j \end{bmatrix}'$ is a 2×1 vector. Let us introduce the stochastic exponential $\xi(\cdot)$ defined by

$$\xi\left(\eta'\tilde{Z}_{\Sigma,t,\tau}\right) = \exp\left(\eta'\tilde{Z}_{\Sigma,t,\tau} - \frac{1}{2}\eta'\left\langle \tilde{Z}_{\Sigma,t,\tau}, \tilde{Z}'_{\Sigma,t,\tau}\right\rangle \eta\right) = \exp\left(\eta'\tilde{Z}_{\Sigma,t,\tau} - \frac{1}{2}\eta'\Sigma_{t,\tau}^{Int}\eta\right). \tag{7.23}$$

Then, we can write (7.22) as

$$\begin{aligned}\tilde{\phi}_t^{LR}(\tau, u_I, u_j) &= \exp(iu'r\mathbf{1}\tau)\cdot E_t^Q\left[\xi\left(iu'\tilde{Z}_{\Sigma,t,\tau}\right)\exp\left(\frac{iu'\Sigma_{t,\tau}^{Int}iu}{2}\right)\exp\left(-\frac{iu'diag\left(\Sigma_{t,\tau}^{Int}\right)}{2}\right)\right] \\ &= \exp(iu'r\mathbf{1}\tau)\cdot E_t^Q\left[\xi\left(iu'\tilde{Z}_{\Sigma,t,\tau}\right)\exp\left(\frac{iu'\left(\Sigma_{t,\tau}^{Int}iu - diag\left(\Sigma_{t,\tau}^{Int}\right)\right)}{2}\right)\right]. \qquad (7.24)\end{aligned}$$

We can define the following change-of-measure

$$\frac{dC}{dQ}(t) \equiv \xi\left(iu'\tilde{Z}_{\Sigma,0,t}\right). \qquad (7.25)$$

Combining (7.24) with the change of measure (7.25), we can write

$$\tilde{\phi}_t^{LR}(\tau, u_I, u_j) = \exp(iu'r\mathbf{1}\tau)E_t^Q\left[\frac{\frac{dC}{dQ}(T)}{\frac{dC}{dQ}(t)}\exp\left(\frac{iu'\left(\Sigma_{t,\tau}^{Int}iu - diag\left(\Sigma_{t,\tau}^{Int}\right)\right)}{2}\right)\right]$$

$$\Rightarrow \tilde{\phi}_t^{LR}(\tau, u_I, u_j) = \exp(iu'r\mathbf{1}\tau)E_t^C\left[\exp\left(\frac{iu'\left(\Sigma_{t,\tau}^{Int}iu - diag\left(\Sigma_{t,\tau}^{Int}\right)\right)}{2}\right)\right].$$

Because

$$\frac{iu'\left(\Sigma_{t,\tau}^{Int}iu - diag\left(\Sigma_{t,\tau}^{Int}\right)\right)}{2} = Tr\left[\Gamma\left(u_I, u_j\right)\Sigma_{t,\tau}^{Int}\right]$$

where

$$\Gamma\left(u_I, u_j\right) \equiv \frac{1}{2}\times\begin{bmatrix} -\left(u_I\right)^2 - iu_I & -u_Iu_j \\ -u_Iu_j & -\left(u_j\right)^2 - iu_j \end{bmatrix},$$

we have

$$\tilde{\phi}_t^{LR}(\tau, u_I, u_j) = \exp(iu'r\mathbf{1}\tau)E_t^C\left[\exp\left(Tr\left[\Gamma\left(u_I, u_j\right)\cdot\Sigma_{t,\tau}^{Int}\right]\right)\right]. \qquad (7.26)$$

While $\Gamma(\cdot,\cdot)$ is function of u_I and u_j, we drop the two input arguments in the rest of the proof for ease of notation. Thus, we now refer to it as Γ. An extension of the multivariate Girsanov theorem to the complex plane implies that under the C-measure, we have

$$dZ_t^C = d\tilde{Z}_t - i\sqrt{\Sigma_t}'udt,$$

where $\tilde{Z}_t \equiv \left[\tilde{Z}_{I,t}\ \tilde{\varepsilon}_{j,t}\right]'$ and

$$dW_t^C = d\tilde{W}_t - i\rho_{lev}\sqrt{\Sigma_t}'\left[u\ \mathbf{0}\right]dt,$$

where $\mathbf{0}$ is a 2×1 vector of zeros. We can now infer the Wishart dynamic under the new measure which satisfies

$$d\Sigma_t = \left(\gamma Q^{'} Q - \tilde{K}\Sigma_t - \Sigma_t \tilde{K}^{'}\right) dt + \sqrt{\Sigma_t} d\tilde{W}_t Q + Q' d\tilde{W}_t' \sqrt{\Sigma_t}^{'}$$

$$\begin{aligned} \Leftrightarrow \quad d\Sigma_t &= \left(\gamma Q^{'} Q - \tilde{K}\Sigma_t - \Sigma_t \tilde{K}^{'}\right) dt + \sqrt{\Sigma_t} \left(d\tilde{W}_t^C + i\rho_{lev} \sqrt{\Sigma_t}^{'} \left[u\ \mathbf{0}\right] dt\right) Q \\ &+ Q' \left(d\tilde{W}_t^C + i\rho_{lev} \sqrt{\Sigma_t}^{'} \left[u\ \mathbf{0}\right] dt\right)^{'} \sqrt{\Sigma_t}^{'} \end{aligned}$$

$$\begin{aligned} \Leftrightarrow \quad d\Sigma_t &= \left(\gamma Q^{'} Q - \tilde{K}\Sigma_t - \Sigma_t \tilde{K}^{'}\right) dt + \sqrt{\Sigma_t} d\tilde{W}_t^C Q + i\rho_{lev} \Sigma_t \left[u\ \mathbf{0}\right] Q dt \\ &+ Q' d\tilde{W}_t^{C'} \sqrt{\Sigma_t}^{'} + i\rho_{lev} Q' \left[u\ \mathbf{0}\right]^{'} \Sigma_t dt \end{aligned}$$

$$\Leftrightarrow d\Sigma_t = \left(\gamma Q^{'} Q - K^C \Sigma_t - \Sigma_t K^{C\prime}\right) dt + \sqrt{\Sigma_t} d\tilde{W}_t^C Q + Q' d\tilde{W}_t^{C'} \sqrt{\Sigma_t}, \tag{7.27}$$

where

$$K^C = \tilde{K} - i\rho_{lev} Q' \left[u\ \mathbf{0}\right]^{'}.$$

We can now make use of the closed-form solution for the moment generating function of $E_t^C[\exp\left(Tr\left[\Gamma \cdot \Sigma_{t,\tau}^{Int}\right]\right)$ to obtain the following expression for $\tilde{\phi}_t^{LR}(\cdot)$,

$$\tilde{\phi}_t^{LR}(\tau, u_I, u_j) = \exp\left(Tr\left[A(\tau) \cdot \Sigma_t\right] + B(\tau)\right), \tag{7.28}$$

with

$$A(\tau) = \left(a^{22}(\tau)\right)^{-1} \cdot \left(a^{21}(\tau)\right) \tag{7.29}$$

where

$$\begin{pmatrix} a^{11}(\tau) & a^{12}(\tau) \\ a^{21}(\tau) & a^{22}(\tau) \end{pmatrix} = \exp\left(\tau \begin{bmatrix} -K^C & -2Q^{'}Q \\ \Gamma & K^{C'} \end{bmatrix}\right)$$

and

$$B(\tau) = -\frac{\gamma}{2} Tr\left[\log\left(a^{22}(\tau)\right) - \tau K^{C^{'}} + i\tau r u^{'} \mathbf{1}\right] \tag{7.30}$$

where

$$\begin{aligned} u &= \left[u_I\ u_j\right]' \\ K^C &= \tilde{K} - i\rho_{lev} Q' \left[u\ \mathbf{0}\right]^{'}, \end{aligned}$$

and

$$\Gamma = \frac{1}{2} \times \begin{bmatrix} -(u_I)^2 - iu_I & -u_I u_j \\ -u_I u_j & -(u_j)^2 - iu_j \end{bmatrix}.$$

Given the characteristic function above the price of a call written on the market index with strike price X is

$$C_t^I(I_t, X, \tau) = I_t \left(\frac{1}{2} - \Pi_t^I \right), \tag{7.31}$$

and the price of a call written on the individual equity is

$$C_t^j(S_t^j, X, \tau) = S_t^j \left(\frac{1}{2} - \Pi_t^j \right), \tag{7.32}$$

where the risk-neutral probabilities Π_t^I and Π_t^j are defined by

$$\Pi_t^I = \frac{e^{-r\tau}}{2\pi} \int_0^\infty \mathrm{Re} \left[\frac{e^{-iu_I \ln X/I_t} \tilde{\phi}_t^{LR}(\tau, u_I - i, 0)}{(u_I)^2 - iu_I} - \frac{e^{iu_I \ln X/I_t} \tilde{\phi}_t^{LR}(\tau, -u_I - i, 0)}{(u_I)^2 + iu_I} \right] du_I \tag{7.33}$$

$$\Pi_t^j = \frac{e^{-r\tau}}{2\pi} \int_0^\infty \mathrm{Re} \left[\frac{e^{-iu_j \ln X/S_t^j} \tilde{\phi}_t^{LR}(\tau, 0, u_j - i)}{(u_j)^2 - iu_j} - \frac{e^{iu_j \ln X/S_t^j} \tilde{\phi}_t^{LR}(\tau, 0, -u_j - i)}{(u_j)^2 + iu_j} \right] du_j. \tag{7.34}$$

F. Term Structure of Risks

We start by deriving the model's prediction for the expected integrated variance-covariance matrix. We then apply this result to find an expression for expected integrated future beta.

An application of Itô's Lemma to $e^{Kt}\Sigma_t e^{tK'}$ where Σ_t follows (2.5) implies

$$\begin{aligned} d\left(e^{Kt}\Sigma_t e^{tK'}\right) &= \left(e^{Kt}K\Sigma_t e^{tK'} + e^{Kt}\Sigma_t K' e^{tK'}\right)dt + e^{Kt}d\Sigma_t e^{tK'} \\ &= \gamma e^{Kt}Q'Qe^{tK'}dt + e^{Kt}\left(\sqrt{\Sigma_t}dW_t Q + Q'dW_t'\sqrt{\Sigma_t}'\right)e^{tK'}. \end{aligned} \tag{7.35}$$

Integrating both sides of the previous equation from t to s with $s > t$ gives

$$e^{Ks}\Sigma_{t,s}e^{sK'} - e^{Kt}\Sigma_t e^{tK'} = \gamma \int_t^s e^{Ku}Q'Qe^{uK'}du + \int_t^s e^{Ku}\left(\sqrt{V_u}dW_u Q + Q'dW_u'\sqrt{V_u}\right)e^{uK'}du, \tag{7.36}$$

which implies that

$$E_t^P\left[e^{Ks}\Sigma_{t,s}e^{sK'}\right] = e^{Kt}\Sigma_t e^{tK'} + \gamma \int_t^s e^{Ku}Q'Qe^{uK'}du. \tag{7.37}$$

Multiplying by e^{-Ks} from the left and $e^{-sK'}$ from the right, we get

$$E_t^P\left[\Sigma_{t,s}\right] = e^{-Ks}\left(e^{Kt}\Sigma_t e^{tK'} + \gamma\int_t^s e^{Ku}Q'Qe^{uK'}du\right)e^{-sK'}ds = e^{-K(s-t)}\Sigma_t e^{-K'(s-t)} + \Gamma_{t,s}, \quad (7.38)$$

where $\Gamma_{t,s} \equiv \gamma\int_t^s e^{-K(s-u)}Q'Qe^{-K'(s-u)}du$. To obtain the model's h-day expected integrated variance-covariance matrix, we need to integrate the previous expression over h-day. This gives

$$\Sigma_{t,h} = \int_t^{t+h/252} E_t^P\left[\Sigma_{t,s}\right]ds = \int_t^{t+h/252}\left(e^{-K(s-t)}\Sigma_t e^{-K'(s-t)} + \Gamma_{t,s}\right)ds. \quad (7.39)$$

We now derive an approximation formula for the h-day ahead model forecast of future realized beta: $\beta_{j,t,h} = \left(\int_t^{t+h/252} E_t^P\left[\beta_{j,s}\right]ds\right)$. A first order Taylor-expansion of spot beta around θ_{jI}/θ_I leads to

$$\beta_{j,s} = \frac{\sigma_{jI,s}}{\sigma_{I,s}^2} = \frac{\theta_{jI}}{\theta_I} + \frac{\left(\sigma_{jI,s} - \theta_{jI}\right)}{\theta_I} - \frac{\left(\sigma_{I,s}^2 - \theta_I\right)\theta_{jI}}{\left(\theta_I\right)^2} + O(2). \quad (7.40)$$

Taking the expectation and ignoring the errors of order greater than two, we get

$$E_t^P\left[\beta_{j,s}\right] \approx \frac{\theta_{jI}}{\theta_I} + \frac{\left(E_t^P\left[\sigma_{jI,s}\right] - \theta_{jI}\right)}{\theta_I} - \frac{\left(E_t^P\left[\sigma_{I,s}^2\right] - \theta_I\right)\theta_{jI}}{\left(\theta_I\right)^2} \quad (7.41)$$

$$\approx \frac{E_t^P\left[\sigma_{jI,s}\right]}{\theta_I} - \frac{\left(E_t^P\left[\sigma_{I,s}^2\right] - \theta_I\right)\theta_{jI}}{\left(\theta_I\right)^2}. \quad (7.42)$$

Integrating the previous expression from t to $t + h/252$, we obtain

$$\beta_{j,t,h} \approx \frac{\sigma_{jI,t,h}}{\theta_I} - \frac{\left(\sigma_{I,t,h}^2 - \theta_I\frac{h}{252}\right)\theta_{jI}}{\left(\theta_I\right)^2} \quad (7.43)$$

where $\sigma_{I,t,h}^2 = \int_t^{t+h/252} E_t^P\left[\sigma_{I,s}^2\right]ds = \left(\Sigma_{t,h}\right)^{(1,1)}$ and $\sigma_{jI,t,h} = \int_t^{t+h/252} E_t^P\left[\sigma_{jI,s}\right]ds = \left(\Sigma_{t,h}\right)^{(2,1)}$.

