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Stock Loan Lotteries and Individual Investor Performance

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Stock loan lotteries and individual investor performance

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Abstract

Individual investors trade excessively, sell winners too soon, and own stocks with lottery features and low expected returns. I propose and model a financial innovation, called stock loan lotteries, that improves individual investor performance. Stock loan lotteries are prize-linked payoffs where the asset pool is an institution's net rebates from lending their clients' shares. I extend the Barberis and Xiong (2009) realization utility model to include stock loan lotteries. Stock loan lottery tickets incentivize prospect theory investors to buy and hold risky assets. Structuring lending fees as lottery tickets can reduce the expected return that investors demand to hold risky assets by at least 2-3%. Stock loan lotteries provide the largest benefits to the poorest investors, who typically exhibit the strongest lottery preferences. Introducing trading costs, leverage constraints, and taxes to the model enhances the benefits of stock loan lotteries. In a laboratory experiment, stock loan lotteries increase risky asset holdings by around 8%. I propose a mechanism for exchanges to structure stock loan lottery tickets as derivative securities.

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1. Introduction

Individual investors trade frequently, hold undiversified portfolios, and earn low returns after controlling for risk and trading costs. What would motivate individual investors to modify their costly behavior? This paper proposes and models a financial innovation, called stock loan lotteries, with the potential to improve individual investor performance. This innovation applies insights from prospect theory to encourage individuals to follow more passive investment strategies.

Barber and Odean (2000) find that individual investors turn over portfolios frequently.[2] In the cross section, higher trading frequency predicts lower average returns net of trading costs. French (2008) estimates that active trading costs US equity investors 67 basis points annually. Odean (1998) shows that the low average returns of individual investors are due to the disposition effect, the tendency to sell winning positions and realize a gain. Weber and Camerer (1998) identify a strong disposition effect in an experimental setting, suggesting that the behavior is not unique to the Odean (1998) sample. The tendency for individuals to trade frequently has substantial economic costs. Odean (1999) shows that the stocks that individuals sell outperform the stocks that individuals buy by more than 3% in the following year.[3] Several studies show that investors with less experience and education exhibit a stronger disposition effect in their trading.[4]

Individual investors often hold portfolios with a small number of individual stocks, hoping to earn extraordinary returns. Kumar (2009) finds that individual investors hold large

[2]In data from a large discount broker between 1991 and 1996, the average household turns over more than 75% of its portfolio annually.

[3]Likewise, Barber et al. (2009) analyze the complete record of Taiwanese equity trading activity over four years and find stocks that individual investors sell outperform stocks that individual investors purchase by 3.8% over the next year.

[4]These include Feng and Seasholes (2005), Dhar and Zhu (2006), Frazzini (2006), Nicolosi et al. (2009), and Seru et al. (2010).

positions in stocks with "lottery features" including low share prices, high volatility and high skew. However, these lottery stocks earn low average returns. Investors who are poor, inexperienced, and uneducated are especially likely to hold lottery stocks. Dorn et al. (2014) provide evidence that when lottery jackpots in a particular state or country increase, trading activity by individual investors in the same state or country falls. Likewise, Barber et al. (2009) document a substantial reduction in the trading of Taiwanese equities following the introduction of legalized gambling. If investors treat lotteries and financial gambling as substitutes, then investors might accept lottery tickets as compensation for not gambling in the financial markets.

Shefrin and Statman (1993) note that understanding the principles of prospect theory is critical to designing successful financial products. Fernandes et al. (2014) analyze over 100 "financial interventions" and show that financial education rarely improves long-term investor behavior. Rather than rely on education, my innovation introduces a product that targets investor preferences and improves financial outcomes. Investors are paid, in the form of lottery tickets, to buy and hold risky assets. Kearney et al. (2010) document a strong demand for "prize-linked" savings accounts and fixed income investments that incorporate lottery payoffs. Stock loan lotteries are prize-linked payoffs where the asset pool is an institution's net rebates from lending their clients' shares. An individual investor and an institution enter into a contract. The investor agrees to hold stocks in his portfolio for multiple periods, and selling stocks early is impossible.[5] When the institution lends shares, it allocates the fees, net of expenses and profits, to the lottery account of the individual investor. Periodically, the institution holds a lottery and pays the winner the entire pool of stock lending fees.

[5]This contract is analogous to a Certificate of Deposit (CD) or a Guaranteed Investment Certificate (GIC).

Typically, investors who borrow shares for short sales are subject to recall risk. The lender can force the borrower to repurchase his shares by the end of the trading day. Investors may be willing to pay a premium to borrow shares that are "locked up" for a long duration. This reduces the borrower's adverse selection problem. The lender may be more likely to recall shares before the stock declines in price. Chuprinin and Ruf (2016) find that recalls predict stock price declines between 2007 and 2013. These recalls can cost informed short sellers around 20% of their profits. Christoffersen et al. (2007) identify a spike in recall activity around the firm's voting record date. This is problematic for institutions who borrow stock to bet on a price decline following a shareholder vote.

I evaluate stock loan lotteries within the Barberis and Xiong (2009) two-period model of realization utility.[6] Investors only derive utility from realized gains and losses. Investors have Tversky and Kahneman (1992) preferences, so they value gains and losses instead of total wealth, react asymmetrically to gains and losses, and evaluate prospects using subjective "decision weights" rather than objective probabilities. Barberis and Xiong (2009) find that prospect theory investors prefer to realize gains over multiple episodes, which can result in a disposition effect. By realizing gains early, prospect theory investors hold smaller average risky-asset positions, reducing their expected future wealth.

I find that prospect theory investors are reluctant to forgo frequent trading for fixed stock loan fees. In practice, many investors choose this option unknowingly. Exchange Traded Funds (ETFs) lend out shares and use the net rebates to reduce expense ratios, increase average returns, and reduce tracking error relative to a benchmark.[7] Prospect theory investors with realization utility overvalue frequent trading as well as lottery tickets. These investors are willing to forgo frequent trading for lottery tickets, resulting in higher expected

[6] The model of realization utility is not the primary model in Barberis and Xiong (2009). However, it is fully specified, and serves as the benchmark model in this paper.

[7] Funds that organize as Unit Investment Trusts (UITs) do not lend their shares.

utility and higher expected returns.

Stock loan lotteries motivate individual investors to hold risky assets with high expected returns. Lotteries can reduce the investor's required risky-asset return by at least 2-3%. Stock loan lotteries provide the greatest benefit to poor investors, who have especially strong lottery preferences. For any particular institution's asset pool, poorer investors a smaller probability of winning a larger relative payoff. This provides the greatest boost in utility terms. Also, poor investors hold large positions in stocks with lottery features and low average returns. Jones and Lamont (2002) show that stocks with low average returns are expensive to short, so poor investors are likely to earn higher average lottery payoffs. Furthermore, stock loan lotteries provide greater benefits after considering real market frictions such as trading costs, leverage constraints, and taxes.

2. Models of Realization Utility with Objective Probabilities

This section evaluates investor allocations and outcomes in three models of realization utility, using objective probabilities to calculate expected utility. First, I describe the two-period model in Barberis and Xiong (2009) and replicate their results. Next, I construct a model where investors receive fixed stock loan fees if they commit to holding shares for two periods. Finally, I construct a model where investors receive stock loan lottery tickets if they commit to holding shares for two periods.

2.1. Baseline Two-Period Model of Realization Utility

There are three dates in the model: $t = 0$, $t = 1$, and $t = 2$. The duration of two periods is calibrated to span one year. Benartzi and Thaler (1995) suggest that the magnitude of the equity premium is consistent with investors who evaluate their performance at an annual frequency. There is a representative investor who allocates his portfolio among two securities. The risk-free asset has an annual gross return of $R_f = 1$. The risky asset has an expected annual gross return of μ and annual standard deviation of σ. If one year consists of two periods, I assume the gross one-period return of the risky asset follows an i.i.d. binomial distribution:

$$\begin{cases} R_t = R_u = \mu^{0.5} + ((\mu^2 + \sigma^2)^{0.5} - \mu)^{0.5} & \pi = 0.5 \\ R_t = R_d = \mu^{0.5} - ((\mu^2 + \sigma^2)^{0.5} - \mu)^{0.5} & 1 - \pi = 0.5 \end{cases}$$

R_u and R_d are gross risky-asset returns in the two states and π is the probability of realizing the up state in a particular period.

The investor maximizes expected utility where the value function, defined over gains and losses, follows the Tversky and Kahneman (1992) cumulative prospect theory functional form:

$$\begin{cases} v(x) = x^{\alpha} & x \geq 0 \\ v(x) = -\lambda(-x)^{\beta} & x < 0 \end{cases}$$

The investor with prospect theory preferences is risk averse over gains and risk seeking over losses, so α and β are restricted to the interval $(0, 1)$. Also, the investor is more sensitive to losses, so $\lambda > 1$.

The investor initially owns wealth W_0. At $t = 0$, the investor chooses to purchase x_0 shares of the risky asset at a price of P_0 per share. The investor is not allowed to have negative wealth at $t = 1$, so x_0 is restricted to the interval: $[0, \frac{W_0}{P_0 * (1 - R_d)}]$.[8] The remainder of the investor's wealth is allocated to the risk-free asset. Therefore, at $t = 1$, the investor's

[8] Because the expected return of the risky asset is higher than the risk-free rate, it is never optimal for the investor to hold a short position in the risky asset.

wealth is distributed:

$$\begin{cases} W_u = W_0 + P_0 x_0 (R_u - 1) & \pi = 0.5 \\ W_d = W_0 + P_0 x_0 (R_d - 1) & 1 - \pi = 0.5 \end{cases}$$

Table 1 summarizes the parameters for the Barberis and Xiong (2009) two-period model of realization utility. At $t = 1$, the investor chooses x_u and x_d, state-contingent positions in the risky asset. If $x_u < x_0$ or $x_d < x_0$, the investor sells some or all of his initial position at $t = 1$, realizing a gain or loss. The investor experiences a burst of prospect theory utility, its magnitude defined by the value function applied to the realized gain or loss. Because of limited liability, the broker will not allow the investor to have negative wealth at $t = 2$. This implies two state-specific nonnegativity constraints: x_u is restricted to the interval: $[0, \frac{W_u}{P_0 * R_u * (1 - R_d)}]$ and x_d is restricted to the interval: $[0, \frac{W_d}{P_0 * R_d * (1 - R_d)}]$. These maximum allocations depend on both the current stock price and, through the investor's wealth, his $t = 0$ allocation to the risky asset. At $t = 2$, the investor liquidates his position in the risky asset and experiences a second burst of prospect theory utility.

The investor chooses x_0, x_u, and x_d to maximize expected prospect theory utility:

$$\max_{x_0, x_u, x_d} E_0[v((x_0 - x_1)(P_1 - P_0)) * 1_{x_1 < x_0} + v(x_1 * (P_2 - P_b)) * 1_{x_1 > 0}]$$

The first term measures the investor's realization utility at $t = 1$, while the second term measures the investor's realization utility at $t = 2$. P_b is the investor's cost basis, the reference price for evaluating gains and losses at $t = 2$. The cost basis depends on whether the investor purchases shares at $t = 1$ and whether the purchase follows an up state or down state:

$$\begin{cases} P_{bu} = P_{bd} = P_0 & x_1 \leq x_0 \\ P_{bu} = \frac{x_0 * P_0 + (x_1 - x_0) * P_0 * R_u}{x_1} & x_1 > x_0 \\ P_{bd} = \frac{x_0 * P_0 + (x_1 - x_0) * P_0 * R_d}{x_1} & x_1 > x_0 \end{cases}$$

The value function at each of the four possible $t = 2$ outcomes (uu, ud, du, dd) can be written in terms of the choice variables:

$$v_{uu}(x_0, x_u, x_d) = v((x_0 - x_u)(P_u - P_0)) * 1_{x_u < x_0} + v(x_u * (P_{uu} - P_{bu})) * 1_{x_u > 0}$$

$$v_{ud}(x_0, x_u, x_d) = v((x_0 - x_u)(P_u - P_0)) * 1_{x_u < x_0} + v(x_u * (P_{ud} - P_{bu})) * 1_{x_u > 0}$$

$$v_{du}(x_0, x_u, x_d) = v((x_0 - x_d)(P_d - P_0)) * 1_{x_d < x_0} + v(x_d * (P_{du} - P_{bd})) * 1_{x_d > 0}$$

$$v_{dd}(x_0, x_u, x_d) = v((x_0 - x_d)(P_d - P_0)) * 1_{x_d < x_0} + v(x_d * (P_{dd} - P_{bd})) * 1_{x_d > 0}$$

Because $\pi = 0.5$, each node of the binomial tree is equally likely, and the investor maximizes the average value associated with each outcome:

$$\max_{x_0, x_u, x_d} 0.25 * (v_{uu} + v_{ud} + v_{du} + v_{dd})$$

I solve the model numerically by calculating $E_0(v)$ for all feasible values of $(x_0,\ x_u,\ x_d)$ and choosing arguments that maximize the value function. Table 2 summarizes the model solutions for different values of μ, holding all other parameter values constant. When the expected gross annual return of the risky asset is below 1.08, the investor's optimal decision is to invest all his wealth in the risk-free asset. Because the gross return of the risk-free asset is calibrated to $R_f = 1$, this conservative investment strategy guarantees $E_0(v) = 0$.[9] Because prospect theory investors are more sensitive to losses than gains, they require a substantial risk premium to invest in the risky asset.

