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Document de recherche

DR 20-06

Decomposing the Systematic and Idiosyncratic Components of the Diffusive and Tail Risks in Individual Equity Options

Publié Août 2020

Ce document de recherche a été rédigée par :

Robert Brooks, University of Alabama
Don M. Chance, Louisiana State University
Mobina Shafaati, Old Dominion University

Decomposing the Systematic and Idiosyncratic Components of the Diffusive and Tail Risks in Individual Equity Options*

Robert Brooks, University of Alabama

Don M. Chance, Louisiana State University

Mobina Shafaati†, Old Dominion University

August 2020

Abstract

This paper introduces an affine jump-diffusion option valuation model for individual equities. The model captures the factor structure in equities and allows for stochastic and jump beta exposure. The right and left tails are disentangled in our framework and exhibit different dynamics. We find that investors require a compensation for their exposure to the negative jumps in the market index, while the positive jumps are not priced. Furthermore, even after accounting for the systematic tail risk, we find a persistent idiosyncratic tail factor in equities, inducing significant fluctuations in the negative jump tails.

JEL Classification: G10, G12, G13.

Keywords: Equity Options, Factor Structure, Variance Risk, Tail Risk, Option Valuation.

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*This research is conducted with high performance computational resources provided by Old Dominion University. We acknowledge the Canadian Derivatives Institute for financial support.

†Corresponding author: Old Dominion University, Department of Finance, 2152 Constant Hall, Norfolk, VA 23508. Phone: +1(757) 683-3563, Email: mshafaat@odu.edu .

1 Introduction

It is supported by the extant literature that markets are not complete and derivative securities are non-redundant assets, offering distinct exposures to volatility and jump risks.[1] Most of our understanding about the priced risks in the option market is restricted to the index options.[2] To analyze the dynamics of the option surface for individual equity options, we need to understand the presence of commonalities in the volatility or jump risks in the cross-section of equities, and specify an option valuation model that captures the common factors. The purpose of this paper is to examine the common factor in the variance and tail risk of the individual equity options by proposing a parametric jump-diffusive model for the risk-neutral return dynamics of individual equities, where stocks are exposed to both systematic and idiosyncratic risk factors, each of which consists of a Gaussian as well as a tail component.

Motivated by recent evidence in the literature that the fluctuations in the left tail of the risk-neutral densities cannot be captured by regular volatility factors, we include a pure tail factor in our model, which is further decomposed into the systematic and idiosyncratic tail risks. Most important, our model specification allows for a separate positive and negative jump intensities, where the negative jump intensity is governed by the tail factor and varies over the time, with innovations that are linked to the volatility and price jumps. In addition, we decompose the total diffusive variance of individual equities into the systematic and idiosyncratic components.

Our empirical results suggest a distinct wedge between the diffusive variance and the tail factor that governs the negative jump intensity. Moreover, we find that both positive and negative jumps in the market index is reflected in the individual equities and increase the right and left jump intensities. In addition, option implied beta, which captures the stock's exposure to the negative

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[1]See Bakshi *et al.* (2000), Buraschi and Jackwerth (2001), Bakshi and Kapadia (2003a), Jones (2006), among many others for more discussion on this topic.

[2]There is a rich literature on the empirical analysis of index options. See for example Longstaff (1995), Jackwerth and Rubinstein (1996), Heston and Nandi (2000), Pan (2002), Jones (2003, 2006), Broadie *et al.* (2007, 2009), Christoffersen *et al.* (2009), Christoffersen *et al.* (2012), Andersen *et al.* (2015a, 2015b, 2020).

jump risk of the market index has a significantly positive relationship with the physical beta, implying that investors should be compensated for bearing the downside risk. There is no such significant link between the positive jump beta and the physical beta.

To derive our model, we synthesize the models proposed by Christoffersen *et al.* (2018), that captures the factor structure in individual equity options, and Andersen *et al.* (2015a, 2015b, 2020) that document the presence of a separately evolving left tail factor governing the left jump intensity and show that including the tail factor improves the characterizations of the option surface dynamics. Our framework extends that of Andersen *et al.* (2015a, 2015b, 2020) by incorporating a factor structure in the price process of individual equities, which allows the diffusive and tail risks of the market index dynamics to impact the price of individual equities. Furthermore, our model expands on that of Christoffersen *et al.* (2018) by including jumps in the systematic as well as the idiosyncratic portion of individual equity returns, where the left jump intensity varies over the time, and differs from the right jump intensity.

Christoffersen *et al.* (2018) document a strong factor structure in equity options, and extend the stochastic volatility model of Heston (1993) to incorporate a CAPM factor structure where the Gaussian variance is decomposed into an idiosyncratic and a systematic component. The authors find that equities with higher option-implied betas have higher implied volatilities, steeper moneyness slope, and a volatility term structure that comoves with the market term structure. The result is consistent with Duan and Wei (2009) that build on Bakshi *et al.* (2003), where implied volatilities are linked to the proportion of systematic risk to total risk. The consistent evidence in the previous studies suggest the necessity of including a factor structure in the option valuation models for individual stocks.

Standard option pricing models mainly specify the dynamics of the equity-index option surface through the stochastic volatility factors.[3] Andersen *et al.* (2015a, 2015b) show that the fluctuations in the left tail of the risk-neutral density of the index options cannot be spanned by

[3]See, for example Bates (1996, 2000, 2003), Pan (2002), Eraker (2004), and Broadie *et al.* (2007).

volatility factors or by intraday realized volatility. The authors suggest that a tail factor, which is distint from the regular voaltility factors, is necessary to capture the dynamics in the left side of the option surface. Separating the aggregate volatility into the tail and volatility factor, Andersen *et al.* (2015b) find that the volatility factor has a strong predictive power for the future return variation, but does not contribute to the equity risk premium, while the pure tail factor has a significant explanatory power for future returns.

To estimate our model, we consider S&P 500 index as a proxy for the market and our sample of individual equities consists of 21 stocks from the constituents of the Dow Jones Industrial Average Index.[4] Due to smaller cross-section of strike prices in individual equity options relative to that of the index options, we employ the more parsimonious modeling in Andersen *et al.* (2020), which contains one diffusive variance factor and a tail factor, and thus mitigates the concern of in-sample overfitting and robustly captures the prominent features across the individual equity options. The model specifies the positive jump intensity to be constant and the negative jump intensity to be time-varying and governed by the tail factor. The jump component captures the price jumps for positive jumps, and the co-jumps in price and state vectors for the negative jumps. A negative price jump of size x translates into positive jumps of x^2 in both variance and tail factor. The price jumps are exponentially distributed and thus have fat tails.