G. Return and Option Log-Likelihoods

Applying Itô's lemma and Euler's discretization to equation (2.3) leads to

$$\begin{bmatrix} \log I_{t+\Delta t} \\ \log S_{t+\Delta t}^j \end{bmatrix} = \begin{bmatrix} \log I_t \\ \log S_t^j \end{bmatrix} + \begin{bmatrix} \mu_{I,t} - \frac{1}{2}\sigma_{I,t}^2 \\ \mu_{j,t} - \frac{1}{2}\sigma_{j,t}^2 \end{bmatrix}\Delta t + \sqrt{\Sigma_{jI,t}}\sqrt{\Delta t}\begin{bmatrix} Z_{t+\Delta t} \\ \varepsilon_{t+\Delta t}^j \end{bmatrix}, \quad (7.44)$$

where Δt is the discretization horizon. We can also discretize the Wishart dynamics using a full truncation Euler scheme

$$\begin{aligned}\Sigma_{jI,t+\Delta t} = \Sigma^{+}_{jI,t} + (\gamma_{jI} Q'_{jI} Q_{jI} - K_{jI}\Sigma^{+}_{jI,t} + \Sigma^{+}_{jI,t} K'_{jI})\Delta t \\ + \sqrt{\Sigma^{+}_{jI,t}} W_{jI,t+\Delta t} Q_{jI} \sqrt{\Delta t} + \left(\sqrt{\Sigma^{+}_{jI,t}} W_{jI,t+\Delta t} Q_{jI} \sqrt{\Delta t}\right)'\end{aligned} \tag{7.45}$$

where $\Sigma^{+}_{jI,t} = U_{jI,t} \times abs(L_{jI,t}) \times U^{T}_{jI,t}$ for which the matrix of eigenvalues $L_{jI,t}$ satisfies $\Sigma_{jI,t} = U_{jI,t} \times L_{jI,t} \times U^{T}_{jI,t}$. Note that in (7.45), $\sqrt{\Sigma^{+}_{jI,t}}$ corresponds to the Cholesky decomposition of $\Sigma^{+}_{jI,t}$ such that

$$\sqrt{\Sigma^{+}_{jI,t}} = \begin{bmatrix} \sqrt{\left(\Sigma^{+}_{jI,t}\right)^{(1,1)}} & 0 \\ \frac{\left(\Sigma^{+}_{jI,t}\right)^{(2,1)}}{\sqrt{\left(\Sigma^{+}_{jI,t}\right)^{(1,1)}}} & \frac{\sqrt{\left(\Sigma^{+}_{jI,t}\right)^{(1,1)}\left(\Sigma^{+}_{jI,t}\right)^{(2,2)} - \left(\left(\Sigma^{+}_{jI,t}\right)^{(2,1)}\right)^{2}}}{\sqrt{\left(\Sigma^{+}_{jI,t}\right)^{(1,1)}}} \end{bmatrix}. \tag{7.46}$$

Together, equations (7.44), (7.45), and (7.46) define the discretized dynamics of index and equity returns and of the variance-covariance matrix. We make use of this in what follows. We divide the filtering and computation of the log-likelihoods in six steps that we now discuss in details.

Step 1 - Sampling: We start the algorithm at time $t = 0$ by setting all the particles $\Sigma^{i}_{jI,0} = \theta_{jI}$ $\forall i \in \{1, 2, ..., N\}$ where θ_{jI} is the long term variance-covariance matrix, i identifies a given particle, and N denotes the total number of particles. Note that at this stage, we set $\mu_{I,t}$ and $\mu_{j,t}$ to their sample average $\hat{\mu}_I$ and $\hat{\mu}_j$.

The algorithm for filtering of the particles at time $t+1$ given time t smoothly resampled particles, $\Sigma^{i}_{jI,t}$, is as follows. From the smoothly resampled particles, we need to simulate forward a new set of particles $\tilde{\Sigma}^{i}_{jI,t+\Delta t}$. To this end, we first filter the vector of shocks to returns based on the bivariate measurement equation

$$\begin{bmatrix} Z^{i}_{t+\Delta t} \\ \varepsilon^{j}_{t+\Delta t} \end{bmatrix} = (\sqrt{\Sigma^{i}_{jI,t}}\sqrt{\Delta t})^{-1} \cdot \left(\begin{bmatrix} \log \frac{I_{t+\Delta t}}{I_t} \\ \log \frac{S_{t+\Delta t}}{S_t} \end{bmatrix} - \begin{bmatrix} \hat{\mu}_I - \frac{1}{2}\left(\sigma^{i,2}_{I,t}\right) \\ \hat{\mu}_j - \frac{1}{2}\left(\sigma^{i,2}_{j,t}\right) \end{bmatrix} \Delta t \right). \tag{7.47}$$

Armed with the shocks $Z^{i}_{t+\Delta t}$ and $\varepsilon^{j}_{t+\Delta t}$, we can generate the matrix of variance shocks $W^{i}_{jI,t+\Delta t}$ consistent with the desired leverage (2.7) as follows

$$W^{i}_{jI,t+\Delta t} = \begin{bmatrix} W^{(1,1)}_{i,I,t+\Delta t} & W^{(1,2)}_{i,I,t+\Delta t} \\ W^{(2,1)}_{i,j,t+\Delta t} & W^{(2,2)}_{i,j,t+\Delta t} \end{bmatrix} = \begin{bmatrix} \rho_{jI,lev} Z^{i}_{t+\Delta t} + \sqrt{1-\rho^{2}_{jI,lev}} B^{(1,1)}_{i,I,t+\Delta t} & B^{(1,2)}_{i,I,t+\Delta t} \\ \rho_{jI,lev} \varepsilon^{j}_{t+\Delta t} + \sqrt{1-\rho^{2}_{jI,lev}} B^{(2,1)}_{i,j,t+\Delta t} & B^{(2,2)}_{i,j,t+\Delta t} \end{bmatrix}, \tag{7.48}$$

where $\rho_{jI,lev}$ is the leverage parameter, and $B^{(1,1)}_{i,I,t+\Delta t}$, $B^{(1,2)}_{i,I,t+\Delta t}$, $B^{(2,1)}_{i,j,t+\Delta t}$, and $B^{(2,2)}_{i,j,t+\Delta t}$ are *i.i.d.*

standard Normal random variables. Note that in this step, we impose the shocks $B^{(1,1)}_{i,I,t+\Delta t}$ and $B^{(1,2)}_{i,I,t+\Delta t}$ to be the same across all equities while $B^{(2,1)}_{i,j,t+\Delta t}$, and $B^{(2,2)}_{i,j,t+\Delta t}$ change from one equity to another. This is important to ensure the consistency of the market variance path across equities.

Given the shocks to variance $W^i_{jI,t+\Delta t}$, we simulate the N particles forward according to

$$\begin{aligned}\tilde{\Sigma}^i_{jI,t+\Delta t} = \Sigma^{i,+}_{jI,t} + (\gamma_{jI}Q'_{jI}Q_{jI} - K_{jI}\Sigma^{i,+}_{jI,t} + \Sigma^{i,+}_{jI,t}K'_{jI})\Delta t \\ + \sqrt{\Sigma^{i,+}_{jI,t}}W^i_{jI,t+\Delta t}Q_{jI}\sqrt{\Delta t} + \left(\sqrt{\Sigma^{i,+}_{jI,t}}W^i_{jI,t+\Delta t}Q_{jI}\sqrt{\Delta t}\right)'. \end{aligned} \tag{7.49}$$

We then ensure positive definiteness by calculating $\tilde{\Sigma}^{i,+}_{jI,t+\Delta t}$ for each particle $\tilde{\Sigma}^i_{jI,t+\Delta t}$.

Step 2 - Importance Sampling: With the N particles $\tilde{\Sigma}^{i,+}_{jI,t+\Delta t}$ in hand, we can infer the probability of occurrence $\tilde{w}^i_{jI,t+\Delta t}$ of each particle from next period log-returns (i.e. the likelihood that next period log-returns at time $t+2\Delta t$ are generated by the particle $\tilde{\Sigma}^{i,+}_{jI,t+\Delta t}$). Conditional on $\tilde{\Sigma}^{i,+}_{jI,t+\Delta t}$, the index and equity log-returns admit a bivariate Normal density. We make use of this to obtain $\tilde{w}^i_{jI,t+\Delta t}$

$$\begin{aligned}\tilde{w}^i_{jI,t+\Delta t} = \frac{1}{\sqrt{(2\pi)^2|\tilde{\Sigma}^{i,+}_{jI,t+\Delta t}|}}\times \\ \exp\left(-\frac{1}{2}\begin{bmatrix}\log(\frac{I_{t+2\Delta t}}{I_{t+\Delta t}}) - \mu_I\Delta t \\ \log(\frac{S_{t+2\Delta t}}{S_{t+\Delta t}} - \mu_j\Delta t)\end{bmatrix}'\left(\tilde{\Sigma}^{i,+}_{jI,t+\Delta t}\right)^{-1}\begin{bmatrix}\log(\frac{I_{t+2\Delta t}}{I_{t+\Delta t}}) - \mu_I\Delta t \\ \log(\frac{S_{t+2\Delta t}}{S_{t+\Delta t}} - \mu_j\Delta t)\end{bmatrix}\right). \end{aligned} \tag{7.50}$$

Based on the $\tilde{w}^i_{t+\Delta t}$, we can obtain the probability of each particle $\breve{w}^i_{t+\Delta t}$ by normalization

$$\breve{w}^i_{jI,t+\Delta t} = \frac{\tilde{w}^i_{jI,t+\Delta t}}{\sum_{i=1}^N \tilde{w}^i_{jI,t+\Delta t}}. \tag{7.51}$$

Step 3 - Resampling: Since we want to maximize the likelihood function as a function of the structural parameters, we apply the algorithm of Malik and Pitt (2011) and resample smoothly $\Sigma^i_{jI,t+\Delta t}$ from the set of particles $\tilde{\Sigma}^{i,+}_{jI,t+\Delta t}$. This ensures that the log-likelihood function is a smooth function of parameters.

Step 4 - Filtered path and returns log-likelihood: We repeat Step 1, 2, and 3 over each week in the sample to obtain $\left\{\Sigma^i_{jI,t}, \tilde{w}^i_{jI,t}\right\}_{t=1,2,\ldots,T}$. Based on the particles and their probability of occurrence, the log-likelihood function is calculated as

$$\mathcal{L}^R(\Theta^P_{jI}) \equiv \sum_{t=1}^T \log\left(\frac{1}{N}\sum_{i=1}^N \tilde{w}^i_{jI,t}(\Theta^P_{jI})\right) \tag{7.52}$$

where $\Theta_{jI}^P = \{\gamma, K, Q, \rho_{lev}\}_{jI}$ are the parameters driving the P-dynamics of the Wishart process. Moreover, the filtered spot variance-covariance matrix at any time t is the average of the smoothly resampled particles:

$$\hat{\Sigma}_{jI,t} = \frac{1}{N}\sum_{i=1}^{N} \Sigma_{jI,t}^i. \tag{7.53}$$

Step 5 - Index and equity options log-likelihoods: Given $\left\{\hat{\Sigma}_{jI,t}\right\}_{t=1,2,\ldots,T}$ and given the risk-neutralization (3.13), the model can be used for pricing index and equity options given a set of parameters $\Theta_{jI}^Q = \{\gamma, \tilde{K}, Q, \rho_{lev}\}_{jI}$. Based on the prices obtained, the model implied volatility errors on any day t for the option contract i written on asset k are calculated as

$$\eta_{i,t}^k = BSIV_{i,t}^{M,k} - BSIV_{i,t}^k(\Theta_{jI}^Q, \hat{\Sigma}_{jI,t}),\ i = 1, \ldots, 5 \tag{7.54}$$

where $k \in \{I, j\}$, $BSIV_{i,t}^{M,k}$ denotes the quoted implied volatility, and $BSIV_{i,t}^k(\Theta_{jI}^Q, \hat{\Sigma}_{jI,t})$ is the implied volatility given the model price. Note that $BSIV$ denotes the Black-Scholes implied volatility.