[9] Setting $R_f = 1$ assumes investors evaluate the performance of the risky investment relative to the risk-free rate instead of relative to 0.

When the expected return of the risky asset is between 1.09 and 1.11, the investor exhibits a disposition effect. He chooses to invest some of his wealth in the risky asset at $t = 0$ and takes some profits at $t = 1$ when the up state occurs. Because the investor has concave realization utility over gains, he prefers to experience gains over multiple bursts. Once the expected return exceeds 12%, the investor prefers to increase his initial investment following the realization of the up state. Because the investor's portfolio appreciates in the first period, he is able to take more risk before exhausting the nonnegative wealth constraint. For assets with sufficiently high expected returns, the marginal expected prospect theory utility of increasing expected gains in the second period exceeds the marginal utility of realizing gains at $t = 1$.

2.2. Two-Period Model of Realization Utility with Fixed Stock Loan Fees

In this model, if the investor purchases shares of the risky asset at $t = 0$, he holds the position until $t = 2$. This commitment allows the centralized exchange to lend shares to institutions who want to short sell the stock. The exchange retains some proportion of the proceeds from lending the shares as a commission and pays the remainder to the investor at $t = 2$. In practice, many investors choose this option unknowingly. Exchange Traded Funds (ETFs) lend out shares and use the rebates to reduce their expense ratios, increase average returns, and reduce tracking error relative to a benchmark.

Since the investor cannot execute closing trades at $t = 1$, there are two new constraints: $x_u \geq x_0$ and $x_d \geq x_0$. The only new parameter in this model is f, the fee rate. If the investor holds x_0 shares of the risky asset at $t = 0$, he receives $f * P_0 * x_0$ at $t = 2$. I consider two values for f: five basis points (0.0005), a realistic fee for US large-cap equities, and 50 basis points (0.005), a realistic fee for US small-cap equities or foreign equities. These parameter estimates are within the range of stock lending fees in the D'Avolio (2002) and

Cohen et al. (2003) data.

Because the investor is guaranteed to receive fees at $t = 2$, x_0 is restricted to the interval: $[0, \frac{W_0}{P_0*(1-R_d^2-f)}]$. This constraint ensures that if the down state occurs in both periods, then the investor would lose 100% of his initial wealth accounting for the stock loan fees he receives at $t = 2$. Since the investor does not receive fees for any $t = 1$ purchases, the restrictions on x_u and x_d are unchanged from the baseline model.

In the fees model, the investor maximizes:

$$\max_{x_0} E_0(v) = 0.25 * (v_{uu} + v_{ud} + v_{du} + v_{dd})$$

The value at each possible $t = 2$ outcome is:

$$v_{uu}(x_0) = v((P_0 * x_0 * f + x_0 * (P_{uu} - P_0))$$

$$v_{ud}(x_0) = v((P_0 * x_0 * f + x_0 * (P_{ud} - P_0))$$

$$v_{du}(x_0) = v((P_0 * x_0 * f + x_0 * (P_{du} - P_0))$$

$$v_{dd}(x_0) = v((P_0 * x_0 * f + x_0 * (P_{dd} - P_0))$$

Table 3 summarizes the investor's optimal allocations and outcomes in the two-period model with fixed stock loan fees. The investor's $t = 0$ allocation is always a corner solution. If the prospective two-period gamble has positive expected utility for some positive allocation, then the gamble has strictly greater expected utility for a larger positive allocation.[10] Therefore, the investor either invests fully in the risky asset, exhausting the nonnegative wealth constraint, or invests fully in the risk-free asset. If the investor has a full position in the risky asset at $t = 0$, he holds the position at $t = 1$ following the realization of a down state. He is prohibited from selling shares and unable to purchase any more because the nonnegative wealth constraint still binds. When the up state is realized

[10] This follows from the functional form of the value function. See Appendix A for details.

at $t = 1$, the nonnegative wealth constraint no longer binds. The investor always chooses to purchase more shares, but never exhausts the new constraint. Eventually the marginal utility of highly weighted losses exceeds the marginal utility over gains.

For very small values of f, there is a small range of moderate μ in which the investor is better off in the baseline model. In general, the investor is better off in the fees model. The investor forgoes the utility advantage of spreading out gains over time. However, he gains utility from the greater expected gains from larger risky-asset positions as well as the guaranteed fees.

2.3. Two-Period Model of Realization Utility with Stock Loan Lotteries

In this model, the investor still promises to hold any shares he purchases at $t = 0$ until $t = 2$. However, instead of receiving a stock loan fee, the investor receives stock loan lottery tickets. At $t = 2$, the exchange holds a lottery and each investor has a 10% chance of winning a jackpot of ten times the stock loan fees.[11] The stock loan fees are net of the institution's expenses and profits, so the lottery itself is actuarially fair. In Kahneman and Tversky (1979) notation, the lottery at $t = 2$ is considered a gamble of $(\frac{f*P_0*x_0}{p}, p; 0, 1-p)$, where p is the probability of winning the lottery.[12] In this model, as in section 2.2, the stock loan fee (f) is either five basis points or 50 basis points. The probability of winning the lottery, p, is 0.1.

Incorporating the lottery payoff at $t = 2$, the investor maximizes:

$$\max_{x_0} E_0(v) = 0.25 * [p * (v_{uuw} + v_{udw} + v_{duw} + v_{ddw}) + (1-p) * (v_{uul} + v_{udl} + v_{dul} + v_{ddl})]$$

In this notation, w is the state where the investor wins the lottery and l is the state where the investor loses the lottery. The investors utility in the outcomes following the realization

[11] In the model, there is only a single risky asset. In practice, an investor's loan fees for all of his stocks would accumulate in his lottery account.

[12] I assume the investor's reference point for the gamble is the baseline model.

of two up states are:

$v_{uuw}(x_0) = v((\frac{f*p_0*x_0}{p} + x_0 * (P_{uu} - P_0))$

$v_{uul}(x_0) = v((x_0 * (P_{uu} - P_0))$

In the contingent value formulas for the six other outcomes $(udw, udl, duw, dul, ddw, ddl)$, the only difference is the share price after two periods, which is either P_{ud}, P_{du}, or P_{dd}.

The non-negative wealth constraint requires that x_0 is restricted to the interval: $[0, \frac{W_0}{P_0*(1-R_d^2)}]$. The worst case scenario is that both periods are down states and the investor doesn't win the lottery. If this scenario plays out, the investor loses 100% of his initial wealth. The optimal strategy for the investor is the same in the fees model and the lottery model. If the expected return is below some threshold, the investor keeps all his wealth in the risk-free asset. If the expected return is above the threshold, the investor exhausts the nonnegative wealth constraint at $t = 0$. If the risky asset earns a positive return in the first period, the investor buys more shares at $t = 1$, but not enough to exhaust the new constraint. If the risky asset earns a negative return in the second period, the $t = 0$ constraint still binds.

Table 4 summarizes the investor's best outcomes in each version of the model with stock loan lotteries. The investor always prefers earning a fixed stock loan fee to participating in a single risky lottery at $t = 2$ with the same expected value. This result follows from the concavity of the prospect theory value function over gains. However, the model assumes that the investor calculates expected utility using objective probabilities. Prospect theory investors overweight low-probability payoffs. This increases the value of implementing stock loan lotteries tremendously.

3. Model Extensions

In this section, I consider important extensions to the two-period models of realization utility. Tversky and Kahneman (1992) show that prospect theory investors calculate expected utility using subjective "decision weights," not probabilities. Investors are willing to pay more than the actuarially fair price for low-probability payoffs, suggesting the potential of introducing lotteries to improve utility. Investors who buy stock loan lottery contracts do not realize gains prematurely and earn higher returns. In an economy with many agents, investors with different levels of wealth will have different Kahneman and Tversky (1979) gambles in the lottery. Poor investors will have greater exposure to lottery features. This regressive feature is appealing because poor investors hold larger proportions of lottery stocks in the data. Finally, lotteries are even more appealing in an economy with market frictions because it is more costly for investors to trade excessively.

3.1. Decision Weights

Tversky and Kahneman (1992) use experimental data to estimate a functional form for $w(p)$, the "weighting function" that converts an objective probability (p) into a decision weight. Wakker et al. (1997) show that these decision weights are necessary to explain the limited willingness to pay for insurance with a nonzero default probability. A lottery is a gamble of the form (1,p;0,1-p).[13] For nonnegative gambles, Tversky and Kahneman (1992) estimate the weighting function as a two-part power function:

$w^{+}(p) = \frac{p^{\gamma}}{(p^{\gamma}+(1-p)^{\gamma})^{1/\gamma}}$

Likewise, for nonpositive gambles, the weighting function is:

$w^{-}(p) = \frac{p^{\delta}}{(p^{\delta}+(1-p)^{\delta})^{1/\delta}}$

Using experimental data, Tversky and Kahneman (1992) estimate $\gamma = 0.61$ and $\delta = 0.69$. These values imply that investors are willing to overpay for both positive and negative

[13]The investor's reference point is receiving a free lottery ticket.

gambles with low probabilities of success. The experimental evidence in Tversky and Kahneman (1992) relies on small wagers. To assuage questions of external validity, Kachelmeier and Shehata (1992) show that for economically large wagers, individuals overpay for both positive and negative payoffs with low probabilities.[14]

Figure 1 shows the potential for stock loan lotteries to increase the investor's maximum expected utility. If an investor with Tversky and Kahneman (1992) preferences receives a certain gain of one dollar, his utility is $v(1) = 1^{0.88} = 1$. Suppose the investor instead receives a free lottery ticket with an expected payoff of one dollar. This lottery ticket is equivalent to a Kahneman and Tversky (1979) gamble $(1/p, p; 0, 1-p)$ with expected utility $E_0(v|p) = w^{+}(p) * v(1/p)$. The dotted line shows the expected utility from the lottery payoff as a function of p. For values of p such that $E_0(v|p) > 1$, the investor prefers the lottery payout to a sure gain. When $p \leq 0.24$, $w(p) > p$, meaning that prospect theory investors are willing to overpay to play these lotteries. For these values of p, the expected utility from the lottery payoff is strictly decreasing in p. This relation holds because the curvature of $w^{+}(p)$ function is stronger than the concavity of the value function over gains in this region. Consistent with this functional form, Cook and Clotfelter (1993) show that lottery ticket sales are positively related to jackpot size.[15] In the context of this model, the optimal lottery is not necessarily the one with the smallest odds of winning the biggest prize. Prospect theory investors place an emphasis on reducing losses. Lotteries that pay off more frequently have more potential to reduce the negative utility from losses in the risky-asset investments.

Table 5 summarizes the investor's best outcomes in two-period models of realization utility,

[14] The authors offer lotteries in China, where the value of the wager comprises a substantial portion of the subject's income.