Our empirical results show that both positive and negative jumps in the market index intensifies the arrival of jumps with the same direction in individual equities. We also document the presence of a separately evolving idiosyncratic tail factor that is highly persistent, which displays a substantial variation over the time, inducing a significant variation in the idiosyncratic left tail. Furthermore, the negative jump beta implied from individual equity options is a closer proxy for the risk factor in the underlying asset.

Closer to our study, Boloorforoosh *et al.* (2020) develop a continuous-time intertemporal CAPM model that allows for a stochastic beta, such that the expected return of a stock depends on

[4]The results for a larger sample of stocks will be provided in the future draft of this paper.

beta's co-movement with the market variance and more generally with the stochastic discount factor. Bégin *et al.* (2019) and Gourier (2016) build on the factor structure option valuation model of Christoffersen *et al.* (2018) and include jumps in the model. Bégin *et al.* (2019) find that idiosyncratic risk explains about 28% of the variation in the expected returns of an average stock, which is mainly due to idiosyncratic jump risk. The authors find the idiosyncratic Gaussian risk is not priced in equities. This finding is consistent with Gourier (2016). Our paper contributes to these studies by analyzing the left and right tails separately and we find that the fear of the downside risk is the priced risk factor in individual equities.

The rest of the paper is organized as follows. Section 2 present the option valuation model. Section 3 describes the data and sample creation procedure. Section 4 explains the estimation procedure. Estimated model parameters and state vector are provided in Section 5. Section 6 compares the option-implied betas with the physical betas, and Section 7 concludes.

2 Model Specification

We specify a parametric model for the risk-neutral return dynamics of individual equities, where stocks are exposed to both systematic and idiosyncratic risk, each of which consist of a Gaussian as well as a tail component. To derive our model, we synthesize the models proposed by Christoffersen *et al.* (2018), which captures the factor structure in equity options, and Andersen *et al.* (2015a, 2015b) that document the presence of a separately evolving left tail factor governing the left jump intensity and show that including the tail factor improves the characterizations of the option surface dynamics. Most important, our model allows for separate dynamics of the left and right jump tails in both market and stock price process. Andersen *et al.* (2015a, 2015b) show that the left and right tails exhibit very different dynamics, and a purely jump driven left tail factor is the major contributor to the left jump intensity and it has no presence in the right jump tail. This framework allows us to examine the price of exposure to market diffusive and tail risks in individual equity options.

The sample of individual equities contains fewer observations in the strike price cross-section, compared to index options, which makes the daily identification of the factors more challenging. Hence, despite the advantages of using two stochastic volatility factors documented in the literature, e.g., Bates (2000), Christoffersen *et al.* (2009), Christoffersen *et al.* (2012), and Ghanbari (2017), we use the risk-neutral model specification in Andersen *et al.* (2020) that contains a single volatility component along with the tail factor. Comparing the left jump factor for S&P 500 across the single and two factors models, Andersen *et al.* (2020) show that a more refined modeling provides robust factor identification,. Therefore, a more parsimonious model mitigates the concern of in-sample overfitting and robustly captures the salient features across the individual equity options, given the lower number of observations available for this class of options.

To specify the dynamics of the market index, we adopt the two factor risk-neutral model in Andersen *et al.* (2020), consisting of a single volatility factor and a tail factor, and extend the model to incorporate a CAPM-like factor structure for individual equity options. The process governing the market index under the risk-neutral measure $\mathbb{Q}$ is given by:

$$\begin{aligned} \frac{dM_t}{M_{t-}} &= (r_t - \delta_t)\, dr + \sqrt{V_t^M}\, dW_t^{\mathbb{Q},M} + \int_{\mathbb{R}} (e^{(x^M)} - 1)\, \tilde{\mu}^{\mathbb{Q},M}(dt, dx^M), \\ dV_t^M &= \kappa_\upsilon^M (\bar{\upsilon}^M - V_t^M)\, dt + \sigma_\upsilon^M \sqrt{V_t^M}\, dB^{\mathbb{Q},M} + \mu_\upsilon^M \int_{\mathbb{R}} (x^M)^2\, 1_{\{x_M<0\}}\, \mu^M(dt, dx^M), \\ dU_t^M &= -\kappa_U^M\, U_t^M\, dt + \mu_U^M \int_{\mathbb{R}} (x^M)^2\, 1_{\{x_M<0\}}\, \mu^M(dt, dx^M), \end{aligned} \tag{1}$$

where $(W_t^{\mathbb{Q},M}, B_t^{\mathbb{Q},M})$ is a two-dimensional Brownian motion with $\operatorname{corr}(W_t^{\mathbb{Q},M}, B_t^{\mathbb{Q},M}) = \rho^M$, which affects the skewness of the index return. A negative correlation $(\rho^M < 0)$ indicates the volatility feedback effect, that is the cost of capital raises with an increase in diffusive risk, resulting in lower asset value (Campbell and Hentschel (1992), and Bekaert and Wu (2000)).[5] More negative

[5]The negative correlation between equity or index returns and volatility is also referred to as the "leverage effect" in the literature. Figlewski and Wang (2000) and Hasanhodzic and Lo (2011) are two studies that question the

correlation generates the so-called implied volatility smirk, in which variance is higher when the spot asset price decreases, and thus out-of-the-money put options are more expensive than out-of-the-money calls.

Parameter r_t is the risk-free interest rate, δ_t is the dividend yield, and κ_υ^M and κ_U^M governs the speed of mean reversion of V_t^M to the long-run mean $\bar{\upsilon}^M$, and that of U_t^M to zero, respectively. The variation of V_t^M around $\bar{\upsilon}^M$ is captured by σ_υ^M. In addition, μ^M is an integer-valued parameter, which counts the number of jumps in the price M, as well as in the state vector (V^M, U^M).