Recall that we observe five contracts for each asset at any point in time. To calculate the options log-likelihood for asset $k \in \{I, j\}$, we define the 1×5 vector of time t implied-volatility errors $\eta_t^k = [\eta_{1,t}^k \eta_{2,t}^k \eta_{3,t}^k \eta_{4,t}^k \eta_{5,t}^k]$ from which we construct the implied volatility moment conditions matrix $\eta^k = \begin{bmatrix} \eta_1^k \\ \vdots \\ \eta_T^k \end{bmatrix}$. Based on η^k, we calculate the sample variance $\hat{V}^k = Var(\eta^k)$ of the five moment conditions and calculate the options log-likelihood for asset k as

$$\mathcal{L}_k^O = \sum_{t=1}^{T} \log\left(\frac{1}{\sqrt{(2\pi)^2|\hat{V}^k|}} \times \exp\left(-\frac{1}{2}\eta_t^k \left(\hat{V}^k\right)^{-1} \eta_t^{k'}\right)\right). \tag{7.55}$$

The last step consists of summing the index and equity options log-likelihoods to get

$$\mathcal{L}^O(\Theta_{jI}^Q) \equiv \mathcal{L}_I^O + \mathcal{L}_j^O. \tag{7.56}$$

Step 6 - Summing the log-likelihoods: The set of structural parameters can be obtained as the solution to the maximization problem

$$\Theta_{jI} = argmax(\mathcal{L}^R(\Theta_{jI}^P) + \mathcal{L}^O(\Theta_{jI}^Q)). \tag{7.57}$$

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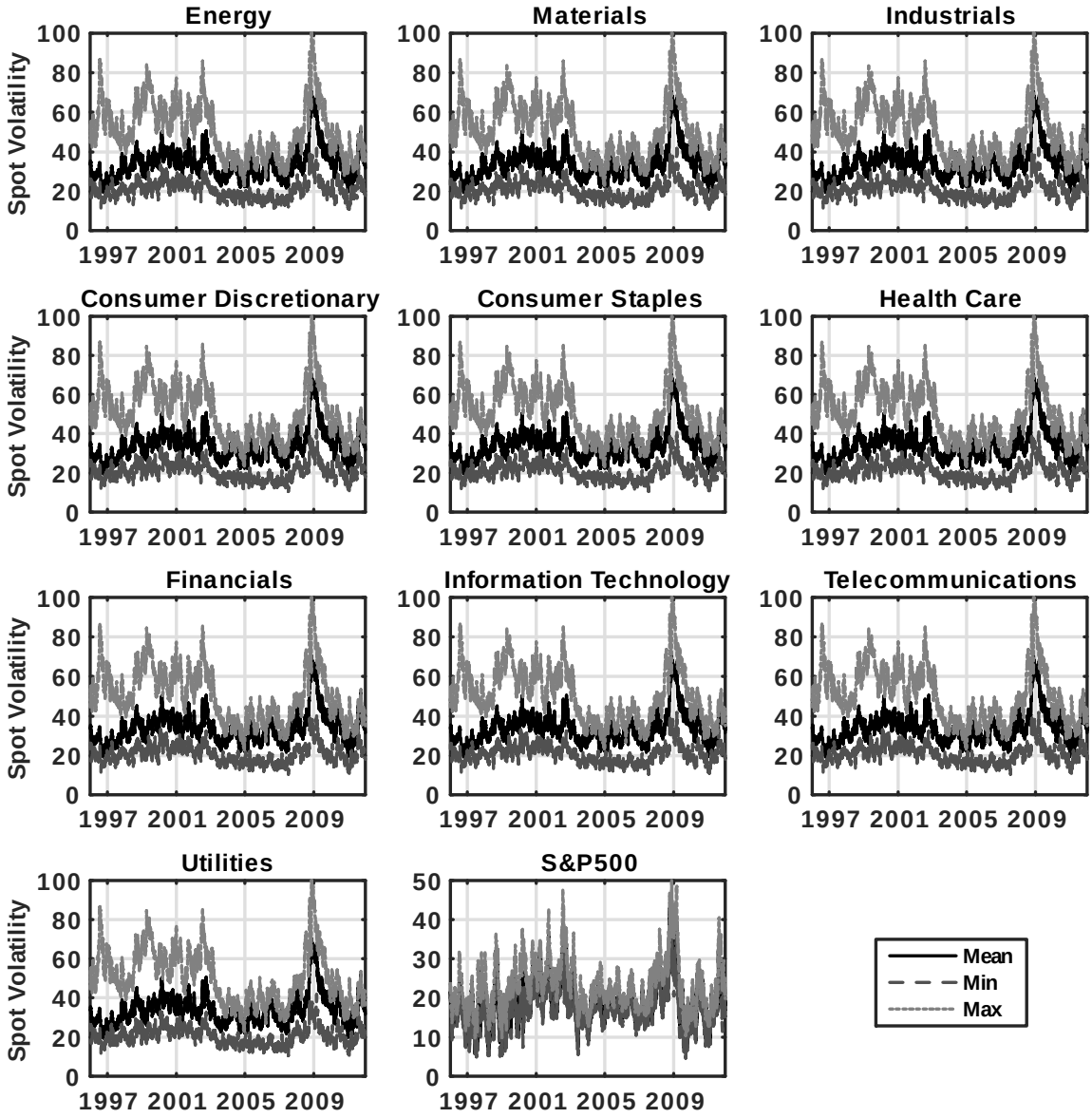
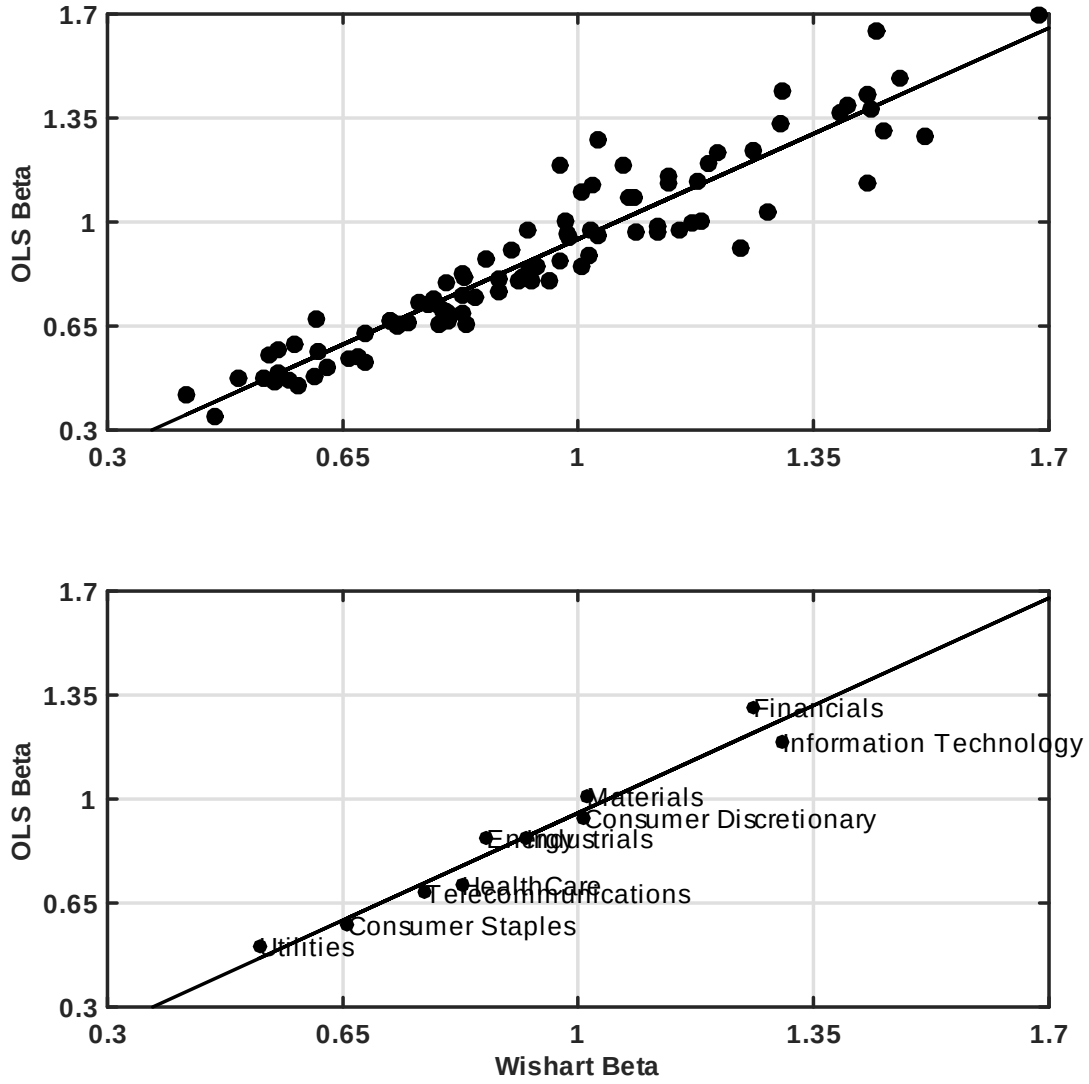
Figure 1: Minimum, Maximum, and Average Spot Volatility, 1996-2011



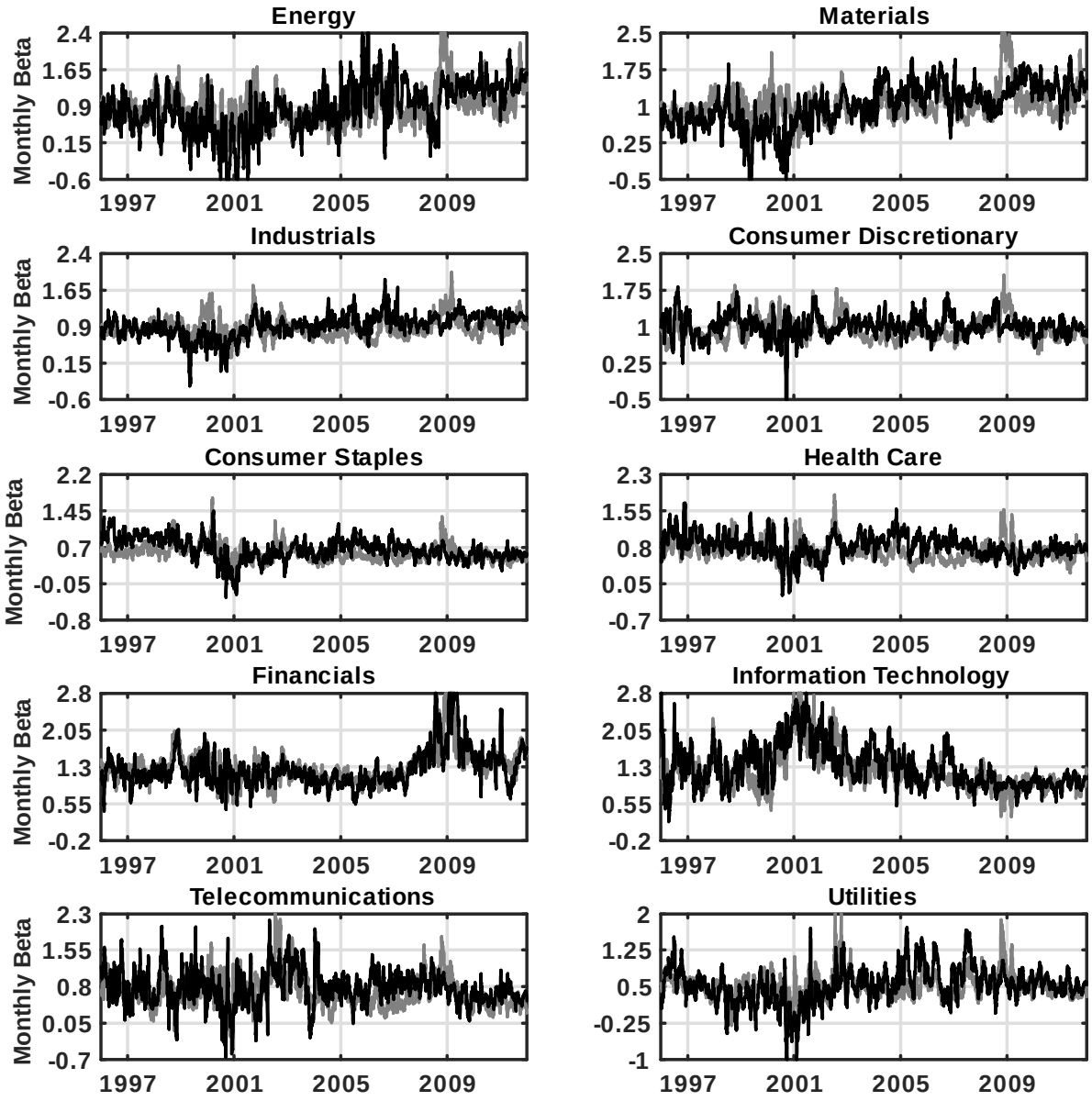
Notes to Figure: We plot the minimum, maximum, and average daily spot volatilities by industry and for the market index. To obtain daily spot market and equity variance estimates, we fix model parameters to their optimal values and apply PF on daily index and equity returns.