[15] A Washington Post article on August 22, 2017 discusses how Powerball changes lottery rules to encourage lower win probabilities and increase demand. The article's title is "How Powerball Manipulated the Odds to Make Another Massive Jackpot."

calculating expected value using both objective probabilities and decision weights. I solve the five model specifications from Section 2. In specification 1, the Barberis and Xiong (2009) baseline model (B), the investor chooses $t = 0$ and $t = 1$ risky asset positions subject to wealth constraints. In the remaining models, the investor cannot liquidate positions at $t = 1$. Specifications 2 and 3 are models with fixed loan fees (F). The investor receives a fee (f) for allowing the exchange to lend shares between $t = 0$ and $t = 2$. In specifications 4 and 5, there is a single lottery at $t = 2$ and the investor has probability p of winning $f * P_0 * x_0/p$. For each specification, the top panel presents the investor's maximum expected prospect theory utility using objective probabilities. The bottom panel presents the investor's maximum expected prospect theory utility calculated using decision weights.

In the baseline model, using decision weights has little effect on optimal allocations because $w^{-}(0.25)$ is only about 1% larger than $w^{+}(0.25)$. The dd outcome always corresponds to a loss and the uu outcome always corresponds to a gain. For the range of μ values, the ud and du outcomes correspond to gains because $R_u * R_d > 1$. In the lottery specifications, the investor's maximum expected utility is greater when prospect theory utility is calculated with decision weights. The investor strongly prefers stock loan lotteries to stock loan fees when utility is calculated with decision weights. On the other hand, the investor slightly prefers fees to lotteries when utility is calculated with objective probabilities. The fixed stock loan fees increase gains or decrease losses by a marginal amount with certainty. On the other hand, investors substantially overvalue the small probability that stock loan lotteries increase gains or decrease losses dramatically.

Relative to the Barberis and Xiong (2009) two-period model, it is much easier to increase expected utility through stock loan lotteries than through stock loan fees. As a result, investors are willing to hold positions in risky assets with lower expected returns. For instance, investors in the baseline model only hold a risky-asset position when the risk premium is at least 10%. When the lending fee is 50 basis points, investors in the fees model only require a risk premium of 9% in the fees model and only 8% in the lotteries model. In section 3.5, I show that it is possible to structure stock loan lotteries to reduce the minimum required risk premium even further. Benartzi and Thaler (1995) argue that the large equity risk premium is due to equity investors reevaluating their portfolios too frequently. Using stock loan lottery tickets to compensate investors to hold risky assets for long horizons has the potential to reduce the equity risk premium and raise stock prices.

3.2. Conditions for Welfare Gains

Barberis (2013) argues that prospect theory complements traditional economic theory. Individuals care mean and variance of wealth as well as gains or losses relative to a reference point. In this section, I argue that introducing stock loan lotteries yields unconditional welfare gains when two conditions hold. First, the investor must experience greater expected prospect theory utility in a model with stock loan lotteries than he experiences in the baseline model. The expected prospect theory utility calculation should use decision weights instead of objective probabilities because utility is a subjective measure of the investor's happiness:

$$E_0^*(v_L) > E_0^*(v_B)$$

Second, if the investor maximizes $E[U(W_2)]$ and has risk-neutral preferences, he must have higher expected wealth at $t = 2$ in the model with stock loan lotteries than he has in the baseline model:[16]

[16]Prospect theory investors are risk averse over gains and risk seeking over losses. The assumption of risk neutrality splits the difference.

$E_0^*[(W_{2,L})] > E_0^*[(W_{2,B})]$

It is imperative to consider the investor's expected wealth. If prospect theory investors only considered expected utility, they prefer to "invest" their entire wealth in lottery tickets! I use objective probabilities to measure the investor's expected wealth at $t = 2$. In the baseline model, the only source of wealth is the terminal portfolio value:

$$E_0[(W_{2,B})] = 0.25 * [W_{uu} + W_{ud} + W_{du} + W_{dd}]$$

In the models with fees or lotteries, the investor earns additional guaranteed or expected wealth on his initial risky-asset allocation:

$$E_0[(W_{2,L/F})] = [P_0 * x_0 * f] + 0.25 * [W_{uu} + W_{ud} + W_{du} + W_{dd}]$$

The terminal wealth at each of the possible $t = 2$ nodes depends on the $t = 0$ and conditional $t = 1$ allocations:

$$W_{uu}(x_0, x_u, x_d) = W_0 + P_0 * x_0 * (R_u^2 - 1) + P_u * (x_u - x_0) * (R_u - 1)$$

$$W_{ud}(x_0, x_u, x_d) = W_0 + P_0 * x_0 * (R_u * R_d - 1) + P_u * (x_u - x_0) * (R_d - 1)$$

$$W_{du}(x_0, x_u, x_d) = W_0 + P_0 * x_0 * (R_d * R_u - 1) + P_d * (x_d - x_0) * (R_u - 1)$$

$$W_{dd}(x_0, x_u, x_d) = W_0 + P_0 * x_0 * (R_d^2 - 1) + P_d * (x_d - x_0) * (R_d - 1)$$

Because $R_u + R_d = 2\sqrt{\mu}$, the expected wealth calculations simplify to:

$$E_0[(W_{2,B})] = W_0 + P_0x_0(\mu - 1) + 0.5[P_u(x_u - x_0) + P_d(x_d - x_0)](\sqrt{\mu} - 1)$$

$$E_0[(W_{2,L/F})] = [P_0x_0f] + W_0 + P_0x_0(\mu - 1) + 0.5[P_u(x_u - x_0) + P_d(x_d - x_0)](\sqrt{\mu} - 1)$$

Figure 2 presents conditions in which introducing stock loan lotteries or fees delivers provides welfare gains. For all points above the solid curve, investors in the model with stock loan lotteries have strictly higher welfare than investors in the baseline model. For these points in (μ, f) space, the utility-maximizing allocation in the model with lotteries produces greater expected utility and greater expected wealth than the utility-maximizing allocation in the baseline model. The solid curve is kinked, first decreasing in μ to some threshold value of μ. For risky-asset returns above this threshold, the investor already wants to maximize risky-asset holdings and any positive f is sufficient to produce greater welfare.

For points above the dotted curve, investors in the model with stock loan fees have strictly higher welfare than investors in the baseline model. The dotted curve is also kinked, but for a wide range of parameter values, introducing stock loan lotteries produces welfare gains, while introducing stock loan fees does not. Graphically, this region is the area between the curves. For moderate values of μ, this range of fees could include a significant proportion of all stocks. For example, investors require a flat fee of 143 basis points to hold stocks with $\mu = 1.08$, but an expected fee of only 30 basis points denominated in lottery tickets. [17] Table 6 suggests that increasing the frequency or lowering the win probability of lotteries would further widen the region of potential welfare gains.

3.3. Implications for Equilibrium with Heterogeneous Agents

Investors benefit from stock loan lotteries as long as the probability of winning the lottery is in the range where the decision weight exceeds the objective probability. Consider a stock loan marketplace with many individual investors. Each investor, i, owns $x_{0,i}$ shares of the risky asset at $t = 0$. An investor's probability of winning the lottery is the ratio of his allocation to the total allocation of all investors, $x_{0,A}$. Since investors in the lottery model who buy the risky asset always exhaust their non-negative wealth constraints, each investor's win probability equals his wealth share:

[17] This range widens for lower values of μ, however the wider range is likely to include fewer stocks since it is rare for US stocks to command high lending fees.

$p = \frac{x_{0,i}}{x_{0,A}} = \frac{W_{0,i}}{W_{0,A}}$

I examine the relative value of stock loan lotteries to different investors by calculating a standardized measure of utility. Define Π as this standardized measure of utility per unit of wealth:

$\Pi_i = \frac{E_0[U(W_{2,i})|p,v(),w^+(p)]}{W_{0,i}}$

Figure 3 shows how standardized utility varies according to wealth share in the fees and lotteries models. The solid curves show standardized utility for models with a single stock loan lottery at $t = 2$ and stock loan fees of five and 50 basis points. The dotted curves show standardized utility for models with stock loan fees of five and 50 basis points. [18] In all four models, standardized utility is strictly downward sloping with respect to wealth share. All these models are regressive in the sense that poor investors benefit disproportionately in utility terms.

Models with stock loan fees are regressive because the prospect theory value function is concave over gains, so the marginal utility of wealth is strictly decreasing with increasing wealth. The lotteries model is even more regressive because of the functional form of the decision weighting function. The ratio $w^+(p)/p$ is strictly decreasing with increasing p. The poorest investors have the lowest probabilities of winning the lotteries and place the highest value on the lottery payoffs relative to the value implied by objective probabilities. For this reason, increasing f provides a larger regressive boost in the lotteries model than in the fees model.

Since the available investments include only a single risky asset, the lottery payoffs are the only source of idiosyncratic returns. The idiosyncratic volatility and idiosyncratic skew of the returns from lottery payoffs are both strictly decreasing with increasing wealth. In the

[18] In all models, the annualized expected gross return of the risky asset (μ), is 1.12.

lotteries model, each investor has probability p of winning a jackpot of $f * W_{0,A}$. Winning the lottery provides a gross return of:

$R_L = \frac{f * W_{0,A}}{W_{0,i}} = \frac{f}{p}$.

Losing the lottery provides a gross return of 0. The idiosyncratic volatility and skew are:

$E(R_L^2) = p * (\frac{f}{p})^2 = \frac{f^2}{p}$

$E(R_L^3) = p * (\frac{f}{p})^3 = \frac{f^3}{p^2}$

Kumar (2009) shows that poor investors have especially strong preferences for stocks with positive idiosyncratic volatility and idiosyncratic skew. Since idiosyncratic volatility and idiosyncratic skew of the lotteries are both strictly decreasing in wealth share, introducing the lotteries effectively targets heterogeneous preferences.

3.4. Market Frictions: Trading Costs, Leverage, and Taxes

The three models of realization utility assume that markets are frictionless. However, excessive trading by individual investors is a problem, in part, because trading is costly. French (2008) estimates that the total cost of active trading ranges from 61 to 74 basis points between 1990 and 2006. Trading costs are relatively constant over time because of two opposing trends. Novy-Marx and Velikov (2016) document a significant decrease in the cost of trading a share of stock over time. On the other hand, French (2008) documents a significant increase in share turnover over time. I model market frictions by introducing a new parameter, ρ, representing round-trip trading costs. Since the two periods in the model correspond to a year, and all positions are closed at $t = 2$, I use the estimates in French (2008) and set $\rho = 0.013$.

Another important market friction is the cost and availability of leverage. In the Barberis and Xiong (2009) model, investors can borrow or lend at the risk-free rate, and the non-negative wealth constraint determines the maximum leverage. In fact, Frazzini and Pedersen (2014) show that leverage constraints lead to high demand and low expected returns for high-beta assets. Barberis and Xiong (2009) acknowledge that while optimal allocations imply the use of substantial leverage, individual investors rarely use leverage. Since the US Federal Reserve Board set a 50% initial margin requirement since 1974, I model leverage constraints by limiting the investor's risky asset investment to twice his wealth.[19] In all three models, this restricts the $t = 0$ allocation to $x_0 \leq \frac{2*W_0}{P_0}$, and the state-contingent investments at $t = 1$ to $x_u \leq \frac{2*W_u}{P_u}$ and $x_d \leq \frac{2*W_d}{P_d}$.

Figure 4 shows the implications of these market frictions on the welfare benefits from stock loan lotteries. For different values of μ, I solve all three models in a perfect market as well as a market with trading costs and leverage constraints. For each value of μ, in each environment, I calculate the minimum fee that produces welfare gains in the fee (f_F) and lottery (f_L) models. For each μ, a proxy for the potential welfare benefits of introducing stock loan lotteries is $max[f_F(\mu) - f_L(\mu), 0]$. The solid line shows the potential for welfare improvement with market frictions. The dotted line shows the potential for welfare improvement with perfect markets.