The corresponding jump intensity under the risk-neutral measure is given by $dt \otimes \pi_t^{\mathbb{Q},M}(dx^M)$, such that the difference $\tilde{\mu}^{\mathbb{Q},M}(dt, dx^M) = \mu^M(dt, dx^M) - dt\,\pi_t^{\mathbb{Q},M}(dx^M)$ is the associated martingale jump measure. The jump component x^M captures the price jumps for positive jumps, and co-jumps that occur simultaneously in the price M and the state variables V^M and U^M, for negative price jumps. Specifically, when there is a negative price jump of size x^M, the two state variables V^M and U^M demonstrate a positive jump that is proportional to $(x^M)^2$. Therefore, based on the index process in Equation (1), the spot variance and tail factor are co-linear with distinct proportionality factors of μ_υ^M and μ_U^M, respectively. The compensator characterizing the conditional jump distribution is given by

$$\frac{\pi_t^{\mathbb{Q},M}(dx^M)}{dx^M} = \left\{ U_t^M .1_{\{x^M<0\}} \lambda_-^M e^{-\lambda_-^M |x^M|} + c_0^{+M} .1_{\{x^M>0\}} \lambda_+^M e^{-\lambda_+^M x^M} \right\} \tag{2}$$

Following Kou (2002) and Andersen *et al.* (2020), the market price jumps are exponential, with separate tail decay parameters of λ_-^M and λ_+^M for negative and positive jumps.[6] The tail decay parameters specify the shape of the tails, while the jump intensity characterize the level of the

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appropriateness of using the "leverage effect" term. See Carr and Wu (2017) for more discussion on the difference between these two effect and the methodology to incorporate either effect in the model.

[6]Parameters λ_- and λ_+ that specify the shape of the jump tails are assumed to be time-invariant, which is questionable based on the nonparametric evidence documented in Bollerslev and Todorov (2014). Examining the model fit to the option characteristics, Andersen *et al.* (2015b) find that the assumption of constant tail decay could lead to the underestimation of the skew during highly turbulent periods. Accounting for this feature could alleviate the mispricing at the cost of taking the model outside the tractable generalized affine framework.

tails.[7] According to Equation (2), the left jump intensity is time-varying and governed by U_t^M, while the right jump intensity is constant and equal to c_0^{+M}. Andersen *et al.* (2015b) find that the tail factor does not contribute directly to the diffusive volatility and has no presence in the right jump intensity. The low availability of deep out-of-the-money calls relative to put options is the possible reason for the weak identification of the tail far into the right side of the option surface (see Andersen *et al.* (2020)).

Next, we propose the stochastic volatility model for individual equity options by extending the market model in Equation (1), so that it captures the factor structure in individual equities. Christoffersen *et al.* (2018) extend the stochastic volatility model of Heston (1993) by decomposing the firm's total variance into a systematic and an idiosyncratic component. Bégin *et al.* (2019) and Gourier (2016) build on this model and include the jump risk in the model. We contribute to these studies by adding a separate factor *U* driving the left jump tail of the risk-neutral distribution, which is recognized as the one with predictive power for the underlying asset returns in Andersen *et al.* (2015a, 2015b and 2020).

This approach allows us to examine whether the idiosyncratic and systematic tail factors are priced in individual equities. In our framework, the diffusive random variation of individual equity returns is related to the Brownian motion that drives the market returns through the coefficient β_{diff}. In addition the jumps of the market returns in the right and left tails can also impact the stock return process through the coefficients β_{jump}^{+} and β_{jump}^{-}. Finally, we propose the following model to capture the dynamics of the individual equity returns:

$$\frac{dS_t}{S_{t-}} = (r_t - \delta_t)\,dt + \overbrace{\sqrt{V_t^S}\,dW_t^{\mathbb{Q},S} + \int_{\mathbb{R}} (e^{(x^S)} - 1)\,\tilde{\mu}^{\mathbb{Q},S}(dt, dx^S)}^{\text{Idiosyncratic Return}} + \overbrace{\beta_{diff}\sqrt{V_t^M}\,dW_t^{\mathbb{Q},M} + \int_{\mathbb{R}} (e^{(x^M)} - 1)\,\tilde{\mu}^{\mathbb{Q},M}(dt, dx^M)}^{\text{Systematic Return}},$$

$$dV_t^S = \kappa_\upsilon^S(\bar{\upsilon}^S - V_t^S)\,dt + \sigma_\upsilon^S\sqrt{V_t^S}\,dB_t^{\mathbb{Q},S} + \mu_\upsilon^S \int_{\mathbb{R}} (x^S)^2\, 1_{\{x^S<0\}}\,\mu^S(dt, dx^S),$$

$$dU_t^S = -\kappa_U^S U_t^S\,dt + \mu_U^S \int_{\mathbb{R}} (x^S)^2\, 1_{\{x^S<0\}}\,\mu^S(dt, dx^S),$$

[7] See Bollerslev and Todorov (2011, 2014) for more details.

$$dV_t^M = \kappa_\upsilon^M (\bar{\upsilon}^M - V_t^M)\, dt + \sigma_\upsilon^M \sqrt{V_t^M}\, dB^{\mathbb{Q},M} + \mu_\upsilon^M \int_{\mathbb{R}} (x^M)^2 1_{\{x^M<0\}} \mu^M(dt, dx^M) \,,$$

$$dU_t^M = -\kappa_U^M U_t^M \, dt + \mu_U^M \int_{\mathbb{R}} (x^M)^2 1_{\{x^M<0\}} \mu^M(dt, dx^M) \,, \tag{3}$$

where the innovation to the idiosyncratic returns, W_t^S, and idiosyncratic volatility, B_t^S, are correlated with coefficient ρ^S. Similar to the market process, having $\rho^S < 0$ implies the presence of the volatility feedback effect between idiosyncratic volatility and returns. We define the idiosyncratic variance as the variance of the residuals obtained after accounting for systematic risk factors, i.e. the systematic diffusive and tail risks. We estimate the idiosyncratic state variables, $\{V_t^S, U_t^S\}$, given the market state variables, $\{V_t^M, U_t^M\}$. In this setting, the stock's betas are considered to be time-invariant.[8] The compensator characterizing the conditional jump distribution are given by Equation (4) for systematic and Equation (5) for idiosyncratic jumps.

$$\frac{\pi_t^{\mathbb{Q},M}(dx^M)}{dx^M} = \beta_{jump}^- U_t^M 1_{\{x^M<0\}} \lambda_-^M e^{-\lambda_-^M |x^M|} + \beta_{jump}^+ c_0^{+M} 1_{\{x^M>0\}} \lambda_+^M e^{-\lambda_+^M x^M} \tag{4}$$

$$\frac{\pi_t^{\mathbb{Q},S}(dx^S)}{dx^S} = U_t^S 1_{\{x^S<0\}} \lambda_-^S e^{-\lambda_-^S |x^S|} + c_0^+ 1_{\{x^S>0\}} \lambda_+^S e^{-\lambda_+^S x^S} \tag{5}$$

The jumps in U and V factors are linked to the negative price jumps for stock and market separately, and we allow the negative jump intensity to vary over the time.