Figure 2: One-month OLS Beta against One-month Expected Integrated Wishart Beta, 1996-2011



Notes to Figure: In the top panel, we scatter plot the average of the daily one-month (i.e. 21 trading days) OLS betas against the average of the daily one-month expected integrated Wishart betas for the 93 firms. In the bottom panel, we average firm level betas by industry and scatter the plot industry averages of OLS betas against the Wishart expected betas.

Figure 3: Time-Series of One-month OLS Beta (grey) and One-month Expected Integrated Wishart Beta (black), 1996-2011



Notes to Figure: We plot the industry averages of the one-month OLS and Wishart expected integrated betas over time.

Table 1: Return and Option Data Summary Statistics, 1996-2011

	Daily Return Statistics Annualized					Wednesday Option Data Statistics					
						Implied Volatility (%)				Moneyness (S/K)	
Index / Industry	Mean (%)	Variance (%)	OLS Beta	OLS Syst. Risk (%)	Corre-lation	All	S/K>1	S/K<1	Vega	S/K>1	S/K<1
S&P 500	6.74	4.38	1	100	1	20.64	24.05	19.78	207.2	1.07	0.97
Energy	19.61	14.51	0.94	29.26	0.54	34.00	36.21	33.45	10.8	1.10	0.95
Materials	16.34	14.40	1.04	35.80	0.59	33.71	36.16	33.09	9.6	1.10	0.95
Industrials	13.59	10.70	0.91	35.56	0.59	29.87	32.03	29.32	10.7	1.09	0.96
Cons. Discretionary	19.44	14.68	1.02	32.01	0.56	34.67	37.07	34.06	7.6	1.10	0.95
Cons. Staples	13.71	7.77	0.62	22.23	0.47	25.75	27.82	25.23	9.4	1.08	0.96
Health Care	17.04	12.73	0.77	23.75	0.48	32.45	34.95	31.83	9.5	1.09	0.95
Financials	16.33	23.36	1.47	42.92	0.65	35.17	38.35	34.37	9.8	1.10	0.95
I. T.	23.13	20.81	1.21	32.96	0.57	39.73	42.51	39.04	9.5	1.12	0.94
Telecom.	9.34	8.54	0.76	30.00	0.55	26.77	28.68	26.26	7.4	1.07	0.96
Utilities	11.98	6.25	0.53	20.16	0.45	22.50	24.77	21.93	7.3	1.06	0.97

Note to Table: The table reports summary statistics of return and option data. We calculate the average, variance, and various OLS statistics from daily stock and index returns. OLS beta, systematic risk and correlation are obtained by regressing stock daily excess returns on the S&P500 excess returns. Note that we report the average of stock level statistics by industry. On each Wednesday, we collect -0.50 delta SPX put option with maturities of 30, 91, and 182 days. For the maturity of 91 days, we further collect -0.25 and -0.75 delta SPX puts to obtain a full representation of the index volatility smile. For each firm, we collect on each Wednesday 0.50 delta call options of maturities of 30, 91, and 182 days. For the maturity of 91 days, we further collect 0.25 and 0.75 delta call to obtain a full representation of stock volatility smile. Based on these data, we compute the average of implied-volatilities for all options, as well as for in- and out-of-the-money separately. S/K>1 corresponds to out-of-the-money index puts and in-the-money equity calls. Reciprocally, S/K<1 corresponds to in-the-money index puts and out-of-the-money equity calls. We further report on option vega and the moneyness (S/K) of in- and out-of-the-money options. Implied-volatilities and vegas are calculated using the Black-Scholes model. The sample period is from January 4th, 1996 to December 30th, 2011.

Table 2: Parameters, Moments, and Model Fit 1996-2011

Panel A: Wishart Structural Parameters, Industry Average

Industry	γ	$K^{(1,1)}$	$K^{(2,1)}$	$K^{(2,2)}$	$Q^{(1,1)}$	$Q^{(2,1)}$	$Q^{(1,2)}$	$Q^{(2,2)}$	ρ_{Lev}	λ_1	λ_2
Energy	4.895	2.34	-0.27	1.58	0.175	-0.032	0.163	0.195	-0.81	-3.45	1.22
Materials	4.925	2.34	-0.40	1.71	0.177	-0.033	0.177	0.181	-0.81	-3.80	-0.01
Industrials	4.859	2.40	0.03	1.54	0.174	-0.023	0.155	0.170	-0.83	-3.63	1.11
Cons. Discretionary	4.964	2.39	-0.21	1.54	0.175	-0.013	0.143	0.214	-0.84	-3.80	0.48
Cons. Staples	5.005	2.43	-0.59	1.50	0.176	-0.032	0.098	0.156	-0.85	-3.53	2.12
Health Care	4.878	2.41	-0.36	1.41	0.174	-0.043	0.144	0.183	-0.85	-3.52	2.05
Financials	4.874	2.32	-0.17	1.40	0.170	0.005	0.196	0.219	-0.85	-3.97	-0.91
I. T.	4.847	2.34	-0.11	1.45	0.175	-0.022	0.201	0.214	-0.80	-3.67	0.61
Telecom.	4.975	2.40	-0.62	1.67	0.174	-0.038	0.111	0.162	-0.85	-4.11	0.80
Utilities	4.981	2.40	-0.01	1.79	0.178	-0.035	0.119	0.150	-0.85	-4.06	1.16

Panel B: Unconditional Second Moments, Risk Premia, and Model Fit

									Log-Likelihood		
Industry	θ_I	θ_j	θ_{Ij}	β_j	VRP_I	VRP_j	CRP_{Ij}	BRP_j	Index Option	Equity Option	Return
Energy	0.034	0.117	0.030	0.883	-0.014	-0.010	-0.012	0.002	10386	9942	3392
Materials	0.034	0.112	0.035	1.036	-0.014	-0.024	-0.019	-0.079	10316	9514	3426
Industrials	0.034	0.086	0.030	0.884	-0.015	-0.013	-0.012	0.002	10239	9841	3511
Cons. Discretionary	0.033	0.118	0.031	0.926	-0.014	-0.020	-0.015	-0.036	10381	9348	3416
Cons. Staples	0.034	0.070	0.021	0.612	-0.014	-0.004	-0.008	0.019	10375	10583	3624
Health Care	0.034	0.115	0.025	0.741	-0.014	-0.006	-0.009	0.029	10335	9779	3441
Financials	0.034	0.179	0.049	1.455	-0.015	-0.066	-0.029	-0.147	10298	7716	3396
I. T.	0.033	0.162	0.040	1.215	-0.013	-0.034	-0.019	-0.060	10453	9107	3280
Telecom.	0.033	0.065	0.021	0.648	-0.015	-0.011	-0.012	-0.055	10353	9781	3612
Utilities	0.035	0.052	0.019	0.548	-0.016	-0.005	-0.009	0.003	10212	9624	3689

Note to Table: In Panel A, we report on parameter estimates for the Wishart P-dynamics. We estimate the model pairwise for the S&P500 index and each stock. The parameters are obtained by maximizing the joint return and option log-likelihoods computed from index and stock return and option data. In Panel B, we present the unconditional moments estimated. We also report on the market and stock variance risk premium (i.e. VRP_I and VRP_j), and on stock covariance and beta risk premium (i.e. CRP and BRP). The three last columns present model fit for return and option data.

Table 3: Explaining the Time-Variation in the One-month Future Realized Beta
1996-2011

Regression Specification: $\beta^{OLS}_{j,t+1,21} = a_0 + a_1 \times \beta^{W}_{j,t,21} + a_2 \times \beta^{OLS}_{j,t,21} + \varepsilon_{j,t+1,21}$

Industry	Intercept	*t-Stat.*	1 mth Exp. Int. Wishart Beta	*t-Stat.*	1 mth OLS Beta	*t-Stat.*	R-Square (%)
Energy	0.389	*5.90*	0.172	*2.84*	0.410	*8.22*	22.90
Materials	0.484	*6.79*	0.111	*2.16*	0.419	*8.56*	21.51
Industrials	0.624	*9.19*	0.085	*1.32*	0.240	*4.57*	9.24
Cons. Discretionary	0.685	*8.94*	0.154	*2.11*	0.172	*3.16*	7.29
Cons. Staples	0.381	*7.24*	0.129	*1.69*	0.314	*5.71*	13.85
Health Care	0.544	*8.65*	0.130	*1.89*	0.248	*4.39*	10.36
Financials	0.488	*5.45*	0.312	*4.84*	0.293	*5.03*	23.98
I. T.	0.657	*7.87*	0.198	*3.08*	0.277	*5.21*	16.82
Telecom.	0.505	*9.52*	0.192	*3.35*	0.163	*2.74*	7.38
Utilities	0.241	*5.93*	0.230	*2.97*	0.320	*5.18*	18.57

Note to Table: The table presents the loadings and t-statistics from regressing one month future realized beta on Wishart model one month expected integrated beta controlling for lagged OLS beta. On each day, we obtain future realized beta by estimating the CAPM regression using the 21 days ahead index and stock excess returns. We compare future realized beta with the one month forecast of the Wishart model on that day computed using previous day filtered spot beta. The one month OLS beta on each day is obtained using the standard methodology of regressing stock excess returns on market excess returns based on the previous 21 daily observations. The t-statistics in italic are calculated using the Newey-West methodology allowing for 21 auto correlated lags in the residuals. The sample period is from January 4th, 1996 to December 30th, 2011. The regression frequency is daily.

Table 4: Wishart and OLS Beta and the Cross-section of Stock Expected Returns, 1996-2011

Panel A: Wishart Beta and the Cross-Section of Stock Expected Returns

	Dependent Variable: Avg. Daily Equity Return Annualized							
	(1)		(2)		(3)		(4)	
	Slope	*t-Stat.*	Slope	*t-Stat.*	Slope	*t-Stat.*	Slope	*t-Stat.*
Unconditional Wishart Beta	0.069	*4.46*						
Average of 1 mth Expected Integrated Wishart Beta			0.075	*5.21*				
Average of 3 mth Expected Integrated Wishart Beta					0.070	*4.83*		
Average of 6 mth Expected Integrated Wishart Beta							0.070	*4.73*
R-Square	13.92%		13.78%		15.57%		15.10%	
Avg. Daily Market Return Annualized	0.067		0.067		0.067		0.067	
Diff. between Slope and Avg. Market Returns	-0.001		-0.008		-0.002		-0.002	

Panel B: OLS Beta and the Cross-Section of Stock Expected Returns

	Dependent Variable: Avg. Daily Equity Return Annualized							
	(5)		(6)		(7)		(8)	
	Slope	*t-Stat.*	Slope	*t-Stat.*	Slope	*t-Stat.*	Slope	*t-Stat.*
Unconditional OLS Beta	0.073	*4.39*						
Average of daily 1 mth OLS Beta			0.023	*3.81*				
Average of daily 3 mth OLS Beta					0.028	*3.77*		
Average of daily 6 mth OLS Beta							0.034	*5.99*
R-Square	13.89%		15.32%		14.71%		19.80%	
Avg. Daily Market Return Annualized	0.067		0.067		0.067		0.067	
Diff. between Slope and Avg. Market Returns	-0.006		0.045		0.040		0.034	

Note to Table: The table reports on the slope coefficients obtained from regressing the cross-section of stock average annualized returns on an intercept and various beta measures over the full sample. In Panel A, we consider four different measures of beta computed using the Wishart model. In Panel B, we consider four different measures of OLS beta. For each Panel, we report the biases of the market premium estimated with the sample average of market return annualized. The sample period is from January 4th, 1996 to December 30th, 2011.

Table 5: Predictive Cross-Sectional Regressions, Various Horizons, 1996-2011

Horizon:	Dependent Variable: $R^j_{t+1,h}$ Monthly (h= 21 days)		Quaterly (h =63 days)		Semi-Annual (h =126 days)	
	Coeff.	*t-Stat.*	Coeff.	*t-Stat.*	Coeff.	*t-Stat.*
Intercept	0.0035	*1.31*	0.0074	*0.91*	0.0001	*0.00*
time-t 1 mth Expected Integrated Wishart Beta	0.0055	*1.93*				
time-t 1 mth OLS Beta	0.0001	*0.04*				
time-t 3 mth Expected Integrated Wishart Beta			0.0179	*2.15*		
time-t 3 mth OLS Beta			-0.0056	*-0.71*		
time-t 6 mth Expected Integrated Wishart Beta					0.0320	*1.70*
time-t 6 mth OLS Beta					-0.0016	*-0.08*
$R^j_{t-h+1,t}$	-0.0144	*-1.08*	0.0071	*0.30*	0.0549	*1.35*
$R^j_{t-2h+1,h}$	-0.0158	*-1.50*	0.0201	*0.96*	0.0225	*0.87*
$R^j_{t-3h+1,h}$	0.0112	*1.12*	0.0459	*2.78*	0.0001	*0.00*
R-Square (%)	18.10		20.78		22.00	
Newey-West Lags	21		63		126	

Note to Table: The table presents predictive cross-sectional Fama-MacBeth estimates obtained by regressing future realized stock excess returns on Wishart and OLS betas controlling for lagged excess returns. We consider three horizons. For a given horizon of h days, we compute future realized stock excess returns on each day by compounding the h days ahead daily excess returns. The daily Wishart beta forecast (i.e. h days expected integrated beta) are calculated based on the spot beta filtered on the previous day. The daily OLS beta measure for a given horizon is obtained based on the standard methodology of regressing past stock excess returns on market excess returns using an estimation window of length equal to the horizon considered. The t-statistics in italic are calculated using the Newey-West methodology allowing for h auto correlated lags. The sample period is from January 4th, 1996 to December 30th, 2011. The regression frequency is daily.