Market frictions increase the potential for lotteries to improve welfare for two reasons. First, trading costs reduce the effective risky-asset return. As Figure 2 shows, it is far easier to persuade investors to buy risky assets with low expected returns by using stock loan lotteries than by using stock loan fees. Second, in the baseline model, investors often choose allocations with $x_u < x_0$ to spread out the realization of gains over multiple episodes. Market frictions make this strategy more costly. This example only considered

[19]Regulation T allows the Federal Reserve to change the margin requirement, but the Federal Reserve has never changed it.

some trading costs. Adding capital gains taxes to the model increases the value of stock loan lotteries even more.

3.5 Determining the Optimal Lottery Structure

The analysis thus far compares the baseline model and fees model to a particular model with stock loan lottery tickets. Specifically, the investor participates in a single lottery with a 10% chance of winning a jackpot that is ten times larger than the fixed stock loan fees. In this section, I consider three alternative lottery structures. In the first alternative, there is a single lottery with a 1% chance of winning a jackpot that is 100 times larger than the fixed stock loan fees. In the two other alternatives, the investor participates in lotteries at both $t = 1$ and $t = 2$. Either the lotteries each have a 10% chance of winning jackpots of five times the fixed fees or each have a 1% chance of winning jackpots of 50 times the fixed fees.

In the model with lotteries at $t = 1$ and $t = 2$, there are 16 possible outcomes. The outcomes depend on whether the up or down state is realized in each period, and whether the investor wins or loses the two lotteries. The investor maximizes:

$$\max_{x_0} 0.25 * [p^2 * (v_{uuww} + v_{udww} + v_{duww} + v_{ddww}) + p(1-p) * (v_{uuwl} + v_{udwl} + v_{duwl} + v_{ddwl} + v_{uulw} + v_{udlw} + v_{dulw} + v_{ddlw}) + (1-p)^2 * (v_{uull} + v_{udll} + v_{dull} + v_{ddll})]$$

Because the investor savors each burst of prospect theory utility distinctly, a lone lottery win at $t = 1$ by itself has a different value than a lone lottery win at $t = 2$ accompanied by realized gains or losses. The four representative value formulas are:

$$v_{uuww}(x_0) = v(\frac{0.5f*p_0*x_0}{p}) + v(\frac{0.5f*p_0*x_0}{p} + x_0 * (P_{uu} - P_0))$$

$$v_{uuwl}(x_0) = v(\frac{0.5f*p_0*x_0}{p}) + v(x_0 * (P_{uu} - P_0))$$

$$v_{uulw}(x_0) = v(\frac{0.5f*p_0*x_0}{p} + x_0 * (P_{uu} - P_0))$$

$v_{uull}(x_0) = v(x_0 * (P_{uu} - P_0))$

Table 6 presents the investor's best outcomes in eight specifications of models with stock loan lotteries. These specifications differ along three dimensions. The fee is either five or 50 basis points, lotteries have either a 1% or 10% win probability, and there is either a lottery at $t = 2$ only or at both $t = 1$ and $t = 2$. Although the Tversky and Kahneman (1992) decision weighting function suggests that investors prefer low-probability lotteries, the concavity of the value function favors higher-probability lotteries.

When the fee is five basis points, investors prefer the 10% lottery. Because the potential lottery payoffs are not very large, investors prefer to win more frequently. When the fee is 50 basis points and the risky asset has a low return, investors prefer a 1% lottery. Investors only hold risky assets with low returns because of how much they value the low-probability lottery payoffs. When the risky asset has a high return, investors prefer the 10% lottery because their marginal utility of winning large lottery payoffs is lower.

Regardless of the stock loan fees and lottery win probability, investors always prefer to participate in two small lotteries at $t = 1$ and $t = 2$. The only way investors can realize gains at $t = 1$ if they can't sell stock is by winning lotteries. Designing a lotteries model with a more optimal lottery structure can significantly change investor behavior. When fixed fees are five basis points, investors require an expected return of 10% to invest in the risky asset, but under the optimal lottery structure, this drops to 8%. When fixed fees are 50 basis points, the required risky asset return is 9%, but this drops to 6% under the optimal lottery structure.

Figure 5 shows how different lottery structures can increase the motivation for individual investors to purchase risky assets. For four different lottery structures and a range of risky-asset returns, this figure shows the minimum fee that motivates the investor to purchase the risky asset. Compared to the baseline model with a single 10% lottery at $t = 2$, adding a second 10% lottery at $t = 1$ motivates investors to purchase risky-assets with lower expected returns. However, investors must have a decent chance of winning these more frequent lotteries. Replacing the 10% lottery with a 1% lottery can substantially reduce the fee needed to generate investment in risky-assets with smaller risk premiums. Investors place a very high value on these lower probability lotteries. Investors are willing to purchase risky assets with low Sharpe ratios if these assets are bundled with low-probability lottery payoffs. These results suggest that regulators could substantially improve individual investor outcomes by optimizing the structure of stock loan lotteries.

4. Implementing Stock Loan Lotteries in Practice

This section addresses practical questions about how to implement stock loan lotteries. Does a laboratory experiment demonstrate whether there is a demand among individual investors for stock loan lotteries? What are the relevant lottery laws and regulations in countries with developed equity markets? Is it possible for exchanges to offer financial securities that replicate lottery payouts?

4.1. Survey Evidence

The Tversky and Kahneman (1992) decision weighting function specifies that prospect theory investors prefer fair lotteries with low win probabilities to certain gains. In the Barberis and Xiong (2009) model of realization utility, providing lottery payoffs to investors as compensation to hold long-term positions leads to better outcomes. In practice, would individuals have demand for stock loan lottery tickets? This section describes a randomized controlled trial (RCT) experiment to test this question.

In this research design, each participant completes six trials. In each trial, he or she allocates a portfolio among a risky stock fund and a risk-free bond fund. For the first three trials, each participants complete the same three allocation tasks. The risk-free bond fund always returns 3% while the risky stock fund has different risk and return characteristics in each trial. In the second set of six trials, subjects in the control group have different allocation tasks than subjects in the treatment group. Subjects in the control group have three new allocation tasks and the risky stock fund has returns that are 1% higher with certainty. Subjects in the treatment group have three new allocation tasks and the risky stock fund has a 1% probability of returns that are 100% higher.

The dependent variable is the change in the percentage of risky stock fund investment after adding the expected 1% return incentive. The coefficient of interest is the lottery treatment dummy. Other independent variables include controls for age, education, gender, and race. Barber and Odean (2001) show that men turn over their portfolios more frequently than women. Kumar (2009) provides evidence that socioeconomic factors explain investment in stocks with lottery features. Feng and Seasholes (2005), Dhar and Zhu (2006), and Frazzini (2006) document a more pronounced disposition effect in the trading activity of less experienced investors. Alevy et al. (2007) show that professional traders outperform students in experimental tasks.

Other control variables include experimental measures of risk tolerance and trust. Subjects complete a test developed by Eckel and Grossman (2002). In this test, subjects choose their favorite lottery from among several alternatives. Participants also complete a survey developed by Weber et al. (2002) to measure financial risk tolerance.[20] Subjects also answer a question about how much they trust the stock market using a Likert scale. Guiso et al. (2008) show that less trusting individuals are less willing to participate in the stock

[20]Weber et al. (2002) ask respondents to indicate their willingness to participate in several investment or gambling activities using a Likert scale. One activity in the investment risk taking questionnaire is: "Investing 5% of your annual income in a very speculative stock." One activity in the gambling risk taking questionnaire is: "Betting a day's income at a high stake poker game."

market.

I select survey participants using Amazon's Mechanical Turk (MTurk) platform. Casler et al. (2013) find that the MTurk community is more demographically diverse than samples recruited on college campuses or via social media. Survey participants will receive $1 for a task which takes an average of 15 minutes to complete. Camerer and Hogarth (1999) review the literature on field experiments with varying levels of financial incentives. The authors find that in similar types of experiments, increasing incentives does not have a significant impact on average behavior. Cryder et al. (2012) show that MTurk participants exhibit standard behavioral biases and their average responses are similar to other survey populations. However, some MTurk participants do not pay attention to or understand the instructions, introducing noise into the responses. I include two "attention check" questions and exclude data from participants who fail either attention check. I also exclude participants who complete the survey very quickly. Finally, I exclude cases when a participant changes their risky stock fund investment by more than 50% since these likely result from data entry errors.

Table 7 summarizes the experimental regression results. In specification 1, the independent variables include an intercept and Lottery Flag, an indicator of whether the subject is in the treatment group. In specification 2, additional controls include the subject's gender, age, education, race, risk tolerance, and trust. In specification 1, the coefficient estimate for the Lottery Flag is 8.83%, significant at the 1% level. Overall, investors in the control group invest 51% of their portfolio in the risky stock fund without stock loan fees and 51% with stock loan fees. Subjects in the treatment group invest 50% of their portfolio in the risky stock fund without stock loan lotteries and 56% with stock loan lotteries. In specification 2, the coefficient estimate for the Lottery Flag is 7.37%, significant at the 5% level. This suggests that the effectiveness of stock loan lotteries is not explained by systematic differences between the treatment and control groups.

Appendix B includes the survey questions for both the control and treatment groups.

4.2. Stock Loan Lottery Tickets as Derivative Securities

How could stock loan lotteries be implemented in practice? The exchange could register to operate the lottery or partner with a private firm that already operates lotteries. Then the exchange could use the stock loan fees to purchase tickets for individual investors. Since individuals are willing to overpay for lottery tickets, lotteries are a form of voluntary taxation. As a result, national governments set laws and regulations for operating any lotteries within its borders. In the United States, the country with the largest developed equity markets, lotteries are subject to the laws of individual states and territories.[21] Other developed countries have more manageable regulatory structures. For instance, there are only five regional organizations that regulate Canadian lotteries. The United Kingdom has a single national commission that issues a single license.

Alternatively, the exchange could structure the lottery tickets as derivative securities. Stock loan lottery tickets can be structured as derivatives with payoffs linked to the daily closing price of a portfolio of securities. For example, a "Pennies Series X" derivative has a payoff of 1 if X is the pennies (hundredths) digit of the closing price of an equity index on a particular day. Because major equity indices are portfolios of a large number of individual stocks in specified proportions, it is not feasible to manipulate the pennies digit of the index price by buying or selling the component securities. Although Harris (1991) and others identify "round number clustering" in individual stock prices, this is unlikely to translate to clustering in a diversified stock portfolio.

[21]Six states (Alabama, Alaska, Hawaii, Mississippi, Nevada, and Utah) prohibit lotteries completely. On the other hand, all the other states coordinate to offer Mega Millions and Powerball.

Figure 6 provides evidence suggesting there is no clustering in the daily closing prices of major diversified equity indices. The black bars show the penny digit frequencies in the closing prices of the S&P 500 index between December 30, 1927 and April 21, 2017. The S&P 500 is a capitalization-weighted index of 500 diversified large US public equities. The gray bars show the penny digit frequencies in the closing prices of the Nikkei 225 index between January 5, 1970 and April 21, 2017. The Nikkei 225 is a price-weighted index of 225 diversified large Japanese public equities. There is no evidence of clustering at zero pennies or any other value. This suggests that a centralized exchange could issue stock loan lottery tickets by buying and issuing financial derivatives instead of holding traditional lottery drawings, potentially bypassing regulations.

5. Conclusion

The tendency for individual investors to trade excessively has large economic consequences. Private firms seek to exploit the psychological biases that lead to costly behavior. Brokers, for example, encourage investors to use mobile apps in order to trade more frequently. Stock loan lotteries can improve individual investor performance. Investors with realization utility need to be compensated to forgo the extra trading necessary to realize gains. It is cheaper to compensate individual investors with stock loan lottery tickets than with stock loan fees because individual investors with prospect theory preferences overvalue lottery payoffs.

Investors participating in stock loan lotteries can experience greater expected utility and earn higher expected returns, while allowing other market participants to earn profits from securities lending and administering the lottery itself. Two promising features of stock loan lotteries are that they provide the greatest utility to the poorest investors and that their value increases in a model with market frictions. The value of stock loan lotteries may be even larger than the theoretical results if, as Dorn et al. (2014) finds, investors who participate in stock loan lotteries reduce their gambling activity outside of the financial markets.