The individual equity option valuation model (3) allows the market diffusive variance to impact the price dynamics of individual equity options through the parameter β_{diff}. Hence, the total diffusive variance of firm S is computed as $V^S + \beta_{diff} V^M$. Furthermore, market jumps can also translate into systematic jumps in individual equities, where parameters β_{jump}^+ and β_{jump}^- serve as scaling factors for market jump intensities.

The model belongs to the popular class of affine jump-diffusion models of Duffie *et al.* (2000), which makes it analytically tractable. The affine characteristics of the model allows us to use the

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[8]Boloorforoosh *et al.* (2020) extend the model in Christoffersen *et al.* (2018) by allowing the only diffusive beta in their model to be time-varying, such that the stock returns depend on the beta's co-movement with market variance.

Fourier Cosine series expansion described in Fang and Oosterlee (2008) to calculate the prices of options. See Appendix A for details.

The novelty of this model is twofold. First, we disentangle the left tail risk from diffusive risk in individual equities, which allows the negative jump intensity to be time varying, while its innovation is directly linked to the volatility and price jumps. Second, our model allows us to evaluate the contribution of the systematic as well as idiosyncratic diffusive and tail risk, by incorporating a factor structure in individual equity option valuation model. In the following sections, we explain the data and sample creation, as well as the estimation procedure.

3 Data and Sample

We obtain the option data from OptionMetrics and consider the S&P 500 index as a proxy for market. Our sample of individual equities consists of 21 constituents of the Dow Jones Industrial Average Index.[9] Due to the favorable properties of implied volatilities in the estimation procedure, we use them rather than option prices (See Renault (1997)). OptionMetrics uses the Cox *et al.* (1979) binomial model to compute implied volatilities for equity options, which accounts for dividend payments for individual equities. Zero-coupon interest rates are also extracted from OptionMetrics and when necessary, we use linear interpolation to find the rate corresponding to an option's maturity.[10] Intraday realized volatility is measured using the high-frequency trading prices from NYSE TAQ database.[11]

Our sample spans from January 2006 to December 2016.[12] In our estimation, we focus on out-of-the-money call and put options with moneyness (S/K) range from 0.7 to 1.3. Because of the put-

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[9]This version of the paper reports the results for only 21 firms in the Dow Jones industrial average index. The results for all the remaining constituents of the index will be available in the future draft of the paper. The missing tickers are AAPL, BA, CVX, GS, HD, IBM, JPM, PFE, and TRV.

[10]OptionMetrics calculates the zero-coupon term structure using ICE IBA LIBOR rates and settlement prices of CME Eurodollar futures.

[11]We use the trade-based intraday volatility measured by WRDS, using the millisecond trade data. The methodology to clean the TAQ data is borrowed from Holden and Jacobsen (2014).

[12]Visa's initial public offering was in March 2008, and its option data coverage starts from April 2008.

call parity, this approach will not cause any loss of information about the implied volatility surface, as we can find the option price of the in-the-money counterpart.[13] In addition, we restrict our analysis to options with maturity between seven to 365 days. To mitigate the beginning- and end-of-week effects, we follow the earlier studies and sample every Wednesday (e.g., Bates (2000), Christoffersen *et al.* (2009) and Andersen *et al.* (2015a, 2015b and 2020)).[14] If a given Wednesday is a holiday, we use the previous business day. In total, we have 574 days for the in-sample analysis.

Several standard filters are imposed to exclude options that may contain errors. We maintain options with non-missing bid or ask prices and require that the option price, measured as the midpoint of bid and ask quotes, to be higher than the bid-ask spread. Options with non-standard settlement, zero open interest, or abnormal bid-ask spread are excluded from our sample. A bid-ask spread is considered as abnormal if it is negative or larger than $5. We further eliminate options that violate the arbitrage condition, or if their corresponding implied volatilities are higher than 150% or lower than 5%.[15]

Table 1 summarizes the main features of our option sample, including the number of quoted calls and puts, along with their moneyness and maturity during the in-sample period. The sample contains 394,706 contracts of index options and 813,854 options contracts for 21 individual equities in our current sample. Put options constitute about 66% of the index and 55% of individual equities sample. The average days to maturity (DTM) is 82 days for index and 95 days for individual equity options. The sample is more dominated by shorter maturity options as the average

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[13]For daily estimation of state factor, we require to have at least the same number of option quotes as the number of parameters within a day, which is 15 for individual equities and 12 for the index option. There is a shortage of enough observations only for a few days in the individual equity options sample, where we use the most liquid options with the highest trading volume, from their in-the-money counterparts.

[14]Wednesdays are less prone to the weekend effects and less likely to be a holiday. See Dumas *et al.* (1998) for further discussion in this regard. In total, we have eight Wednesdays that are holidays, which are replaced with the previous days.

[15]We define the no-arbitrage condition as $\max(0, S_t - Ke^{-r\tau}) \le C_{t,k,\tau} \le S_t$ for call options, and for put options $\max(0, Ke^{-r\tau} - S_t) \le P_{t,k,\tau} \le K$.

of the median value of days to maturity is 52 days for index options and 60 days for individual equity options. Moneyness is measured as the stock price divided by strike price (*S/K*). Index options have the average moneyness of 0.94 for calls and 1.12 for puts. The corresponding values for individual equity options are 0.92 and 1.10, respectively. The median and mean of the options are close in terms of their moneyness.[16]

4 Estimation Procedure

This section explains our approach of estimating the market and equity model in Equations (1) and (3), which allows us to understand the role of the diffusive and jump beta in the valuation of individual equities. We first derive the market index dynamic by estimating its model parameters and period-by-period state vector. Next, we estimate those of the individual equities, by taking the market dynamic as given.