Table 6: Hedging Systematic Risk, Various Horizons, 1996-2011

Regression Specification: $R^{j}_{t+1,h}$ - $\beta_{j,t,h} \times R^{I}_{t+1,h} = c_{j,h} + d_{j,h} \times R^{I}_{t+1,h} + \varepsilon_{j,t+1,h}$

Horizon:	Monthly *(h=21 days)*				Quaterly (h = 63 days)			
	Wishart Beta		OLS Beta		Wishart Beta		OLS Beta	
BRMSE All Firms	*14.97%*		*16.37%*		*21.08%*		*22.51%*	
Industry	abs(d)	*t-Stat.*	abs(d)	*t-Stat.*	abs(d)	*t-Stat.*	abs(d)	*t-Stat.*
Energy	0.160	*1.59*	0.090	*1.05*	0.195	*1.46*	0.184	*1.12*
Materials	0.081	*0.86*	0.170	*1.50*	0.193	*1.29*	0.232	*1.27*
Industrials	0.110	*1.41*	0.115	*1.59*	0.162	*1.34*	0.135	*1.19*
Cons. Discretionary	0.151	*1.47*	0.170	*1.53*	0.235	*1.26*	0.251	*1.21*
Cons. Staples	0.085	*0.99*	0.088	*1.28*	0.144	*1.29*	0.120	*1.13*
Health Care	0.113	*1.14*	0.114	*1.27*	0.151	*1.13*	0.137	*1.05*
Financials	0.099	*0.72*	0.154	*0.85*	0.128	*0.65*	0.227	*0.89*
I. T.	0.098	*0.88*	0.065	*0.64*	0.152	*1.03*	0.138	*0.90*
Telecom.	0.144	*1.98*	0.095	*1.35*	0.182	*1.66*	0.179	*1.83*
Utilities	0.181	*2.33*	0.104	*1.44*	0.189	*1.86*	0.103	*1.51*

Horizon:	Semi-Annual (h = 126 days)			
	Wishart Beta		OLS Beta	
BRMSE All Firms	*27.36%*		*28.60%*	
Industry	abs(d)	*t-Stat.*	abs(d)	*t-Stat.*
Energy	0.197	*1.13*	0.218	*1.28*
Materials	0.242	*1.34*	0.302	*1.53*
Industrials	0.216	*1.46*	0.182	*1.30*
Cons. Discretionary	0.320	*1.27*	0.335	*1.18*
Cons. Staples	0.163	*1.21*	0.135	*0.98*
Health Care	0.152	*0.95*	0.142	*0.90*
Financials	0.147	*0.78*	0.235	*1.01*
I. T.	0.251	*1.22*	0.207	*1.14*
Telecom.	0.120	*1.01*	0.146	*1.13*
Utilities	0.206	*1.54*	0.177	*1.56*

Note to Table: The table presents the loadings obtained from regressing future realized stock returns minus models' beta forecasts times future realized market returns on future realized market returns over the full sample. For each horizon, we report on the industry average of the absolute slope coefficients and their t-statistics computed using Newey-West methodology allowing for h auto correlated lags of the residuals. For each model and each horizon, we also report on the beta root mean squared errors calculated as the square root of the cross-sectional average of the squared of the slope estimates. The regression frequency is daily.

Table 7: Hedging Systematic Risk, Various Horizons, 2007-2009

Regression Specification: $R^{j}_{t+1,h}$ - $\beta_{j,t,h} \times R^{I}_{t+1,h} = c_{j,h} + d_{j,h} \times R^{I}_{t+1,h} + \varepsilon_{j,t+1,h}$

Horizon:	Monthly *(h=21 days)*				Quaterly (h = 63 days)			
	Wishart Beta		OLS Beta		Wishart Beta		OLS Beta	
BRMSE All Firms	*25.79%*		*33.62%*		*36.16%*		*44.01%*	
Industry	abs(d)	*t-Stat.*	abs(d)	*t-Stat.*	abs(d)	*t-Stat.*	abs(d)	*t-Stat.*
Energy	0.281	*1.44*	0.184	*1.17*	0.251	*1.31*	0.422	*1.66*
Materials	0.254	*1.22*	0.410	*1.92*	0.487	*1.59*	0.645	*2.11*
Industrials	0.098	*0.83*	0.187	*1.65*	0.102	*0.66*	0.114	*0.68*
Cons. Discretionary	0.206	*1.05*	0.293	*1.44*	0.326	*1.70*	0.412	*1.64*
Cons. Staples	0.112	*0.67*	0.121	*0.95*	0.171	*1.13*	0.163	*1.50*
Health Care	0.165	*0.91*	0.156	*1.15*	0.254	*1.28*	0.196	*1.19*
Financials	0.330	*1.14*	0.493	*1.34*	0.466	*1.16*	0.600	*1.38*
I. T.	0.178	*1.34*	0.122	*1.04*	0.212	*1.46*	0.234	*1.92*
Telecom.	0.267	*2.30*	0.110	*1.24*	0.405	*2.89*	0.336	*3.12*
Utilities	0.194	*1.29*	0.061	*0.50*	0.167	*1.15*	0.078	*0.85*

Horizon:	Semi-Annual (h = 126 days)			
	Wishart Beta		OLS Beta	
BRMSE All Firms	*43.48%*		*47.84%*	
Industry	abs(d)	*t-Stat.*	abs(d)	*t-Stat.*
Energy	0.210	*1.40*	0.305	*1.62*
Materials	0.730	*3.69*	0.791	*3.32*
Industrials	0.170	*1.21*	0.206	*1.60*
Cons. Discretionary	0.466	*2.73*	0.542	*2.66*
Cons. Staples	0.168	*1.31*	0.172	*1.63*
Health Care	0.292	*1.91*	0.254	*2.01*
Financials	0.471	*1.73*	0.571	*1.74*
I. T.	0.265	*2.43*	0.275	*2.42*
Telecom.	0.341	*3.37*	0.392	*5.12*
Utilities	0.124	*1.15*	0.172	*1.55*

Note to Table: The table presents the loadings obtained from regressing future realized stock returns minus models' beta forecasts times future realized market returns on future realized market returns over the 2007-2009 sample period. For each horizon, we report on the industry average of the absolute slope coefficients and their t-statistics computed using the Newey-West methodology allowing for h auto correlated lags of the residuals. For each model and each horizon, we also report on the beta root mean squared errors calculated as the square root of the cross-sectional average of the squared of slope estimates. The regression frequency is daily.

Table 8: Firm-Specific News and the Cross-section of Betas and Covariances.
2000-2011

Panel A: The Cross-Section of Wishart and OLS Betas and Covariances, Weekly Horizon

Dependent Variable: Cross-Section of	1 week Exp. Int. Wishart $\beta^{W}_{j,t,5}$		1 week Exp. Int. Wishart $\sigma^{W}_{Ij,t,5}$		1 week OLS $\beta^{OLS}_{j,t,5}$		1 week OLS $\sigma^{OLS}_{Ij,t,5}$	
	Coeff.	*t-Stat.*	Coeff.	*t-Stat.*	Coeff.	*t-Stat.*	Coeff.	*t-Stat.*
Intercept	0.1488	*18.28*	0.0052	*14.04*	0.7329	*28.68*	0.0348	*11.28*
$FirmNews_{j,t}$	-1.0712	*-10.43*	-0.0366	*-9.41*	-0.4189	*-1.07*	-0.0405	*-2.17*
$\beta^{W/OLS}_{j,t-4,5}$ /$\sigma^{W/OLS}_{Ij,t,t-4,5}$	0.9071	*207.64*	0.9091	*154.05*	0.2526	*24.18*	0.2941	*18.72*
R-Square (%)	81.19		80.75		14.75		14.75	
Newey-West Lags	5		5		5		5	

Panel B: The Cross-Section of Wishart and OLS Betas and Covariances, Monthly Horizon

Dependent Variable: Cross-Section of	1 mth Exp. Int. Wishart $\beta^{W}_{j,t,21}$		1 mth Exp. Int. Wishart $\sigma^{W}_{Ij,t,21}$		1 mth OLS $\beta^{OLS}_{j,t,21}$		1 mth OLS $\sigma^{OLS}_{Ij,t,21}$	
	Coeff.	*t-Stat.*	Coeff.	*t-Stat.*	Coeff.	*t-Stat.*	Coeff.	*t-Stat.*
Intercept	0.2888	*14.53*	0.0105	*11.47*	0.4267	*18.50*	0.0189	*4.81*
$FirmNews_{j,t}$	-1.0287	*-5.78*	-0.0360	*-5.51*	-0.0415	*-0.22*	-0.0164	*-1.09*
$\beta^{W/OLS}_{j,t-20,21}$ /$\sigma^{W/OLS}_{Ij,t,t-20,21}$	0.7528	*60.06*	0.7671	*42.09*	0.5551	*27.15*	0.6583	*14.44*
R-Square (%)	57.54		57.64		35.93		35.93	
Newey-West Lags	21		21		21		21	

Panel C: The Cross-Section of Wishart and OLS Betas and Covariances, Quaterly Horizon

Dependent Variable: Cross-Section of	3 mth Exp. Int. Wishart $\beta^{W}_{j,t,63}$		3 mth Exp. Int. Wishart $\sigma^{W}_{Ij,t,63}$		3 mth OLS $\beta^{OLS}_{j,t,63}$		3 mth OLS $\sigma^{OLS}_{Ij,t,63}$	
	Coeff.	*t-Stat.*	Coeff.	*t-Stat.*	Coeff.	*t-Stat.*	Coeff.	*t-Stat.*
Intercept	0.3320	*11.11*	0.0119	*11.12*	0.2902	*8.12*	0.0147	*2.42*
$FirmNews_{j,t}$	-0.7984	*-3.60*	-0.0284	*-3.62*	-0.1005	*-0.71*	-0.0192	*-1.34*
$\beta^{W/OLS}_{j,t-62,63}$ /$\sigma^{W/OLS}_{Ij,t,t-62,63}$	0.6994	*31.56*	0.7459	*20.44*	0.7121	*20.18*	0.8616	*9.17*
R-Square (%)	50.02		50.62		51.51		51.51	
Newey-West Lags	63		63		63		63	

Note to Table: The table presents the cross-sectional result of regressing betas and covariances against firm-specific news index (ESS/1000) and lagged variables. The daily Wishart beta/covariance forecast (i.e. h-day expected integrated beta/covariance) are calculated based on the spot variables filtered on the previous day. The daily OLS beta/covariance measure for a given horizon is computed from past excess returns using an estimation window of length equal to the horizon considered. The coefficients reported are Fama-MacBeth coefficients. We consider three horizons. The t-statistics in italic are calculated using the Newey-West methodology allowing for h auto correlated lags in the residuals. The sample period is from January 3rd, 2000 to December 30th, 2011. The regression frequency is daily.