The Barberis and Xiong (2009) model has a time horizon of one year. In reality, many investors have long-term financial goals. To the extent that investors care about their children or heirs, a recursive argument suggests investors optimize over an infinite time horizon. Barberis and Xiong (2012) and Henderson (2012) consider the implications of realization utility in an infinite-horizon model framework. Evaluating the implications of introducing stock loan lotteries in one of these models can yield new insights about the potential for this financial innovation.

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Table 1: Baseline Model Parameters This table lists the important parameters, symbols, and calibrated values for the Barberis and Xiong (2009) two-period model of realization utility. The investor has W_0 in wealth at $t = 0$. He allocates his wealth between the risk-free asset, which has a normalized gross return of $R_f = 1$, and a risky asset. The risky asset has an annualized mean return of μ and an annualized standard deviation of σ. The risky asset return in each period has an i.i.d. binomial distribution with an equal probability of returning R_u in the up state and returning R_d in the down state. R_u and R_d are chosen so that the risky asset return has an annual mean of μ and standard deviation of σ. The investor chooses x_0, the $t = 0$ risky asset allocation, x_u, the $t = 1$ risky asset allocation following the up state, and x_d, the risky asset allocation following the down state. The investor maximizes $E_0(v)$, the total expected value at $t = 1$ and $t = 2$, and $v()$ is the Tversky and Kahneman (1992) value function applied to realized gains and losses.

Parameter	Symbol	Value(s)
Initial Wealth	W_0	40
Risk-Free Asset: Annualized Gross Return	R_f	1
Risky Asset: Initial Price	P_0	40
Risky Asset: Annualized Mean Return	μ	1.05-1.15
Risky Asset: Annualized SD Return	σ	0.3
Value Function: Concavity	α	0.88
Value Function: Convexity	β	0.88
Value Function: Sensitivity	λ	2.25

Table 2: Optimal Allocations and Best Outcomes in the Baseline Model For different values of μ, the annualized expected gross return of the risky asset, this table lists the investor's optimal allocations and best outcome in the baseline Barberis and Xiong (2009) two-period model of realization utility. In all cases, σ, the annualized standard deviation of risky asset returns, is 0.3. Table 1 lists and describes the other important model parameters. The model assumes there are two periods in a year, and the gross one-period return of the risky asset follows an i.i.d. binomial distribution with equal probabilities of realizing a return of R_u in the up state and R_d in the down state. The choice variables are the risky asset allocations at $t = 0$ (x_0) and the risky asset allocations at $t = 1$ following the realization of the up (x_u) or down (x_d) states. $E_0(v)$ is expected $t = 0$ cumulative prospect theory utility. The investor experiences utility at $t = 1$ and $t = 2$ if he realizes gains or losses, and the bursts of future prospect theory utility are not discounted by time. The value function for prospect theory utility uses the functional form and parameter values in Tversky and Kahneman (1992).

μ	R_u	R_d	x_0^*	x_u^*	x_d^*	$E_0^*(v)$
1.05	1.230	0.820	0	0	0	0
1.06	1.234	0.826	0	0	0	0
1.07	1.238	0.831	0	0	0	0
1.08	1.241	0.837	0	0	0	0
1.09	1.245	0.843	3.4	2.6	3.4	0.01
1.10	1.249	0.848	3.6	2.8	3.6	1.09
1.11	1.253	0.854	3.7	3.0	3.7	2.22
1.12	1.257	0.860	3.8	5.4	3.8	3.66
1.13	1.261	0.865	4.0	6.0	4.0	5.29
1.14	1.265	0.871	4.1	6.6	4.1	7.01
1.15	1.269	0.876	4.2	7.2	4.4	8.95

Table 3: Optimal Allocations and Best Outcomes in the Stock Loan Fees Model For different values of μ, the annualized expected gross return of the risky asset, this table lists the investor's optimal allocations and best outcome in various two-period models of realization utility. The left panel shows results from the baseline (B) model. The choice variables are the risky asset allocations at $t = 0$ (x_0) and the risky asset allocations at $t = 1$ following the realization of the up (x_u) or down (x_d) states. $E_0(v)$ is the expected $t = 0$ cumulative prospect theory utility the investor experiences at $t = 1$ and $t = 2$ if he realizes gains or losses, and the bursts of prospect theory utility are not discounted by time. The value function for prospect theory utility uses the functional form and parameter values in Tversky and Kahneman (1992). The center and right panels show results from models with stock loan fees (F). The investor is not allowed to execute closing trades at $t = 1$. The investor receives a lending fee of f at $t = 2$ for allowing the exchange to lend shares between $t = 0$ and $t = 2$. The center panel shows the investor's optimal allocations and best outcome when f is five basis points. The right panel shows the investor's optimal allocations and best outcome when f is 50 basis points. The asterisks denote cases in which the investor's maximum expected utility in the model with stock loan fees is greater than the investor's maximum utility in the baseline model.

Model	B				F(5)				F(50)			
μ	x_0^*	x_u^*	x_d^*	$E_0^*(v)$	x_0^*	x_u^*	x_d^*	$E_0^*(v)$	x_0^*	x_u^*	x_d^*	$E_0^*(v)$
1.05	0	0	0	0	0	0	0	0	0	0	0	0
1.06	0	0	0	0	0	0	0	0	0	0	0	0
1.07	0	0	0	0	0	0	0	0	0	0	0	0
1.08	0	0	0	0	0	0	0	0	0	0	0	0
1.09	3.4	2.6	3.4	0.01	0	0	0	0	3.5	4.2	3.5	*0.22
1.10	3.6	2.8	3.6	1.09	3.6	4.6	3.6	0.93	3.6	4.7	3.6	*1.48
1.11	3.7	3.0	3.7	2.22	3.7	5.0	3.7	*2.27	3.8	5.2	3.8	*2.87
1.12	3.8	5.4	3.8	3.66	3.8	5.5	3.8	*3.75	3.9	5.7	3.9	*4.39
1.13	4.0	6.0	4.0	5.29	4.0	6.0	4.0	*5.37	4.1	6.2	4.1	*6.07
1.14	4.1	6.6	4.1	7.01	4.1	6.6	4.1	*7.16	4.2	6.9	4.2	*7.92
1.15	4.2	7.2	4.4	8.95	4.3	7.4	4.3	*9.16	4.4	7.6	4.4	*9.99

Table 4: Best Outcomes in the Stock Loan Lotteries Model For different values of μ, the annualized expected gross return of the risky asset, this table lists the investor's best outcome in two models of realization utility with stock loan lotteries. The choice variables are the risky asset allocations at $t = 0$ (x_0) and the risky asset allocations at $t = 1$ following the realization of the up (x_u) or down (x_d) states. The investor is not allowed to sell any risky-asset holdings at $t = 1$. The table shows $E_0^*(v)$, the investor's optimal expected $t = 0$ cumulative prospect theory utility at $t = 1$ and $t = 2$ if he realizes gains or losses. Prospect theory utility is determined by realized gains and losses at $t = 1$ and $t = 2$ and not discounted by time. The value function for prospect theory utility uses the functional form and parameter values in Tversky and Kahneman (1992). The investor receives a fee of f for allowing the exchange to lend shares between $t = 0$ and $t = 2$. The fee is structured as a single lottery at $t = 2$ and the investor has probability p of winning $f * P_0 * x_0/p$. The asterisks denote cases in which the investor's maximum expected utility in the model with stock loan lotteries is greater than the investor's maximum utility in the baseline model.

p	0.1	0.1
f (BPs)	5	50
Model	L(2)	L(2)
$\mu = 1.05$	0	0
$\mu = 1.06$	0	0
$\mu = 1.07$	0	0
$\mu = 1.08$	0	0
$\mu = 1.09$	0	*0.13
$\mu = 1.10$	0.93	*1.44
$\mu = 1.11$	*2.27	*2.80
$\mu = 1.12$	*3.74	*4.28
$\mu = 1.13$	*5.36	*5.92
$\mu = 1.14$	*7.15	*7.73
$\mu = 1.15$	*9.14	*9.74

Table 5: Best Outcomes in Models with Decision Weights For different values of μ, the annualized expected gross return of the risky asset, this table lists the investor's best outcome in various two-period models of realization utility. Specification 1 is the baseline model in Barberis and Xiong (2009). The choice variables are the risky asset allocations at $t = 0$ (x_0) and the risky asset allocations at $t = 1$ following an up (x_u) or down (x_d) return in the first period. Specifications 2 and 3 are models where the investor chooses a single risky asset allocation at $t = 0$ (x_0) and is not allowed to trade shares at $t = 1$. The investor receives a fee of f for allowing the exchange to lend shares between $t = 0$ and $t = 2$. Specifications 4 and 5 are models where the stock loan fee is pooled into a lottery, and the investor has probability p of winning the lottery. There is a single lottery (L) at $t = 2$ where the investor has probability p of winning $f * P_0 * x_0/p$. The table shows $E_0(v)$, the investor's $t = 0$ cumulative prospect theory utility from realized gains and losses at $t = 1$ and $t = 2$. In the top panel, the investor maximizes expected prospect theory utility using objective probabilities. In the bottom panel, the investor maximizes prospect theory utility using decision weights. The value function and decision weighting function use the functional form and parameter values in Tversky and Kahneman (1992).

	1	2	3	4	5
p				0.1	0.1
f (BPs)		5	50	5	50
Model	B	F(5)	F(50)	L(2)	L(2)
$\mu = 1.05$	0	0	0	0	0
$\mu = 1.06$	0	0	0	0	0
$\mu = 1.07$	0	0	0	0	0
$\mu = 1.08$	0	0	0	0	0
$\mu = 1.09$	0.01	0	0.22	0	0.20
$\mu = 1.10$	1.09	0.93	1.48	0.93	1.44
$\mu = 1.11$	2.22	2.27	2.87	2.27	2.80
$\mu = 1.12$	3.66	3.75	4.39	3.74	4.28
$\mu = 1.13$	5.29	5.37	6.07	5.36	5.92
$\mu = 1.14$	7.01	7.16	7.92	7.15	7.73
$\mu = 1.15$	8.95	9.16	9.99	9.14	9.74
$\mu = 1.05$	0	0	0	0	0
$\mu = 1.06$	0	0	0	0	0
$\mu = 1.07$	0	0	0	0	0
$\mu = 1.08$	0	0	0	0	0.77
$\mu = 1.09$	0	0	0.09	0.82	2.48
$\mu = 1.10$	1.11	0.92	1.56	2.63	4.34
$\mu = 1.11$	2.43	2.48	3.17	4.61	6.36
$\mu = 1.12$	4.08	4.20	4.94	6.77	8.59
$\mu = 1.13$	5.92	6.08	6.90	9.16	11.03
$\mu = 1.14$	8.01	8.17	9.06	11.82	13.74
$\mu = 1.15$	10.34	10.50	11.50	14.71	16.74

Table 6: Optimal Lottery Structure in the Stock Loan Lotteries Model For different values of μ, the annualized expected gross return of the risky asset, this table lists the investor's best outcome in various two-period models of realization utility. The investor chooses a risky asset allocation (x_0) at $t = 0$ and must hold this allocation until $t = 2$. The investor can also add to his risky asset allocation at $t = 1$. The investor receives a fee (f) on his $t = 0$ risky asset allocation for holding the risky asset for two periods. In some specifications, there is a single lottery at $t = 2$ [L(2)] and the investor has probability p of winning $f * P_0 * x_0/p$. Alternatively, there are lotteries at both $t = 1$ and $t = 2$ [L(1,2)] and the investor has probability p of winning $0.5 * f * P_0 * x_0/p$ in each lottery. The values in the table are $E_0^*(v)$, the investor's maximum total expected $t = 0$ cumulative prospect theory utility at $t = 1$ and $t = 2$. Utility is measured over realized gains or losses, and the bursts of prospect theory utility are not discounted by time. The value function and decision weighting function use the functional form and parameter values in Tversky and Kahneman (1992).