For both market and equity model estimation, we follow Bates (2000), Santa-Clara and Yan (2010) and Christoffersen *et al.* (2018), and treat the variance and tail factors as parameters to be estimated.[17] Given the period-by-period state vector realization, we estimate the model parameters, while we use the optimization approach in Andersen *et al.* (2015a) in both steps.

Let N_t^j denote the number of options available at time t for asset j, where $j = M$ for market index proxied by S&P 500 and $j = S$ for individual equities. We observe option prices at the end of the trading day $t = 1, \dots, T$, based on which implied volatility is calculated. The parameter vector Ψ^j and state vector (V_t^j, U_t^j) are obtained from optimization of the following objective function (See Andersen *et al.* (2015a)):

[16]The median of moneyness is 0.95 (1.11) for call (put) index options and the average of the medians of moneyness is 0.93 (1.09) for call (put) individual equity options.

[17]Several approaches for estimating stochastic volatility models have been proposed in the literature. Jacquier *et al.* (1994) use Markov chain Monte Carlo to estimate a discrete-time stochastic volatility model. Pan (2002) uses generalized method of moments (GMM) to estimate the objective and risk-neutral parameters from returns and option prices. Serban *et al.* (2008) estimation strategy is based on simulated maximum likelihood using the expectation-maximization (EM) algorithm and a particle filter.

$$(\{V_t^j, U_t^j\}_{t=1}^T, \Psi^j) = \arg\min \sum_{t=1}^{T} \left\{ \frac{\text{Option Fit}_t^j + \theta \times \text{Vol Fit}_t^j}{V_t^{ATM}} \right\},$$

$$\text{Vol Fit}_t^j = \begin{cases} \left(RVol_t^j - \sqrt{V_t^M} \right)^2 & \text{for market index } (j = M) \\ \left(RVol_t^j - \sqrt{V_t^S + \beta_{diff}^2 V_t^M} \right)^2 & \text{for stocks } (j = S) \end{cases} \quad (6)$$

The optimization problem minimizes the distance between the implied volatility obtained from market and the volatility implied from our model, i.e. Option Fit, while it penalizes for Vol Fit, which is measured as the discrepancies between the diffusive spot volatilities inferred from the model and the realized volatilities ($RVol_t^j$) estimated from high-frequency data of the underlying asset. According to the market and equity models in (1) and (3), the spot diffusive variance of the market is V_t^M and that of the stock is $V_t^S + \beta_{diff}^2 V_t^M$. The tuning parameter θ penalizes the spot volatility values that deviates largely from their corresponding realized volatilities, which also favors the no-arbitrage condition satisfaction. We follow Andersen (2015a, 2020) and use a moderate fixed value of 0.05. Moreover, V_t^{ATM} is the implied volatility of the shortest maturity option on date *t* that is the closest to be at-the-money. We standardize the loss function by V_t^{ATM}, and thus use a different weight for options on low and high volatility days.

The market model is estimated in two steps. First, on each day *t*, we estimate state vector (V_t^M, U_t^M) given a set of starting values $\Psi_0^M \equiv \{\kappa_\upsilon^M, \bar{\upsilon}^M, \sigma_\upsilon^M, \rho^M, \kappa_U^M, \mu_\upsilon^M, \mu_U^M, c_0^{+M}, \lambda_-^M, \lambda_+^M\}$. Thus, the option fit is:

$$\text{Option Fit}_t^M = \frac{1}{N_t^M} \sum_{i=1}^{N_t^M} \left(IV_t^{CRR}(k_i, \tau_i) - IV(k_i, \tau_i, V_t^M, U_t^M, \Psi^M) \right)^2 \quad (7)$$

where N_t^M is the number of options available for S&P 500 on date *t*. $IV_t^{CRR}(k_i, \tau_i)$ denotes the implied volatility of option *i* with strike price of *k* and maturity of τ. OptionMetrics calculates this measure following Cox *et al.* (1979). Correspondingly, $IV(k_i, \tau_i, V_t^M, U_t^M, \Psi^M)$ is the implied volatility inferred from the model (1), given the starting values of the parameters Ψ^M. Next, we take the state vector estimated in the first step as given and estimate the model parameters over the entire sample period.[18]

The model for individual equities is estimated following a similar two-step process, with the difference that we take the market state vector and model parameters $\{\Psi^M, V_t^M, U_t^M\}$ as given in the model (3). Hence, the option fit for individual equities is:

$$\text{Option Fit}_t^S = \frac{1}{N_t^S}\sum_{i=1}^{N_t^S}\left(IV_t^{CRR}(k_i,\tau_i) - IV(k_i,\tau_i,V_t^M,U_t^M,\Psi^M,V_t^S,U_t^S,\Psi^S)\right)^2 \tag{8}$$

In the first step, state vector (V_t^S, U_t^S) is estimated on each date and next the individual equity parameters $\Psi^S \equiv \left\{\beta_{diff}^S, \beta_{jump}^{+S}, \beta_{jump}^{-S}, \kappa_\upsilon^S, \bar{\upsilon}^S, \sigma_\upsilon^S, \rho^S, \kappa_U^S, \mu_\upsilon^S, \mu_U^S, c_0^{+S}, \lambda_-^S, \lambda_+^S\right\}$ are obtains taking the state vector as given.

5 Estimation Results

In this section, we examine the characterization of the option surface dynamics of market index and individual equities, based on model (1) and (3). We present the estimation results for 21 firms in our sample over the period of 2006-2016. Considering the limited number of observations available for individual equity options in the cross-section of strike price, we do not aim to specify a perfect option valuation model for this class of options, but rather a more parsimonious model that allows us to evaluate the systematic and idiosyncratic tail risk in individual equities. Following Andersen *et al.* (2015b, 2020), our model contains a separately evolving tail factor *U*, which affects only the negative jumps, and does not contribute to either positive jumps or the diffusive volatility.

[18]To ease the computational burden in the second step, where we run the optimization over the entire sample period to estimate the parameter vector, we shrink our sample by retaining the most liquid options from all over the volatility surface. For that, on each Wednesday and for each maturity, we select the most liquid options from 12 moneyness buckets, which range from 0.7 to 1.3 with steps of 0.05. See Appendix A1 for summary statistics of the sample used in the second step.