Table 9: Aggregate Uncertainty and Co-movements in Betas and Covariances.
2000-2011

Panel A: Principal Component of Wishart and OLS Betas and Covariances, Weekly Horizon

Dependent Variable: PC Factor of	1 week Exp. Int. Wishart $\beta^{W}_{j,t,5}$		1 week Exp. Int. Wishart $\sigma^{W}_{Ij,t,5}$		1 week OLS $\beta^{OLS}_{j,t,5}$		1 week OLS $\sigma^{OLS}_{Ij,t,5}$	
	Coeff.	*t-Stat.*	Coeff.	*t-Stat.*	Coeff.	*t-Stat.*	Coeff.	*t-Stat.*
Intercept	0.5745	*52.59*	-0.0087	*-4.62*	0.9575	*66.27*	-0.0927	*-4.92*
$MacroNews_t$	-0.3662	*-2.42*	-0.1373	*-5.60*	0.0720	*0.35*	-0.4788	*-3.68*
Vix_t	1.3901	*37.67*	0.2364	*32.87*	-0.0854	*-2.27*	0.7422	*7.52*
R-Square (%)	75.30		71.77		0.51		44.06	
Newey-West Lags	5		5		5		5	

Panel B: Principal Component of Wishart and OLS Betas and Covariances, Monthly Horizon

Dependent Variable: PC Factor of	1 mth Exp. Int. Wishart $\beta^{W}_{j,t,21}$		1 mth Exp. Int. Wishart $\sigma^{W}_{Ij,t,21}$		1 mth OLS $\beta^{OLS}_{j,t,21}$		1 mth OLS $\sigma^{OLS}_{Ij,t,21}$	
	Coeff.	*t-Stat.*	Coeff.	*t-Stat.*	Coeff.	*t-Stat.*	Coeff.	*t-Stat.*
Intercept	0.5903	*38.81*	-0.0045	*-1.92*	0.9576	*55.91*	-0.0959	*-4.69*
$MacroNews_t$	-0.4464	*-2.72*	-0.1218	*-4.84*	0.1138	*0.64*	-0.2307	*-2.11*
Vix_t	1.3915	*22.23*	0.2113	*20.51*	-0.0816	*-1.31*	0.6928	*5.81*
R-Square (%)	77.17		71.78		0.95		65.07	
Newey-West Lags	21		21		21		21	

Panel C: Principal Component of Wishart and OLS Betas and Covariances, Quaterly Horizon

Dependent Variable: PC Factor of	3 mth Exp. Int. Wishart $\beta^{W}_{j,t,63}$		3 mth Exp. Int. Wishart $\sigma^{W}_{Ij,t,63}$		3 mth OLS $\beta^{OLS}_{j,t,63}$		3 mth OLS $\sigma^{OLS}_{Ij,t,63}$	
	Coeff.	*t-Stat.*	Coeff.	*t-Stat.*	Coeff.	*t-Stat.*	Coeff.	*t-Stat.*
Intercept	0.6318	*33.85*	0.0040	*1.68*	0.9633	*37.53*	-0.0745	*-3.89*
$MacroNews_t$	-0.4718	*-2.79*	-0.0946	*-4.04*	0.0949	*0.39*	-0.1597	*-1.68*
Vix_t	1.2965	*15.24*	0.1607	*15.54*	-0.0998	*-1.05*	0.5778	*5.01*
R-Square (%)	79.27		72.03		1.58		63.46	
Newey-West Lags	63		63		63		63	

Note to Table: We regress the principal component factor of models' betas and covariances against the macro-economic news index and VIX for three horizons. The daily Wishart beta/covariance forecast (i.e. *h* days expected integrated beta/covariance) are calculated based on the spot variables filtered on the previous day. The daily OLS beta/covariance measure for a given horizon is computed from past excess returns using an estimation window of length equal to the horizon considered. For each variable, the principal component factor is obatined by summing the first three principal components multiplied by the average loading of stocks for that component. The t-statistics in italic are calculated using Newey-West methodology allowing for *h* auto correlated lags in the residuals. The sample period is from January 3rd, 2000 to December 30th, 2011. The regression frequency is daily.

Table A.1.A: Return Summary Statistics, 1996-2011

			Daily Return Data Annualized				
Company Name	Ticker	Industry Classification	Mean (%)	Variance (%)	Beta	Syst. Risk (%)	Correlation
Apple	AAPL	Information Technology	38.06	25.92	1.12	21.36	0.46
Abbott	ABT	Health Care	12.38	6.99	0.59	21.69	0.47
Adobe Systems	ADBE	Information Technology	22.45	27.21	1.32	28.14	0.53
Archer Daniels Midland	ADM	Consumer Staples	11.94	11.62	0.78	23.15	0.48
Automatic Data Processing	ADP	Information Technology	12.88	7.62	0.80	36.85	0.61
American Electronic Power	AEP	Utilities	8.52	6.85	0.59	22.40	0.47
Aflac	AFL	Financials	21.33	17.58	1.20	35.73	0.60
Allergan	AGN	Health Care	20.98	10.16	0.73	23.27	0.48
American International Group	AIG	Financials	4.45	41.55	1.52	24.38	0.49
Allstate	ALL	Financials	10.74	12.88	1.06	38.22	0.62
Amgen	AMGN	Health Care	16.56	13.83	0.85	22.68	0.48
Apache	APA	Energy	21.47	15.59	0.90	22.98	0.48
Anadarko Petroleum	APC	Energy	20.48	17.27	0.98	24.14	0.49
American Express	AXP	Financials	18.09	16.36	1.45	56.04	0.75
Boing	BA	Industrials	11.58	11.34	0.93	33.21	0.58
Bank of America	BAC	Financials	9.20	25.29	1.57	42.76	0.65
Baxter International	BAX	Health Care	12.38	8.76	0.61	18.73	0.43
Bristol-Myers Squibb	BMY	Health Care	11.93	9.46	0.74	25.40	0.50
Caterpillar	CAT	Materials	20.24	12.66	1.08	40.64	0.64
Celgene	CELG	Health Care	44.55	36.98	1.05	13.17	0.36
Colgate-Palmolive	CL	Consumer Staples	15.34	6.81	0.57	21.08	0.46
Capital One Financial	COF	Financials	26.54	29.81	1.65	39.98	0.63
Costco Companies	COST	Consumer Staples	21.22	11.81	0.88	29.03	0.54

Note to Table: The table reports important summary statistics of return data. We calculate the average, variance, and various OLS statistics from daily stock and index returns. OLS beta, systematic risk and correlation are obtained by regressing daily excess stock returns on the excess S&P 500 returns. The sample period is from January 4th, 1996 to December 30th, 2011.

Table A.1.B: Return Summary Statistics, 1996-2011

			Daily Return Data Annualized				
Company Name	Ticker	Industry Classification	Mean (%)	Variance (%)	Beta	Syst. Risk (%)	Correlation
Cisco Systems	CSCO	Information Technology	20.14	20.71	1.41	41.97	0.65
CSX	CSX	Industrials	14.92	12.52	0.99	34.31	0.59
Chevron	CVX	Energy	15.79	7.59	0.81	37.84	0.62
Du Pont de Nemours	DD	Materials	9.74	9.89	0.99	43.44	0.66
Deere & Co.	DE	Industrials	18.19	13.88	1.04	34.11	0.58
Danaher	DHR	Health Care	20.16	8.32	0.87	40.00	0.63
Disney Walt	DIS	Consumer Discretionary	10.87	11.57	1.05	42.14	0.65
Dow Chemical	DOW	Materials	11.57	13.26	1.07	37.60	0.61
Duke Energy	DUK	Utilities	11.00	6.83	0.54	18.74	0.43
EMC	EMC	Information Technology	28.18	26.19	1.38	31.77	0.56
Emerson Electric	EMR	Industrials	12.52	9.12	1.00	48.02	0.69
EOG Resources	EOG	Energy	22.42	17.09	0.91	21.29	0.46
Exelon	EXC	Utilities	13.98	7.61	0.60	20.62	0.45
Ford Motor	F	Consumer Discretionary	13.15	20.94	1.16	27.95	0.53
Fedex	FCX	Industrials	21.88	26.14	1.14	21.63	0.47
Freeport-McMoran	FDX	Materials	15.58	11.80	0.97	34.78	0.59
General Electric	GE	Industrials	10.58	10.77	1.19	57.61	0.76
Gilead Sciences	GILD	Health Care	32.84	28.59	1.01	15.55	0.39
General Mills	GIS	Consumer Staples	11.27	3.77	0.38	16.61	0.41
Halliburton	HAL	Energy	20.28	23.44	1.15	24.85	0.50
Home Depot	HD	Consumer Discretionary	16.60	12.61	1.09	41.33	0.64
Honeywell International	HON	Industrials	13.47	12.50	1.09	41.70	0.65
HP	HWP	Information Technology	12.87	17.19	1.13	32.51	0.57

Note to Table: The table reports important summary statistics of return data. We calculate the average, variance, and various OLS statistics from daily stock and index returns. OLS beta, systematic risk and correlation are obtained by regressing daily excess stock returns on the excess S&P 500 returns. The sample period is from January 4th, 1996 to December 30th, 2011.

Table A.1.C: Return Summary Statistics, 1996-2011

			Daily Return Data Annualized				
Company Name	Ticker	Industry Classification	Mean (%)	Variance (%)	Beta	Syst. Risk (%)	Correlation
International Business Machines	IBM	Information Technology	19.31	9.58	0.91	37.88	0.62
Intel Corp	INTC	Information Technology	18.09	18.21	1.32	41.85	0.65
Johnson Controls	JCI	Consumer Discretionary	18.57	11.62	1.02	38.98	0.62
Johnson and Johnson	JNJ	Health Care	11.61	5.17	0.57	27.71	0.53
JP Morgan	JPM	Financials	16.37	19.44	1.55	54.09	0.74
Kimberly Clark	KMB	Consumer Staples	9.68	5.97	0.52	19.85	0.45
Coca-Cola	KO	Consumer Staples	9.04	6.19	0.58	23.94	0.49
Eli Lilly	LLY	Health Care	10.34	9.41	0.75	26.29	0.51
Lockheed Martin	LMT	Industrials	10.98	8.63	0.55	15.16	0.39
Lowe's Companies	LOW	Consumer Discretionary	18.94	14.25	1.09	36.59	0.60
McDonald's	MCD	Consumer Discretionary	14.54	7.25	0.60	21.71	0.47
McKesson	MCK	Health Care	15.00	13.09	0.68	15.52	0.39
Medtronic	MDT	Health Care	12.81	9.54	0.72	24.06	0.49
Minnesota Mining and Manufacturing	MMM	Industrials	11.73	6.84	0.77	38.21	0.62
Philip Morris/Altria	MO	Consumer Staples	18.39	8.82	0.51	12.95	0.36
Merck & Co	MRK	Health Care	9.39	9.35	0.77	27.78	0.53
Morgan Stanley	MS	Financials	19.56	32.25	1.96	52.43	0.72
Nextera Energy	NEE	Utilities	12.45	5.58	0.54	22.91	0.48
Nike	NKE	Consumer Discretionary	18.29	12.38	0.82	23.72	0.49
Norfolk Southern	NSC	Industrials	14.72	12.31	0.98	33.98	0.58
Oracle	ORCL	Information Technology	25.90	24.71	1.33	31.27	0.56
Occidental Petroleum	OXY	Energy	22.63	11.87	0.94	32.86	0.57
Pepsico	PEP	Consumer Staples	11.10	6.60	0.54	19.44	0.44

Note to Table: The table reports important summary statistics of return data. We calculate the average, variance, and various OLS statistics from daily stock and index returns. OLS beta, systematic risk and correlation are obtained by regressing daily excess stock returns on the excess S&P 500 returns. The sample period is from January 4th, 1996 to December 30th, 2011.

Table A.1.D: Return Summary Statistics, 1996-2011

			Daily Return Data Annualized				
Company Name	Ticker	Industry Classification	Mean (%)	Variance (%)	Beta	Syst. Risk (%)	Correlation
Pfizer	PFE	Health Care	11.67	8.98	0.82	32.50	0.57
Procter and Gamble	PG	Consumer Staples	12.40	6.33	0.57	22.22	0.47
PNC Financial Services	PNC	Financials	15.97	16.59	1.25	41.55	0.64
Praxair	PX	Materials	18.28	10.03	0.90	35.70	0.60
Qualcomm	QCOM	Information Technology	34.52	28.80	1.31	26.24	0.51
Raytheon	RTN	Industrials	8.54	11.30	0.59	13.40	0.37
Starbucks	SBUX	Consumer Discretionary	28.67	18.72	1.07	26.76	0.52
Schlumberger	SLB	Energy	18.85	15.99	1.06	30.79	0.55
Southern	SO	Utilities	13.96	4.41	0.40	16.11	0.40
State Street	STT	Financials	21.01	21.86	1.48	44.06	0.66
Stryker	SYK	Health Care	18.69	10.73	0.72	21.30	0.46
AT&T	T	Telecommunications	8.76	8.94	0.79	30.83	0.56
Target	TGT	Consumer Discretionary	20.92	13.72	1.07	36.39	0.60
TJX Companies	TJX	Consumer Discretionary	28.13	13.53	0.92	27.46	0.52
Thermo Fisher Scientific	TMO	Health Care	8.77	11.37	0.88	30.16	0.55
Time Warner	TWX	Consumer Discretionary	25.11	24.86	1.29	29.13	0.54
Texas Instruments	TXN	Information Technology	22.01	22.74	1.30	32.70	0.57
United Healthcare	UNH	Health Care	19.68	15.72	0.80	17.89	0.42
Union Pacific	UNP	Industrials	15.93	9.15	0.83	32.97	0.57
United Technologies	UTX	Industrials	17.88	8.91	0.95	44.81	0.67
Verizon Communications	VZ	Telecommunications	9.93	8.15	0.74	29.16	0.54
Walgreens Boots Alliance	WAG	Consumer Staples	14.89	9.26	0.75	26.28	0.51
Wal-Mart Stores	WMT	Consumer Staples	15.49	8.26	0.75	29.95	0.55
Exxon	XON	Energy	14.97	7.19	0.80	39.31	0.63

Note to Table: The table reports important summary statistics of return data. We calculate the average, variance, and various OLS statistics from daily stock and index returns. OLS beta, systematic risk and correlation are obtained by regressing daily excess stock returns on the excess S&P 500 returns. The sample period is from January 4th, 1996 to December 30th, 2011.