	1	2	3	4	5	6	7	8
p	0.01	0.1	0.01	0.1	0.01	0.1	0.01	0.1
f (BPs)	5	5	5	5	50	50	50	50
Model	L(2)	L(2)	L(1,2)	L(1,2)	L(2)	L(2)	L(1,2)	L(1,2)
$\mu = 1.05$	0	0	0	0	0	0	0	0
$\mu = 1.06$	0	0	0	0	0	0	0.40	0
$\mu = 1.07$	0	0	0	0	1.11	0	1.84	0.55
$\mu = 1.08$	0	0	0	0.49	2.50	0.77	3.36	2.51
$\mu = 1.09$	0.41	0.82	1.04	2.57	4.01	2.48	5.01	4.65
$\mu = 1.10$	1.96	2.63	2.71	4.83	5.65	4.34	6.81	6.97
$\mu = 1.11$	3.66	4.61	4.54	7.30	7.44	6.36	8.77	9.51
$\mu = 1.12$	5.52	6.77	6.55	10.02	9.40	8.59	10.93	12.32
$\mu = 1.13$	7.57	9.16	8.76	13.00	11.56	11.03	13.30	15.41
$\mu = 1.14$	9.84	11.82	11.21	16.30	13.89	13.74	15.89	18.67
$\mu = 1.15$	12.34	14.71	13.92	19.97	16.56	16.74	18.76	22.45

Table 7: Experimental Evidence of Demand for Stock Loan Lotteries This table shows results from a laboratory experiment that tests whether individuals would participate in stock loan lotteries. Subjects are randomly assigned to the control group or treatment group. Subjects in both groups participate in six investment allocation tasks. In the first set of three tasks, the subjects allocate their portfolio between a risk-free bond fund and a risky stock fund. In all three trials, the survey explicitly states the guaranteed return of the bond fund and the expected return and variance of the stock fund. In the second set of three tasks, subjects in the control group are presented the same allocation problems, but the returns of the stock fund are 1% higher. In the second set of three tasks, subjects in the control group are presented the same allocation problems, but the stock fund has a 1% probability of returns that are 100% higher. The dependent variable is the difference in a subject's stock fund allocation when stock loan fees or lotteries are added. In specification 1, the independent variables include an intercept and Lottery Flag, an indicator of whether the subject is in the treatment group. In specification 2, additional controls include the subject's gender, age, race, risk tolerance, trust, and education level. Lottery Flag is measured in percent. T-statistics are in parentheses.

	1	2
Lottery Flag	8.83	7.37
	(2.74)	(2.09)
Controls	Y	N
N	60	60
Adj. R^2	0.099	0.192

Figure 1: Utility Gains from Lottery Payoffs This figure presents potential utility gains from compensating prospect theory investors with lottery payoffs. The weighting function for nonnegative gambles in Tversky and Kahneman (1992) is a two-part power function: $w^+(p) = \frac{p^\gamma}{(p^\gamma+(1-p)^\gamma)^{1/\gamma}}, \gamma = 0.61$. A certain gain of one dollar provides utility of $v(1) = 1^{0.88} = 1$, where $v()$ indicates the Tversky and Kahneman (1992) value function. A free lottery ticket with an expected payoff of one dollar is a Kahneman and Tversky (1979) gamble $(1/p, p; 0, 1-p)$ with expected utility $E_0(v|p) = w^+(p) * v(1/p)$. The dotted line shows the expected utility as a function of p. For values of p such that $E_0(v|p) > 1$, the investor prefers the lottery payout to a sure gain.

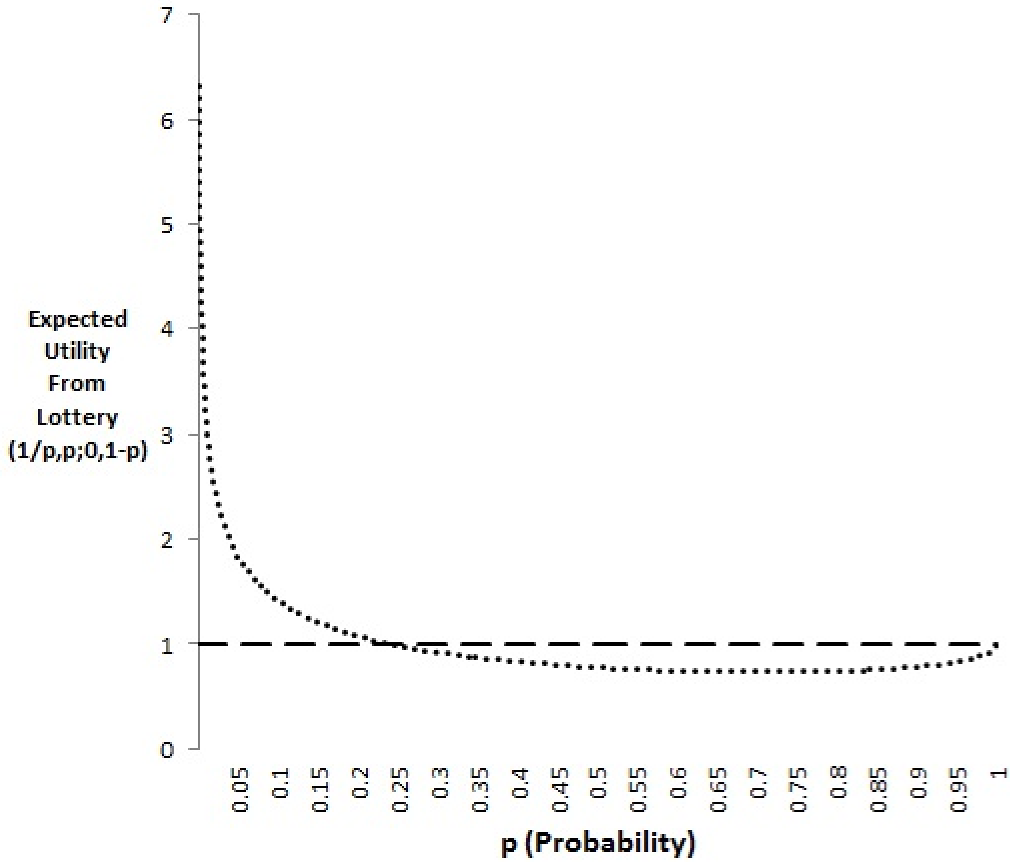


Figure 2: Conditions for Welfare Gains This figure shows conditions in which stock loan fees or stock loan lotteries produce welfare gains. In any model, an investor chooses allocations to maximize prospect theory utility. An investor has unconditionally greater welfare if his allocations in one model produce higher expected utility and higher expected wealth than his allocations in another model. For all points above the dotted curve, the investor has unconditionally greater welfare in the model with stock loan fees than in the baseline model of realization utility. For all points above the solid line, the investor has unconditionally greater welfare in the model with stock loan lotteries than in the baseline model of realization utility. In the model with stock loan lotteries, there is a single lottery at $t = 2$ with a probability $p = 0.1$ of winning the lottery. The area between the curves are conditions in which stock loan lotteries produce welfare gains and stock loan fees do not.

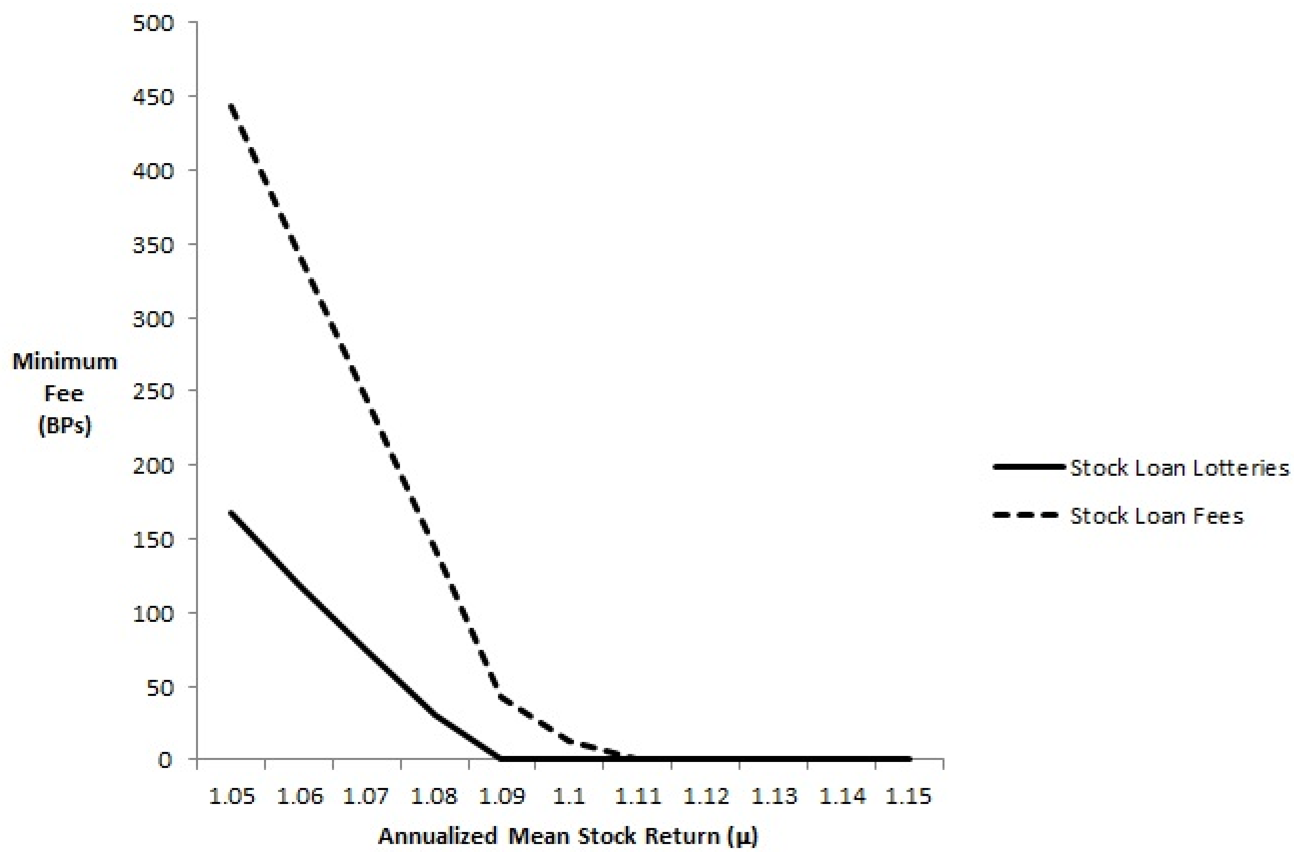


Figure 3: Regressive Features of Stock Loan Lotteries This figure shows how introducing stock loan lotteries is regressive in that it provides disproportionate benefits to poor investors. In a model with stock loan lotteries, the probability of winning the lottery is identical to the investor's wealth share. This figure shows how a standardized measure of utility, $E[U(W_i)]/W_i$, varies with the investor's wealth share. The solid lines show standardized utility for models with a single stock loan lottery at $t = 2$ and expected lottery payoffs of five and 50 basis points. The dotted lines show standardized utility for models with guaranteed stock loan fees of five and 50 basis points. For all models, the annualized expected gross return of the risky asset (μ), is 1.12.