5.1 Parameter Estimates

Table 2 reports the point estimates of the parameters in model (3), using weekly observations on Wednesdays or a neighboring trading day if Wednesday is holiday. The risk-neutral variance υ^M is 0.0125 for the market index. The mean value of the idiosyncratic risk-neutral variance, υ^S, across all stocks is 0.0319, which drops to 0.0021 if we exclude AXP which has an abnormally high variance. Consistent with Andersen *et al.* (2015a, 2015b and 2020), we find that *V* is a rapidly moving variance factor in both the market index and individual equities. The estimate of the risk-neutral mean reversion of the variance factor for the market index, κ_υ^M , is 9.355. The mean of idiosyncratic κ_υ^S is 12.0206, with the maximum of 46.2782 for CAT, and the minimum of 0.0007 for AXP, implying that idiosyncratic variance is highly persistent in AXP, and thus the associated parameters, ρ^S, κ_υ^S, and μ_υ^S, for this specific stock, are poorly estimated using options with maturity of up to one year. The volatility of variance factor for market index σ_υ^M is 0.5973 which is consistent with Andersen *et al.* (2020). The volatility of idiosyncratic variance σ_υ^S ranges from 0.3837 to 0.9889, with an average value of 0.8887. Overall, the variance of market index is larger, less volatile, and more persistent than the average of idiosyncratic variance factors in our sample of 21 stocks.

The parameters controlling the tail factor in both market index, U^M and individual equities, U^S are provided in Table 2. The mean reversion of *U* factor for the market index, κ_U^M, is 0.7596 and the average of mean reversion across the 21 stocks in our sample is 1.6052, with the minimum of 0.4930 for CSCO and maximum of 1.9812 for CAT. Consistent with Andersen *et al.* (2015a, 2015b and 2020), we find that the tail factor *U* is highly persistent in market index and stocks, with half-life of approximately one year for S&P 500 and average of five month for individual equities. Comparing our results with those of Christoffersen *et al.* (2018), we find that the mean reversion of idiosyncratic variance across stocks is far stronger. The more transient identification of the variance factor, *V*, is compensated by the presence of the very persistent idiosyncratic tail factor, U^S. Hence, by including the idiosyncratic *U* factor, we decompose the volatility and jump intensity, while the latter has the potential to impact the option prices across both short- and long-term maturities.

Our model allows for a separate left and right jump intensities, as specified in Equation (4) and (5). The price jumps are exponentially distributed with separate tail decay parameters for positive and negative jumps, i.e. λ_- and λ_+. The estimates of the tail decay parameters in market model (1), is 23.3512 for negative and 78.2494 for positive jumps, implying that jump tails is significantly fatter for negative jumps compared to positive jumps. This finding is also in line with those of Bollerslev and Todorov (2011), using a nonparametric specification, as well as Andersen *et al.* (2015a, 2015b and 2020). The average of idiosyncratic decay parameters is 32.5739 for negative jumps, λ_-^S, and 75.8919 for positive jumps, λ_+^S. Among the stocks, CSCO (CAT) represents the heaviest negative (positive) jump tail.

In market and equity models (1) and (3), state variables U^M and V^M for market index and U^S and V^S for stocks, display jumps proportional to $(x^M)^2$ and $(x^S)^2$, which correspond to negative price jumps of size x^M and x^S, respectively. The proportionality factors of V and U in the market index process, μ_v^M and μ_U^M, are very close. Table 2 reports similar finding across individual equities for the proportionality factors of μ_v^S and μ_U^S. Next section elaborates on the extracted state variables and jump intensities in the market and equities. Furthermore, in the following, we will provide the estimates for β_{diff}, β_{jump}^+ and β_{jump}^-, which perform as scaling factors that capture the stock's exposure to the market diffusive and tail risks.

5.2 State Vector Dynamics and Jump Intensities

This section explores the state vectors and jump intensities in our model. We start by analyzing the extracted series of the Gaussian variance factors, V^M and V^S. Tables 3 provides the summary statistics. The average of the market diffusive variance, V^M, from 2006 to 2016, is 0.006 and the average of idiosyncratic diffusive variance, V^S, across the 21 stocks in our sample is 0.008. The variance factors demonstrate a low level of persistency, where the first order autocorrelation is about 0.24 for both market and stocks, and the autocorrelation of order 15 drops to an average of 0.082 across all assets.

In the individual equity model (3), β_{diff} captures the stock's exposure to the market Gaussian risk. The total diffusive variance of firm S consists of an idiosyncratic component V^S, and a systematic component $\beta_{diff}V^M$. All option implied betas are reported in Table 4. The average of diffusive betas across the 21 stocks in our sample is 0.3555. The lowest diffusive beta is 0.006 estimated for AXP and the highest is 0.7541 for PG. The last column of Table 4 provides the mean of the systematic diffusive risk ratio (SR) series over the sample period. For each individual equity *S*, SR is given by:

$$SR^S = \frac{1}{T}\sum_{t=1}^{T}\frac{(\beta_{diff}^S)^2 V^M}{V^S + (\beta_{diff}^S)^2 V^M}$$

Where T is the length of diffusive variance time series. $\left(\beta_{diff}^S\right)^2 V^M$ is the systematic diffusive variance of stock S and $\left(\beta_{diff}^S\right)^2 V^M + V^S$ is its total diffusive variance. Table 3 shows that, on average, the systematic diffusive variance construct about 16.7% of the total diffusive variance in individual equities, ranging from 0.71% for AXP and UNH to 39.69% for PG. The SR measure is a strongly related to the diffusive beta, β_{diff}, with correlation of 0.98 in the cross-section of 21 stocks.

We followed Andersen *et al.* (2020) to specify the market model (1) and the compensator characterizing its condition jump distribution according to Equation (2), in which the right jump intensity is equal to the constant parameter C_0^{+M}. The right jump intensity estimates are provided in Table 2. The average positive jump arrival per year for market index is 7.4772, which is close to the corresponding estimate in Andersen *et al.* (2020).