Table A.2.A: Wednesday Option Summary Statistics, 1996-2011

		Wednesday Option Data					
		Implied Volatility (%)				Moneyness (S/K)	
Company Name	Ticker	All	In-the-money	Out-of-the-money	Vega	In-the-money	Out-of-the-money
Apple	AAPL	48.19	50.33	47.65	16.69	1.14	0.92
Abbott	ABT	25.32	27.44	24.78	8.79	1.07	0.96
Adobe Systems	ADBE	46.64	49.46	45.94	7.86	1.13	0.93
Archer Daniels Midland	ADM	32.62	34.59	32.12	4.07	1.10	0.95
Automatic Data Processing	ADP	25.27	27.37	24.74	8.40	1.08	0.96
American Electronic Power	AEP	22.39	24.75	21.80	6.85	1.06	0.97
Aflac	AFL	32.16	34.70	31.52	7.95	1.09	0.95
Allergan	AGN	33.28	35.41	32.74	12.53	1.10	0.95
American International Group	AIG	36.81	39.39	36.17	12.72	1.10	0.94
Allstate	ALL	29.43	30.95	29.00	8.03	1.07	0.95
Amgen	AMGN	34.14	37.16	33.38	11.11	1.10	0.95
Apache	APA	36.86	39.05	36.32	11.62	1.11	0.95
Anadarko Petroleum	APC	38.00	40.39	37.41	10.34	1.11	0.94
American Express	AXP	33.95	37.26	33.12	10.44	1.10	0.95
Boing	BA	31.31	33.82	30.68	10.79	1.09	0.95
Bank of America	BAC	34.29	38.28	33.29	9.18	1.10	0.95
Baxter International	BAX	27.89	30.00	27.36	9.17	1.08	0.96
Bristol-Myers Squibb	BMY	28.64	31.14	28.02	8.30	1.08	0.96
Caterpillar	CAT	32.93	35.31	32.34	11.47	1.10	0.95
Celgene	CELG	59.36	62.84	58.50	7.26	1.16	0.90
Colgate-Palmolive	CL	24.12	26.00	23.65	12.29	1.07	0.96
Capital One Financial	COF	44.94	49.19	43.88	10.40	1.13	0.93
Costco Companies	COST	31.68	33.80	31.16	9.08	1.09	0.95

Note to Table: The table reports important statistics of the option data. For each firm, we collect on each Wednesday 0.50 delta call options of maturities of 30, 91, and 182 days. For the maturity of 91 days, we further collect 0.25 and 0.75 delta call to obtain a full representation of stock volatility smile. Based on these data, we compute the average of implied-volatilities for all options, as well as for in- and out-of-the-money separately. S/K>1 corresponds to out-of-the-money index puts and in-the-money equity calls. Reciprocally, S/K<1 corresponds to in-the-money index puts and out-of-the-money equity calls. We further report on option vega and the moneyness (S/K) of in- and out-of-the-money options. Implied-volatilities and vegas are calculated using the Black-Scholes model. The sample period is from January 4th, 1996 to December 30th, 2011.

Table A.2.B: Wednesday Option Summary Statistics, 1996-2011

		Wednesday Option Data					
		Implied Volatility (%)				Moneyness (S/K)	
Company Name	Ticker	All	In-the-money	Out-of-the-money	Vega	In-the-money	Out-of-the-money
Cisco Systems	CSCO	39.88	43.32	39.02	6.69	1.12	0.94
CSX	CSX	32.32	34.67	31.73	7.63	1.10	0.95
Chevron	CVX	24.71	26.65	24.23	14.10	1.07	0.96
Du Pont de Nemours	DD	28.07	30.60	27.44	9.17	1.08	0.96
Deere & Co.	DE	33.85	36.19	33.26	10.70	1.10	0.95
Danaher	DHR	28.84	31.06	28.29	10.82	1.09	0.96
Disney Walt	DIS	30.47	32.81	29.88	6.70	1.09	0.96
Dow Chemical	DOW	31.91	34.84	31.17	9.34	1.09	0.95
Duke Energy	DUK	23.49	26.14	22.83	5.90	1.07	0.97
EMC	EMC	43.92	46.75	43.22	5.46	1.13	0.93
Emerson Electric	EMR	26.43	28.62	25.88	10.68	1.08	0.96
EOG Resources	EOG	38.90	41.14	38.34	9.93	1.11	0.94
Exelon	EXC	25.09	27.47	24.49	8.68	1.07	0.96
Ford Motor	F	42.11	43.75	41.58	3.65	1.10	0.93
Fedex	FCX	47.59	49.88	47.01	7.27	1.13	0.93
Freeport-McMoran	FDX	31.83	33.97	31.30	13.20	1.09	0.95
General Electric	GE	29.04	32.36	28.19	8.73	1.09	0.96
Gilead Sciences	GILD	47.52	50.64	46.74	8.64	1.13	0.93
General Mills	GIS	20.51	21.96	20.15	9.69	1.06	0.97
Halliburton	HAL	42.06	44.62	41.43	7.02	1.12	0.94
Home Depot	HD	31.99	34.94	31.25	7.57	1.10	0.95
Honeywell International	HON	32.76	34.36	32.38	7.73	1.09	0.95
HP	HWP	37.22	39.93	36.54	8.22	1.11	0.95

Note to Table: The table reports important statistics of the option data. For each firm, we collect on each Wednesday 0.50 delta call options of maturities of 30, 91, and 182 days. For the maturity of 91 days, we further collect 0.25 and 0.75 delta call to obtain a full representation of stock volatility smile. Based on these data, we compute the average of implied-volatilities for all options, as well as for in- and out-of-the-money separately. S/K>1 corresponds to out-of-the-money index puts and in-the-money equity calls. Reciprocally, S/K<1 corresponds to in-the-money index puts and out-of-the-money equity calls. We further report on option vega and the moneyness (S/K) of in- and out-of-the-money options. Implied-volatilities and vegas are calculated using the Black-Scholes model. The sample period is from January 4th, 1996 to December 30th, 2011.

Table A.2.C: Wednesday Option Summary Statistics, 1996-2011

		Wednesday Option Data					
		Implied Volatility (%)				Moneyness (S/K)	
Company Name	Ticker	All	In-the-money	Out-of-the-money	Vega	In-the-money	Out-of-the-money
International Business Machines	IBM	28.51	31.13	27.86	20.24	1.09	0.96
Intel Corp	INTC	37.34	40.19	36.63	8.09	1.11	0.94
Johnson Controls	JCI	31.58	33.89	31.00	10.71	1.09	0.95
Johnson and Johnson	JNJ	21.93	24.28	21.34	12.10	1.07	0.97
JP Morgan	JPM	34.28	38.18	33.31	9.46	1.10	0.95
Kimberly Clark	KMB	22.48	24.30	22.02	11.10	1.06	0.97
Coca-Cola	KO	23.27	25.26	22.77	9.85	1.07	0.97
Eli Lilly	LLY	29.07	31.75	28.40	11.01	1.09	0.96
Lockheed Martin	LMT	29.68	30.58	29.39	12.56	1.07	0.95
Lowe's Companies	LOW	35.41	37.15	34.91	7.35	1.09	0.95
McDonald's	MCD	26.27	28.40	25.74	8.30	1.08	0.96
McKesson	MCK	32.01	34.34	31.42	9.04	1.10	0.95
Medtronic	MDT	28.87	31.15	28.30	9.10	1.09	0.96
Minnesota Mining and Manufacturing	MMM	25.14	27.42	24.57	15.83	1.07	0.96
Philip Morris/Altria	MO	28.36	30.99	27.70	8.14	1.08	0.96
Merck & Co	MRK	27.86	30.44	27.21	10.44	1.08	0.96
Morgan Stanley	MS	40.36	44.02	39.38	9.76	1.11	0.94
Nextera Energy	NEE	21.51	23.36	21.05	9.92	1.06	0.97
Nike	NKE	33.13	35.60	32.51	11.67	1.10	0.95
Norfolk Southern	NSC	31.81	33.96	31.28	7.82	1.09	0.95
Oracle	ORCL	42.40	45.75	41.56	4.76	1.12	0.94
Occidental Petroleum	OXY	31.33	33.50	30.79	8.86	1.09	0.95
Pepsico	PEP	23.40	25.63	22.84	9.27	1.07	0.97

Note to Table: The table reports important statistics of the option data. For each firm, we collect on each Wednesday 0.50 delta call options of maturities of 30, 91, and 182 days. For the maturity of 91 days, we further collect 0.25 and 0.75 delta call to obtain a full representation of stock volatility smile. Based on these data, we compute the average of implied-volatilities for all options, as well as for in- and out-of-the-money separately. S/K>1 corresponds to out-of-the-money index puts and in-the-money equity calls. Reciprocally, S/K<1 corresponds to in-the-money index puts and out-of-the-money equity calls. We further report on option vega and the moneyness (S/K) of in- and out-of-the-money options. Implied-volatilities and vegas are calculated using the Black-Scholes model. The sample period is from January 4th, 1996 to December 30th, 2011.

Table A.2.D: Wednesday Option Summary Statistics, 1996-2011

		Wednesday Option Data					
		Implied Volatility (%)				Moneyness (S/K)	
Company Name	Ticker	All	In-the-money	Out-of-the-money	Vega	In-the-money	Out-of-the-money
Pfizer	PFE	28.78	31.29	28.15	7.53	1.08	0.96
Procter and Gamble	PG	22.37	24.52	21.83	13.71	1.07	0.97
PNC Financial Services	PNC	30.82	33.77	30.08	9.85	1.09	0.95
Praxair	PX	28.03	30.16	27.50	10.62	1.08	0.96
Qualcomm	QCOM	46.21	49.19	45.46	10.43	1.13	0.93
Raytheon	RTN	28.59	30.49	28.11	8.06	1.08	0.96
Starbucks	SBUX	38.87	41.41	38.24	5.75	1.11	0.94
Schlumberger	SLB	36.11	38.21	35.59	12.65	1.11	0.95
Southern	SO	20.00	22.11	19.47	5.36	1.05	0.97
State Street	STT	34.69	37.76	33.92	10.69	1.10	0.95
Stryker	SYK	30.87	32.92	30.36	9.61	1.09	0.96
AT&T	T	27.47	28.86	27.06	6.60	1.06	0.96
Target	TGT	34.37	36.95	33.72	9.11	1.10	0.95
TJX Companies	TJX	34.93	37.55	34.28	5.58	1.10	0.95
Thermo Fisher Scientific	TMO	31.95	34.13	31.40	6.35	1.10	0.95
Time Warner	TWX	42.29	45.35	41.53	7.08	1.12	0.93
Texas Instruments	TXN	41.47	44.17	40.80	7.99	1.12	0.94
United Healthcare	UNH	35.36	38.09	34.67	10.08	1.11	0.95
Union Pacific	UNP	28.39	30.58	27.85	12.21	1.08	0.96
United Technologies	UTX	27.14	29.39	26.58	13.67	1.08	0.96
Verizon Communications	VZ	26.07	28.50	25.47	8.16	1.07	0.96
Walgreens Boots Alliance	WAG	27.74	30.07	27.16	6.71	1.08	0.96
Wal-Mart Stores	WMT	26.66	28.88	26.10	9.14	1.08	0.96
Exxon	XON	24.04	26.15	23.51	12.19	1.07	0.97

Note to Table: The table reports important statistics of the option data. For each firm, we collect on each Wednesday 0.50 delta call options of maturities of 30, 91, and 182 days. For the maturity of 91 days, we further collect 0.25 and 0.75 delta call to obtain a full representation of stock volatility smile. Based on these data, we compute the average of implied-volatilities for all options, as well as for in- and out-of-the-money separately. S/K>1 corresponds to out-of-the-money index puts and in-the-money equity calls. Reciprocally, S/K<1 corresponds to in-the-money index puts and out-of-the-money equity calls. We further report on option vega and the moneyness (S/K) of in- and out-of-the-money options. Implied-volatilities and vegas are calculated using the Black-Scholes model. The sample period is from January 4th, 1996 to December 30th, 2011.