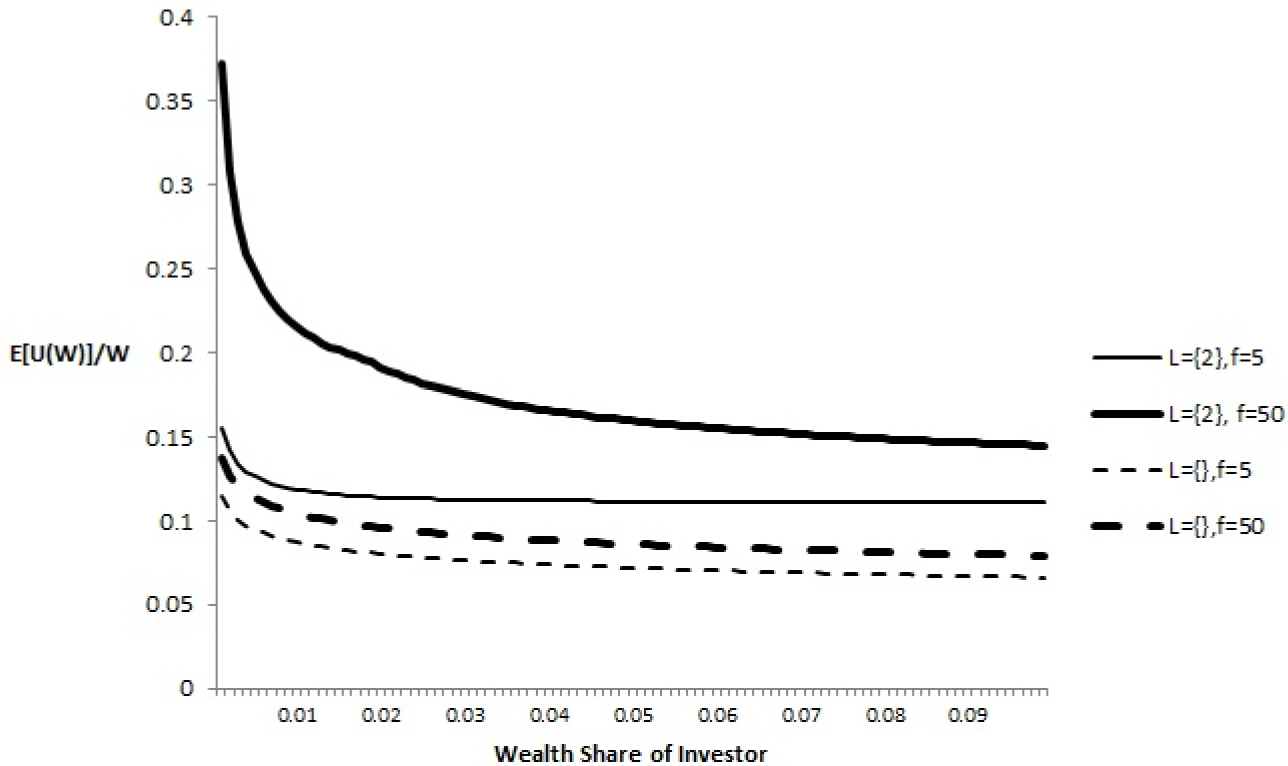


Figure 4: Stock Loan Lotteries with Trading Costs and Leverage Constraints This figure shows changes in the conditions for welfare gains after introducing trading costs and leverage constraints. An investor has unconditionally greater welfare if the utility-maximizing allocation provides strictly higher expected utility and strictly higher expected wealth. There are three two-period models of realization utility: the baseline Barberis and Xiong (2009) model, a model with fixed stock loan fees, and a model with a single stock loan lottery at $t = 2$ with probability $p = 0.1$ of winning the lottery. For different values of μ, the annualized expected gross return of the risky asset, I solve all three models in a perfect market as well as a market with frictions. The market frictions include 1.3% round-trip trading costs and a maximum leverage ratio of 2. For each value of μ, in each environment, I calculate f_F and f_L, the minimum fee required to provide investors in the fee and lottery models unconditionally greater welfare than in the baseline model. For each μ, the potential for welfare improvement by introducing stock loan lotteries is $max[f_F(\mu) - f_L(\mu), 0]$. The solid line shows the potential for welfare improvement with market frictions, while the dotted line shows the potential for welfare improvement with perfect markets.

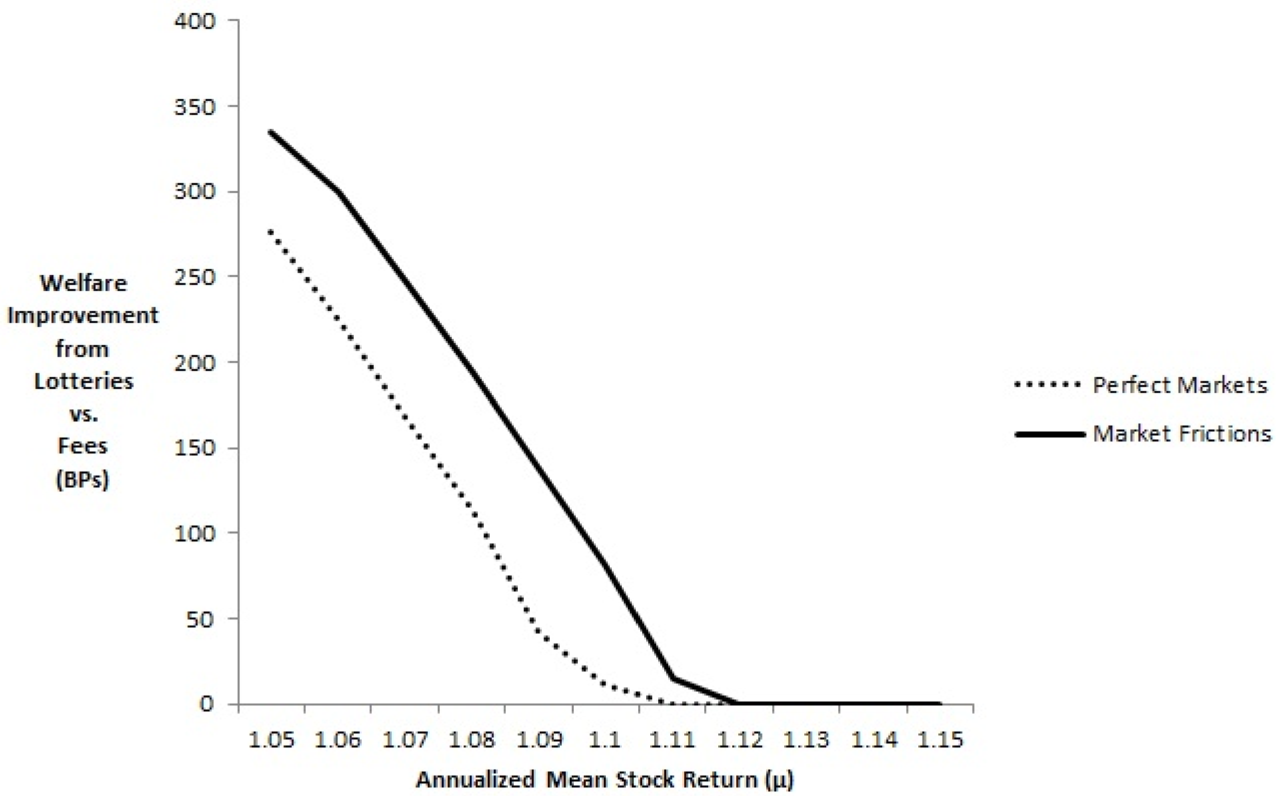


Figure 5: Comparing Stock Loan Lottery Structures This figure compares the effectiveness of various stock loan lottery structures. The horizontal axis shows μ, the annualized expected gross return of the risky asset. The vertical axis shows the annual net stock loan fee (f). The fee is measured in basis points and structured as one or two lottery payouts. For four different lottery structures and a range of risky-asset returns, this figure shows the minimum fee that motivates the investor to purchase the risky asset. The thick solid line corresponds to a single lottery at $t = 2$ with a 10% win probability. The thin solid line corresponds to a single lottery at $t = 2$ with a 1% win probability. The thick dotted line corresponds to lotteries at $t = 1$ and $t = 2$ that each have a 10% win probability. The thin dotted line corresponds to lotteries at $t = 1$ and $t = 2$ that each have a 1% win probability.

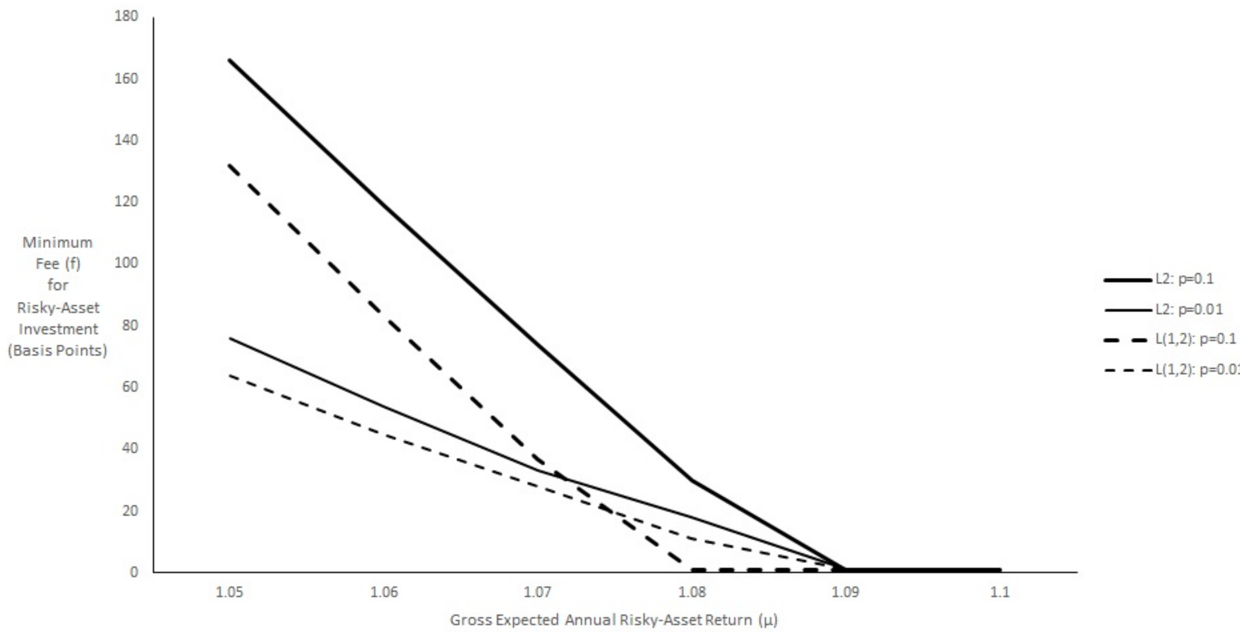
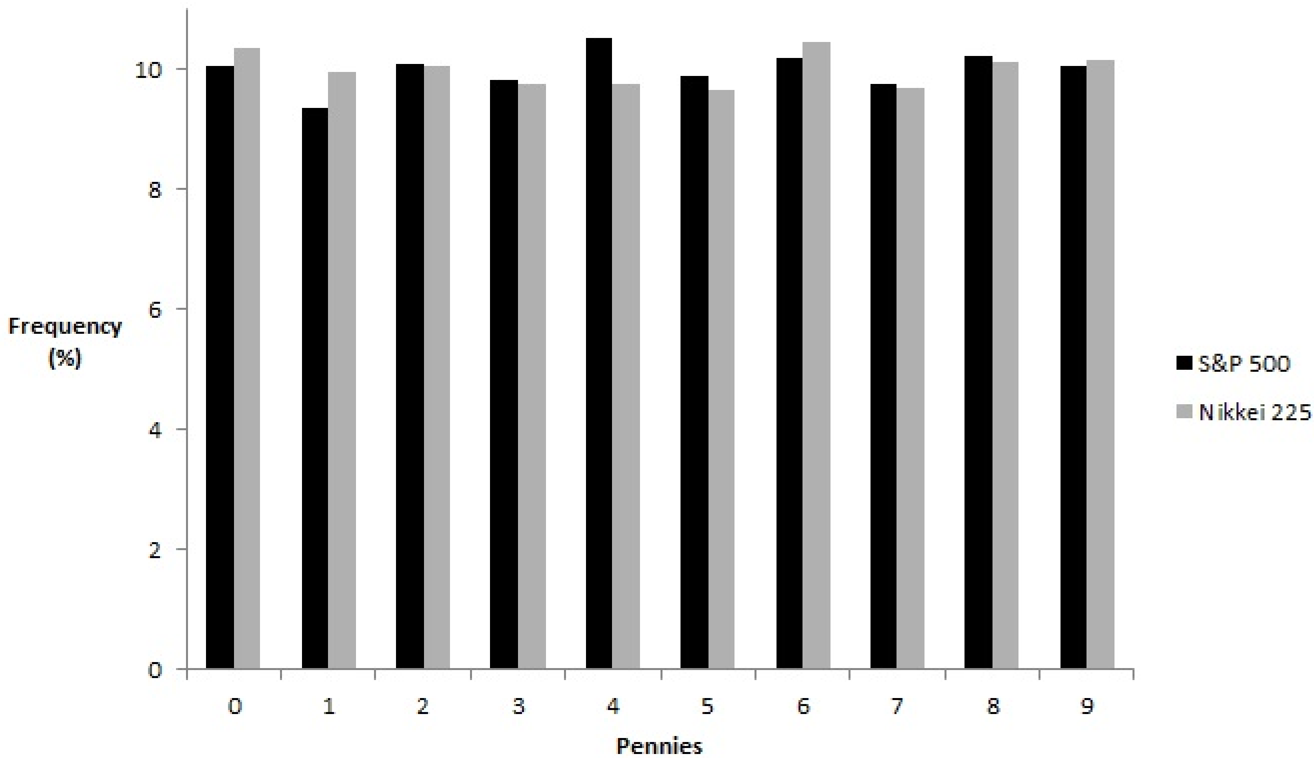


Figure 6: Stock Loan Lotteries as Derivative Securities This figure shows how lottery tickets could be structured as derivative securities. A "Pennies Series X" derivative has a payoff of 1 if the pennies digit of the daily closing price of a diversified index on a particular day is X. This histogram shows that there is no evidence of clustering in equity index daily closing prices. The black bars show the frequency of each of the pennies digits in the daily closing prices of the S&P 500 index between December 30, 1927 and April 21, 2017. The S&P 500 index is a capitalization-weighted index of 500 diversified large US public equities. The gray bars show the frequency of each of the pennies digits in the daily closing prices of the Nikkei 225 index between January 5, 1970 and April 21, 2017. The Nikkei 225 index is a price-weighted index of 225 diversified large Japanese public equities. Daily closing prices are from Bloomberg.