The left jump intensity in Equation (2) is governed by the tail factor U_t^M. This specification is motivated by recent evidence that a tail factor, which is distinct from the regular volatility factors, is necessary to capture the dynamics in the left side of the option surface (See Andersen *et al.* (2015b)). The summary statistics for the extracted *U* factor is reported in Table 5. The mean of the time series U_t^M implies the average negative jump arrival of 4.064 per year in the market index.

The standard deviation of U_t^M is 2.293 which stimulates substantial fluctuations in the negative jump tail. The skewness and kurtosis of the market *U* factor is lower than those of the variance factor. Table 5 documents that the *U* factor demonstrates a high degree of autocorrelation, with AC(1) equal to 0.607 and it remains high, even up to order 30 which is equal to 0.220.

The individual equity option valuation model (3) contains two jump components: systematic and idiosyncratic. The compensator characterizing the idiosyncratic conditional jump distribution takes the form of Equation (5). Similar to the market index specification, the idiosyncratic positive jump is constant and equal to C_0^{+S}. The average idiosyncratic positive jump arrival across all 21 stocks is 11.4289 per year, which is higher than the positive jump intensity of the market. CAT has an abnormally positive jump intensity of 73.9575, while the average of C_0^{+S} is 8.3525 across other stocks in our sample.

In addition, positive jumps in the market index can impact individual equity option surface through the positive jump beta, β_{jump}^{+}. These parameter work as a scaling factor for the market positive jump intensity. Table 4 reports the option implied beta estimates. The average of β_{jump}^{+} across all stocks in our sample is 2.5281, with the minimum estimate of 0.1972 for CAT and maximum estimate of 7.1314 for INTC. These results imply that positive price jumps in the market index intensifies the average arrival of positive jumps in individual equities by 2.53 times. The scaled systematic intensities for the positive jumps, $\beta_{jump}^{+}C_0^{+M}$, is on average 18.903, across all individual equities.

The negative jump intensity of individual equities is specified separately for the systematic and idiosyncratic jump components in Model (3). For each individual equity *S*, the idiosyncratic left intensity is governed by U_t^S, and the systematic intensity is governed by the scaled measure of the market tail factor, i.e. $\beta_{jump}^{-}U_t^M$. Table 5 documents the summary statistics of the *U* factor. The average of idiosyncratic tail factor, U_t^S, is equal to 3.978, which is very close to the negative jump intensity of the market index. JNJ displays the lowest negative jump frequency and the highest negative jump arrival is documented for CSCO.

The average of standard deviation of U_t^S across stocks is 3.137, which is significantly higher than that of the market, indicating a substantial degree of variation over time in individual equities. Interestingly, JNJ and CSCO show the lowest and highest fluctuations in the negative jump tails, respectively. The skewness and kurtosis of the U_t^S series is much higher than those of the market index. The average skewness (excess kurtosis) of U_t^S across stocks is 4.002 (23.532) compared to 1.092 (0.167) for the market. The autocorrelation values in the last three columns show that the idiosyncratic tail factor, U_t^S, is less persistent the tail factor of the market index.

Finally, the negative price jumps of the market index can affect the individual equities option surface, and the parameter β_{jump}^- captures the role of the market tail factor in the negative jump intensities in systematic jump component in Model (3). The negative jump beta estimates are reported in Table 4. The average of β_{jump}^- is 2.2749, suggesting that the negative jumps in the market index level intensifies the frequency of the negative jumps in individual equities by more than two times the market's negative jump intensity. Although, CSCO reported the highest idiosyncratic negative jump intensity, its estimated negative jump beta is the lowest, resulting in the lowest left jump intensity in the systematic jump component. CAT has the highest negative jump beta, while its positive jump beta was the lowest. In the following, we compare the risk-neutral betas obtained from the option data with those estimated using physical stock returns.

6 Exploring the model

6.1 Physical and Risk-Neutral Betas

The last column of Table 4 reports β_{mkt}, which is obtained by estimating the market model, using monthly returns of the S&P 500 index and individual equities over the entire sample period, i.e. from 2006 to 2016. Figure 1 compares the estimated risk-neutral and physical betas, while we should note that due to the small size of our individual equity option sample, which contains only 21 stocks, we cannot generalize our finding to the cross-section of all stocks.[19]

[19]A more comprehensive analysis will be provided in the future version of the paper.

Panel A of Figure 1 demonstrates a surprisingly negative relationship between the diffusive beta, β_{diff}, and the market beta, β_{mkt}. This finding can be justified by the significantly positive correlation between the negative jump beta and the market beta. Previous studies, such as Serban *et al.* (2008) and Christoffersen *et al.* (2018), documented that the risk-neutral and physical betas are close, and our findings contribute to these studies by showing that the negative jump beta behaves more closely to the market betas estimated by stock returns.

The significant positive correlation between the physical beta and the negative jump beta, β_{jump}^{-}, implies that stocks with higher negative jump beta carry a positive risk premium since investors require should be awarded for bearing the downside risks. This finding is in line with Andersen *et al.* (2015b), which show that the tail factor has a significant predictive power for equity risk premium. Panel C of Figure 1 shows that the positive jump betas do not vary with the market beta, and Panel D displays the negative relationship between negative jump betas and diffusive betas, although the magnitude of diffusive betas are substantially smaller than their negative jump counterparts. Table 6 reports the correlations between all estimated betas.

7 Conclusion

This paper proposes a parametric risk-neutral volatility model for individual equities, which captures the exposure of stocks to the risk factors in the market index and measures the idiosyncratic factors affecting the individual equity options surface. In our framework, the instantaneous risk-neutral returns consist of systematic and idiosyncratic components, each of which includes a diffusive variance factor and a separately evolving tail factor to analyze option price dynamics. The model belongs to the affine jump-diffusion class of models. The novel feature of this study is the jump specification of the systematic and idiosyncratic jump components. The right and left tails have separate jump intensities. The positive jump intensity is constant, while the negative jump intensity is governed by the time-varying tail factor.

We find that the negative jump beta implied from equity options is positively related to the physical beta, while the positive jump beta does not display a significant correlation. This implies that investors require a compensation only for their exposure to the negative jumps in the market index. In addition, we document a highly persistent idiosyncratic tail factor in individual equities, even after accounting for the market index jumps.

Appendix A: Option Pricing Model

This section describes the option valuation for the model outlined in Section 2. Our four-factor model remains within the class of affine jump-diffusion models of Duffie *et al.* (2000). Therefore, we can obtain the option prices using the Fourier-Cosine series expansion (FCSE), following Fang and Oosterlee (2008). Correspondingly, we need to find the conditional characteristic function of the logarithm of the equity price.