Table A.3.A: Model's Parameters, 1996-2011

Ticker	γ	$K^{(1,1)}$	$K^{(2,1)}$	$K^{(2,2)}$	$Q^{(1,1)}$	$Q^{(2,1)}$	$Q^{(1,2)}$	$Q^{(2,2)}$	ρ_{Lev}	λ_1	λ_2
AAPL	5.00	2.35	-0.18	0.83	0.170	-0.045	0.173	0.207	-0.81	-3.55	1.80
ABT	5.00	2.30	0.01	1.30	0.162	-0.061	0.132	0.130	-0.87	-3.35	2.20
ADBE	4.99	2.27	-0.35	1.23	0.171	-0.028	0.227	0.214	-0.78	-3.03	1.99
ADM	4.99	2.51	-1.12	1.63	0.197	-0.030	0.095	0.199	-0.84	-4.04	2.00
ADP	4.65	2.31	-0.81	1.43	0.181	-0.012	0.087	0.157	-0.82	-3.18	4.50
AEP	5.00	2.30	-0.44	1.92	0.163	-0.059	0.136	0.134	-0.87	-3.39	2.23
AFL	4.95	2.25	-0.93	1.72	0.171	-0.022	0.211	0.180	-0.82	-3.63	0.27
AGN	5.00	2.30	-1.33	1.45	0.162	-0.061	0.124	0.178	-0.89	-3.21	2.57
AIG	5.00	2.25	-0.92	1.11	0.161	-0.089	0.304	0.188	-0.75	-3.02	1.67
ALL	4.85	2.40	1.47	1.58	0.177	0.037	0.137	0.239	-0.82	-3.21	-2.30
AMGN	4.90	2.28	-0.82	2.01	0.171	-0.036	0.158	0.228	-0.81	-2.95	1.85
APA	4.80	2.33	-0.77	1.42	0.176	-0.026	0.130	0.240	-0.84	-3.72	2.04
APC	5.00	2.27	0.29	1.40	0.158	-0.069	0.217	0.175	-0.81	-3.29	1.88
AXP	4.75	2.36	-0.12	1.48	0.179	0.027	0.165	0.221	-0.85	-3.84	0.48
BA	4.95	2.34	-0.43	1.38	0.175	-0.025	0.148	0.145	-0.77	-3.79	1.30
BAC	4.93	2.29	-0.10	0.90	0.169	0.047	0.132	0.225	-0.75	-4.38	-2.95
BAX	5.03	2.81	0.61	1.55	0.191	-0.015	0.117	0.189	-0.83	-4.18	0.58
BMY	5.04	2.45	-1.36	1.54	0.163	-0.111	0.146	0.129	-0.89	-3.09	2.80
CAT	4.90	2.32	0.53	1.83	0.176	-0.007	0.178	0.219	-0.80	-3.49	-2.16
CELG	5.00	2.29	-1.78	0.91	0.162	-0.063	0.180	0.272	-0.87	-2.84	3.39
CL	5.00	2.31	-1.01	1.51	0.165	-0.057	0.090	0.145	-0.88	-3.24	2.34
COF	4.75	2.32	-0.67	1.52	0.165	-0.069	0.321	0.172	-0.76	-4.62	-0.89
COST	4.59	2.57	0.00	0.70	0.191	-0.011	0.108	0.143	-0.85	-4.63	3.09

Note to Table: We report on parameter estimates for the Wishart P-dynamics and the price of risks. We estimate the model pairwise for the S&P500 index and each stock. The parameters are obtained by maximizing the joint return and option log-likelihoods computed from index and stock return and option data.

Table A.3.B: Model's Parameters, 1996-2011

Ticker	γ	$K^{(1,1)}$	$K^{(2,1)}$	$K^{(2,2)}$	$Q^{(1,1)}$	$Q^{(2,1)}$	$Q^{(1,2)}$	$Q^{(2,2)}$	ρ_{Lev}	λ_1	λ_2
CSCO	4.82	2.39	0.05	0.98	0.176	-0.040	0.201	0.155	-0.83	-5.36	-2.18
CSX	5.00	2.32	-0.12	1.37	0.166	-0.054	0.177	0.141	-0.83	-3.80	0.70
CVX	5.03	2.40	-0.99	2.74	0.173	-0.040	0.140	0.185	-0.80	-3.90	0.71
DD	4.80	2.42	-1.18	1.97	0.200	-0.036	0.135	0.175	-0.82	-4.08	-0.77
DE	5.04	2.29	0.37	1.48	0.166	-0.048	0.219	0.142	-0.76	-4.04	0.59
DHR	4.75	2.37	0.00	0.89	0.181	0.017	0.097	0.153	-0.82	-3.56	4.77
DIS	4.94	2.44	0.90	2.04	0.176	0.039	0.138	0.257	-0.84	-3.80	1.77
DOW	5.00	2.35	0.05	1.89	0.169	-0.047	0.233	0.153	-0.81	-4.31	0.08
DUK	5.07	2.44	-0.20	2.16	0.196	-0.039	0.139	0.172	-0.85	-3.72	1.68
EMC	4.94	2.30	-0.41	1.56	0.173	-0.026	0.254	0.232	-0.78	-3.49	1.13
EMR	4.65	2.23	1.35	1.46	0.172	0.046	0.134	0.206	-0.87	-1.68	-5.68
EOG	5.00	2.32	-0.69	1.35	0.167	-0.053	0.157	0.208	-0.85	-3.50	2.32
EXC	4.95	2.45	0.05	1.58	0.177	-0.037	0.127	0.146	-0.86	-4.44	0.91
F	5.00	2.43	-1.31	1.50	0.177	-0.026	0.135	0.244	-0.83	-3.86	1.09
FCX	5.02	2.33	-0.71	1.15	0.173	-0.024	0.176	0.226	-0.83	-3.75	2.81
FDX	4.78	2.39	0.31	1.43	0.191	0.027	0.117	0.234	-0.82	-3.51	3.01
GE	4.95	2.53	-0.63	1.93	0.181	-0.027	0.186	0.141	-0.82	-4.53	2.59
GILD	5.01	2.28	-0.86	0.91	0.160	-0.067	0.183	0.223	-0.87	-2.77	2.38
GIS	5.92	3.39	-0.41	2.79	0.195	0.005	0.075	0.185	-0.89	-3.55	4.56
HAL	4.90	2.26	-0.43	1.64	0.170	-0.039	0.255	0.226	-0.77	-3.08	1.27
HD	4.95	2.44	1.80	1.80	0.175	0.044	0.153	0.264	-0.86	-3.06	-3.40
HON	4.95	2.45	1.24	1.59	0.176	0.038	0.146	0.241	-0.90	-3.68	-2.45
HWP	5.03	2.30	0.40	1.83	0.171	-0.029	0.227	0.223	-0.77	-3.85	1.71

Note to Table: We report on parameter estimates for the Wishart P-dynamics and the price of risks. We estimate the model pairwise for the S&P500 index and each stock. The parameters are obtained by maximizing the joint return and option log-likelihoods computed from index and stock return and option data.

Table A.3.C: Model's Parameters, 1996-2011

Ticker	γ	$K^{(1,1)}$	$K^{(2,1)}$	$K^{(2,2)}$	$Q^{(1,1)}$	$Q^{(2,1)}$	$Q^{(1,2)}$	$Q^{(2,2)}$	ρ_{Lev}	λ_1	λ_2
IBM	4.75	2.32	0.60	1.63	0.175	0.042	0.104	0.217	-0.83	-2.41	-1.04
INTC	4.75	2.36	-0.49	1.77	0.178	-0.031	0.237	0.197	-0.74	-4.34	-1.03
JCI	4.89	2.44	-0.92	1.51	0.190	-0.045	0.159	0.129	-0.82	-4.50	-1.17
JNJ	5.00	2.35	0.27	1.49	0.170	-0.044	0.118	0.119	-0.87	-3.85	0.64
JPM	4.75	2.29	0.04	1.44	0.169	0.057	0.130	0.270	-0.93	-2.94	-2.18
KMB	4.98	2.52	0.09	0.79	0.182	-0.009	0.067	0.114	-0.86	-4.27	0.77
KO	4.80	2.29	-0.16	1.47	0.175	-0.032	0.099	0.147	-0.87	-3.61	1.27
LLY	4.34	2.63	0.81	0.76	0.199	0.009	0.120	0.160	-0.83	-4.59	1.19
LMT	5.03	2.43	-0.89	1.59	0.154	-0.117	0.142	0.124	-0.91	-2.54	3.03
LOW	4.95	2.45	0.78	1.33	0.177	0.035	0.117	0.235	-0.86	-3.43	-2.47
MCD	5.00	2.49	0.09	1.09	0.181	-0.007	0.085	0.157	-0.86	-4.58	2.07
MCK	5.00	2.34	-0.93	1.74	0.168	-0.050	0.140	0.223	-0.87	-3.28	2.10
MDT	4.71	2.44	0.22	1.10	0.183	-0.025	0.126	0.171	-0.86	-3.72	2.46
MMM	4.27	2.66	0.41	1.27	0.202	-0.009	0.133	0.160	-0.83	-4.74	4.03
MO	4.90	2.20	-1.42	2.08	0.154	-0.075	0.129	0.208	-0.87	-2.62	2.29
MRK	5.00	2.37	-1.52	1.57	0.151	-0.095	0.132	0.118	-0.91	-3.29	2.17
MS	4.96	2.39	-0.45	0.93	0.169	0.055	0.137	0.222	-0.91	-6.43	2.38
NEE	4.80	2.31	0.64	1.53	0.175	-0.030	0.122	0.131	-0.83	-3.91	0.31
NKE	5.01	2.36	-0.10	1.67	0.174	-0.030	0.143	0.213	-0.83	-3.96	2.07
NSC	4.90	2.26	-0.31	1.68	0.169	-0.041	0.210	0.131	-0.77	-3.46	0.47
ORCL	4.95	2.22	-1.15	1.67	0.173	-0.018	0.224	0.231	-0.84	-3.35	0.66
OXY	4.76	2.28	-0.45	1.19	0.176	-0.021	0.118	0.163	-0.77	-2.91	1.69
PEP	4.91	2.24	-0.57	1.06	0.160	-0.066	0.100	0.121	-0.92	-2.95	2.59

Note to Table: We report on parameter estimates for the Wishart P-dynamics and the price of risks. We estimate the model pairwise for the S&P500 index and each stock. The parameters are obtained by maximizing the joint return and option log-likelihoods computed from index and stock return and option data.

Table A.3.D: Model's Parameters, 1996-2011

Ticker	γ	$K^{(1,1)}$	$K^{(2,1)}$	$K^{(2,2)}$	$Q^{(1,1)}$	$Q^{(2,1)}$	$Q^{(1,2)}$	$Q^{(2,2)}$	ρ_{Lev}	λ_1	λ_2
PFE	4.65	2.47	0.45	1.21	0.187	-0.005	0.130	0.180	-0.79	-4.16	-0.05
PG	4.95	2.19	-0.89	1.99	0.169	-0.029	0.129	0.149	-0.75	-3.09	1.55
PNC	4.95	2.44	1.76	1.52	0.174	0.052	0.165	0.286	-1.00	-3.55	-3.75
PX	4.91	2.27	-0.70	1.71	0.168	-0.050	0.165	0.133	-0.79	-3.35	0.00
QCOM	4.44	2.55	1.56	0.95	0.194	-0.003	0.214	0.269	-0.80	-3.92	-2.10
RTN	4.80	2.32	-1.36	1.92	0.152	-0.093	0.161	0.197	-0.88	-3.03	2.56
SBUX	5.00	2.32	-0.78	1.12	0.165	-0.055	0.162	0.186	-0.87	-3.56	1.98
SLB	4.85	2.45	-0.73	1.02	0.179	-0.033	0.126	0.168	-0.82	-3.81	2.50
SO	5.08	2.52	-0.08	1.79	0.180	-0.011	0.070	0.167	-0.87	-4.81	0.65
STT	4.85	2.25	-1.82	1.84	0.171	-0.047	0.255	0.185	-0.90	-4.11	-1.82
SYK	4.40	2.62	0.70	1.14	0.198	0.000	0.123	0.202	-0.84	-4.20	2.44
T	4.95	2.46	-0.37	1.69	0.179	-0.026	0.112	0.172	-0.84	-4.58	0.38
TGT	4.90	2.27	-0.35	2.00	0.170	-0.038	0.192	0.217	-0.81	-3.58	0.53
TJX	5.02	2.22	-0.97	1.24	0.163	-0.051	0.138	0.180	-0.85	-2.87	2.15
TMO	5.05	2.34	-0.27	2.37	0.183	-0.064	0.213	0.194	-0.83	-3.71	1.07
TWX	4.95	2.49	-1.43	1.61	0.181	-0.010	0.151	0.278	-0.81	-4.57	0.60
TXN	5.00	2.33	-0.46	2.03	0.168	-0.051	0.265	0.249	-0.78	-3.88	1.30
UNH	5.03	2.30	-0.36	1.98	0.174	-0.066	0.209	0.240	-0.85	-3.16	2.29
UNP	4.95	2.48	0.23	1.44	0.182	-0.006	0.121	0.173	-0.84	-4.26	2.64
UTX	4.88	2.47	0.27	1.46	0.182	0.015	0.116	0.179	-0.83	-4.17	1.69
VZ	5.00	2.34	-0.88	1.65	0.168	-0.049	0.110	0.151	-0.85	-3.63	1.22
WAG	5.01	2.29	-0.53	1.31	0.170	-0.034	0.103	0.157	-0.84	-3.38	2.43
WMT	4.99	2.25	-0.44	1.18	0.172	-0.008	0.081	0.153	-0.82	-3.43	0.39
XON	4.83	2.39	1.59	1.87	0.198	0.025	0.160	0.197	-0.81	-3.39	-2.62

Note to Table: We report on parameter estimates for the Wishart P-dynamics and the price of risks. We estimate the model pairwise for the S&P500 index and each stock. The parameters are obtained by maximizing the joint return and option log-likelihoods computed from index and stock return and option data.