Appendix A: Mathematical Properties of the Value Function

The investor maximizes $E_0 v(x)$, where $v(x)$ has the functional form:

$$\begin{cases} v(x) = x^\alpha & x \geq 0, 0 < \alpha < 1 \\ v(x) = -\lambda(-x)^\alpha & x < 0, 0 < \alpha < 1, \lambda > 1 \end{cases}$$

Suppose the investor accepts some gamble (G) with potential gains $(g_1, g_2 \ldots g_m)$ and potential losses $(l_1, l_2 \ldots l_m)$. This implies:

$E_0 v(x; G) = \sum_{i=1}^{m} p(g_i)v(g_i) + \sum_{j=1}^{n} p(l_j)v(l_j) > 0$

Applying the functional form of the value function:

$E_0 v(x; G) = \sum_{i=1}^{m} p(g_i)(g_i)^\alpha - \lambda \sum_{j=1}^{n} p(l_j)(-l_j)^\alpha > 0$

Consider a proportionately larger gamble $(kG, k > 1)$. This gamble has expected value:

$E_0 v(x; kG) = \sum_{i=1}^{m} p(g_i)v(kg_i) + \sum_{j=1}^{n} p(l_j)v(kl_j)$

Applying the functional form of the value equation to the larger gamble:

$E_0 v(x; kG) = k^\alpha \sum_{i=1}^{m} p(g_i)(g_i)^\alpha - \lambda k^\alpha \sum_{j=1}^{n} p(l_j)(-l_j)^\alpha$

$E_0 v(x; kG) = k^\alpha [\sum_{i=1}^{m} p(g_i)(g_i)^\alpha - \lambda \sum_{j=1}^{n} p(l_j)(-l_j)^\alpha] = k^\alpha E_0 v(x; G)$

Since $k > 1$, $0 < \alpha < 1$, and $E_0 v(x; G) > 0$,

$E_0 v(x; Kg) > E_0 v(x; g) > 0$

So given that the investor accepts G, he always prefers kG. Therefore, an investor who chooses a positive risky asset allocation always chooses to exhaust the nonnegative wealth constraint.

QED

Appendix B: Survey Text

Survey Participation Consent

You are invited to participate in this online research survey entitled Discovering Investor Preferences. You are included in this survey because you are at least 18 years old and you have a strong track record of completing tasks on Amazon Mechanical Turk. The number of subjects to be enrolled in the study will be 50-100.

The survey may take approximately 5-10 minutes to complete. Your participation is voluntary. If you do not wish to participate in this survey, do not respond to this online survey. Completing this survey indicates that you are voluntarily giving consent to participate in the survey. I expect the study to last 1-2 days.

Participation in this study has no cost to you. You will receive $1.00 following the completion of the task. You will not receive any partial payment if you do not complete the survey. At the end of the survey, you will receive a message with an Amazon Mechanical Turk Code. Enter your code into a box on the Amazon Mechanical Turk Workers (HIT) website to receive payment.

The purpose of this research study is to better understand the investment decision-making process. There are no risks or discomforts associated with this survey. There may be no direct benefit to you, however, by participating in this study, you may help us understand investor preferences so that we can introduce innovative financial products.

Your response will be kept confidential. We will store the data in a secure computer file and the file will be destroyed once the data has been published. Any part of the research that is published as part of this study will not include your individual information. If you have any questions about the survey, you can contact Jordan Moore, the principal investigator, at moorejs@rowan.edu, but you do not have to give your personal identification.

Voluntary Consent by Participant

Please complete the checkboxes below.

To participate in this survey, you must be 18 years or older.

Completing this survey indicates that you are voluntarily giving consent to participate in the survey.

Instructions

Thank you for participating in this survey!

The following pages will ask you to make simple economic decisions. Please answer the questions as if you were making these decisions with your own money.

Allocation Task: Control Group (Fees)

Trial 1:

You contribute a portion of every paycheck to your retirement plan. The plan offers two funds. What percent would you invest in each fund? Your choices must total 100%.

Risk-Free Bond Fund: 100% chance of a 3% gain
Risky Stock Fund: 50% chance of a 10% loss; 50% chance of a 20% gain

Trial 2:

You contribute a portion of every paycheck to your retirement plan. The plan offers two funds. What percent would you invest in each fund? Your choices must total 100%.

Risk-Free Bond Fund: 100% chance of a 3% gain
Risky Stock Fund: 50% chance of a 8% loss; 50% chance of a 30% gain

Trial 3:

You contribute a portion of every paycheck to your retirement plan. The plan offers two funds. What percent would you invest in each fund? Your choices must total 100%.

Risk-Free Bond Fund: 100% chance of a 3% gain
Risky Stock Fund: 50% chance of a 6% loss; 50% chance of a 40% gain

Trial 4:

You contribute a portion of every paycheck to your retirement plan. The plan offers two funds. What percent would you invest in each fund? Your choices must total 100%.

Risk-Free Bond Fund: 100% chance of a 3% gain
Risky Stock Fund: 50% chance of a 9% loss; 50% chance of a 21% gain

Trial 5:

You contribute a portion of every paycheck to your retirement plan. The plan offers two funds. What percent would you invest in each fund? Your choices must total 100%.

Risk-Free Bond Fund: 100% chance of a 3% gain
Risky Stock Fund: 50% chance of a 7% loss; 50% chance of a 31% gain

Trial 6:

You contribute a portion of every paycheck to your retirement plan. The plan offers two funds. What percent would you invest in each fund? Your choices must total 100%.

Risk-Free Bond Fund: 100% chance of a 3% gain
Risky Stock Fund: 50% chance of a 5% loss; 50% chance of a 41% gain

Allocation Task: Treatment Group (Lotteries)

Trial 1:

You contribute a portion of every paycheck to your retirement plan. The plan offers two funds. What percent would you invest in each fund? Your choices must total 100%.

Risk-Free Bond Fund: 100% chance of a 3% gain
Risky Stock Fund: 50% chance of a 10% loss; 50% chance of a 20% gain

Trial 2:

You contribute a portion of every paycheck to your retirement plan. The plan offers two funds. What percent would you invest in each fund? Your choices must total 100%.

Risk-Free Bond Fund: 100% chance of a 3% gain
Risky Stock Fund: 50% chance of a 8% loss; 50% chance of a 30% gain

Trial 3:

You contribute a portion of every paycheck to your retirement plan. The plan offers two funds. What percent would you invest in each fund? Your choices must total 100%.

Risk-Free Bond Fund: 100% chance of a 3% gain
Risky Stock Fund: 50% chance of a 6% loss; 50% chance of a 40% gain

Trial 4:

You contribute a portion of every paycheck to your retirement plan. The plan offers two funds. What percent would you invest in each fund? Your choices must total 100%.

Risk-Free Bond Fund: 100% chance of a 3% gain
Risky Stock Fund: 49.5% chance of a 10% loss; 49.5% chance of a 20% gain; 1% chance of a 105% gain

Trial 5:

You contribute a portion of every paycheck to your retirement plan. The plan offers two funds. What percent would you invest in each fund? Your choices must total 100%.

Risk-Free Bond Fund: 100% chance of a 3% gain
Risky Stock Fund: 49.5% chance of a 8% loss; 49.5% chance of a 30% gain; 1% chance of a 111% gain

Trial 6:

You contribute a portion of every paycheck to your retirement plan. The plan offers two funds. What percent would you invest in each fund? Your choices must total 100%.

Risk-Free Bond Fund: 100% chance of a 3% gain
Risky Stock Fund: 49.5% chance of a 6% loss; 49.5% chance of a 40% gain; 1% chance of

a 117% gain

Risk Aversion Test [From Eckel and Grossman (2002)]

Which lottery would you most prefer to participate in?
50% chance of $2; 50% chance of $70
50% chance of $20; 50% chance of $44
50% chance of $16; 50% chance of $52
50% chance of $24; 50% chance of $36
50% chance of $12; 50% chance of $60
50% chance of $28; 50% chance of $28

Demographic Questions

Thank you for completing the survey.

Following, you will find a few questions regarding your demographic details. All information will be kept anonymous and not used for purposes outside our study.

Highest Level of Education Attained? (Less than high school, High school graduate, Some college, 2 year degree, 4 year degree, Professional degree, Graduate/Master's degree, Doctorate, Prefer not to answer)

Do you currently invest in the stock market either directly or through retirement accounts (401k, IRA, etc.)? (Yes, No, Prefer not to answer)

Do you currently invest in mutual funds? (Yes, No, Not sure, Prefer not to answer)

Do you rent or own your current residence? (Own, Rent, Live with family and friends, Prefer not to answer)

In what state do you live? (All 50 states, Other, Prefer not to answer)

Are you confident that the stock market is fair and that you will not be cheated when investing? (Likert scale)

What is your employment status? (Employed full time, Employed part time, Unemployed looking for work, Unemployed not looking for work, Retired, Student, Disabled, Homemaker, Prefer not to answer)

Please select Asia from the items below. (*Attention Check 1*) (Africa, Asia, Australia, North America, South America, Europe, Other)

What is your annual income (in thousands of $)? (¡50K, 50K-100K, 100K-250K, ¿50K, Prefer not to answer)

What is your current age? (¡18, 18-20, 21-25, 26-30, 31-35, 36-40, 41-45, 46-50, 51-55, 56-60, 61-65, ¿65, Prefer not to answer)

What is your marital status? (Married, Widowed, Divorced, Separated, Never married, Co-habitating, Prefer not to answer)

Generally speaking, when interacting with people, would you agree that most individuals are trustworthy? (Likert scale)

What is your gender identity? (Male, Female, Other, Prefer not to answer)

What race do you identify with? (White, Black or African American, American Indian or Alaska Native, Asian, Native Hawaiian or Pacific Islander, Latino or Hispanic, Other, Prefer not to answer)

Did you notice anything unusual about this survey? (Yes, No)

Risk Aversion Questionnaire [from Weber et al. (2002)]

Please indicate your likelihood of engaging in the below activity or behavior:
Betting a day's income at the horse races. (Likert scale)
Investing 10% of your annual income in a moderate growth mutual fund. (Likert scale)
Betting a day's income at a high stakes poker game. (Likert scale)
Investing 5% of your annual income in a very speculative stock. (Likert scale)
Betting a day's income on the outcome of a sporting event. (Likert scale)
Investing 5% of your annual income in a conservative stock. (Likert scale)
Investing 10% of your annual income in government bonds. (Likert scale)
Gambling a week's income at a casino. (Likert scale) Please choose "Not Sure" for this question. (*Attention Check 2*) (Likert scale)