Assume that the characteristic function of the log-price process takes the following exponential form:

$$\phi(u, y_t, V_t^S, V_t^M, U_t^S, U_t^M, \tau) = \mathbb{E}_t^{\mathbb{Q}}\left[e^{u\, y_{t+\tau}}\right] \tag{A.1}$$

where $y_t = \log(S_t)$ and $u \in \mathbb{C}$. Since the model in Equation (3) is in the affine class, we can rewrite Equation (A.1) as:

$$\phi(u, y_t, V_t^S, V_t^M, U_t^S, U_t^M, \tau) = e^{\alpha(u,\tau)+\beta_1(u,\tau)V_t^S+\beta_2(u,\tau)V_t^M+\beta_3(u,\tau)U_t^S+\beta_4(u,\tau)U_t^M+u\, y_t} \tag{A.2}$$

Our goal is to determine the coefficients $\alpha(u,\tau)$, $\beta_1(u,\tau)$, $\beta_2(u,\tau)$, $\beta_3(u,\tau)$, and $\beta_4(u,\tau)$. Since the solution is not available in closed form, we can find the solution by solving a system of ordinary differential equations (ODEs).[20] We use the Feynman-Kac theorem to derive the following equation, where the prime sign indicates the derivative with respect to time-to-maturity, τ, and for brevity, we suppress the subscript *t* of the state variables:

$$-\alpha' - \beta_1' V^S - \beta_2' V^M - \beta_3' U^S - \beta_4' U^M + u\Big[r_t - \delta_t - \frac{1}{2}\left(V^S + \beta_{diff}^2 V^M\right) - U^S(\Theta_n^S(u,0,0)-1) -$$

$$c_0^{+S}(\Theta_p^S(u)-1) - \beta_{jump}^- U^M(\Theta_n^M(u,0,0)-1) - \beta_{jump}^+ c_0^{+M}(\Theta_p^M(u)-1)\Big] + \beta_1(\kappa_\upsilon^S(\bar{\upsilon}^S - V^S)) +$$

$$\beta_2(\kappa_\upsilon^M(\bar{\upsilon}^M - V^M)) - \beta_3\kappa_U^S U^S - \beta_4\kappa_U^M U^M + \frac{1}{2}u^2(V^S + \beta_{diff}^2 V^M) + \frac{1}{2}\beta_1^2(\sigma_\upsilon^S)^2 V^S + \frac{1}{2}\beta_2^2(\sigma_\upsilon^M)^2 V^M +$$

$$u\,\beta_1\sigma_\upsilon^S\rho^S V^S + u\,\beta_2\sigma_\upsilon^M\rho^M\beta_{diff}V^M + U^S(\Theta_n^S(u,\beta_1,\beta_3)-1) + c_0^{+S}(\Theta_p^S(u)-1) +$$

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[20]For a comprehensive discussion on this topic, see Duffie *et al.* (2000) and Duffie *et al.* (2003).

$$\beta_{jump}^{-} U^M (\Theta_n^M (u,\beta_2,\beta_4)-1) + \beta_{jump}^{+} c_0^{+M} (\Theta_p^M (u)-1) = 0 \tag{A.3}$$

where,

$$\Theta_n^{S/M}(q_0,q_1,q_2) = \int_{-\infty}^{0} e^{q_0 z + q_1 \mu_\upsilon^{S/M} z^2 + q_3 \mu_u^{S/M} z^2} \lambda_{-}^{S/M} e^{z \lambda_{-}^{S/M}} dz \tag{A.4}$$

$$\Theta_p^{S/M}(q_0) = \int_{0}^{+\infty} e^{q_0 z} \lambda_{+}^{S/M} e^{-z \lambda_{+}^{S/M}} dz \tag{A.5}$$

By collecting the terms containing V^M, V^S, U^M, and U^S, and setting their coefficients equal to zero, we obtain the following system of ODEs.

$$\alpha' = u\left[r-\delta - c_0^{+S}(\Theta_p^S(u)-1) - \beta_{jump}^{+} c_0^{+M}(\Theta_p^M(u)-1)\right] + \beta_1 \kappa_\upsilon^S \bar{\upsilon}^S + \beta_2 \kappa_\upsilon^M \bar{\upsilon}^M + c_0^{+S}(\Theta_p^S(u)-1) + \beta_{jump}^{+} c_0^{+M}(\Theta_p^M(u)-1)$$

$$\beta_1' = -\frac{1}{2}u - \beta_1 \kappa_\upsilon^S + \frac{1}{2}u^2 + \frac{1}{2}\beta_1^2 (\sigma_\upsilon^S)^2 + u\beta_1 \sigma_\upsilon^S \rho^S$$

$$\beta_2' = -\frac{1}{2}u\beta_{diff}^2 - \beta_2 \kappa_\upsilon^M + \frac{1}{2}u^2 \beta_{diff}^2 + \frac{1}{2}\beta_2^2 (\sigma_\upsilon^M)^2 + u\beta_2 \sigma_\upsilon^M \rho^M \beta_{diff}$$

$$\beta_3' = u\left[-(\Theta_n^S(u,0,0)-1)\right] - \beta_3 \kappa_U^S + (\Theta_n^S(u,\beta_1,\beta_3)-1)$$

$$\beta_4' = u\left[-\beta_{jump}^{-}(\Theta_n^M(u,0,0)-1)\right] - \beta_4 \kappa_U^M + \beta_{jump}^{-}(\Theta_n^M(u,\beta_2,\beta_4)-1) \tag{A.6}$$

Once we obtain the characteristic function of equity prices by solving this system of ODEs, we can compute the put option price with strike K and maturity $\tau = T - t$ as:

$$P_t^j(K,\tau) = S_t^j\,(e^k e^{-r\tau}(1-\Pi_2^j) - (1-\Pi_1^j)) \tag{A.7}$$

where $k = \log(K/S_t)$, and the risk-neutral probabilities Π_1^j and Π_2^j are defined as:

$$\Pi_1^j = \frac{1}{2} + \frac{1}{\pi}\int_0^{+\infty} \mathrm{Re}\left[\frac{e^{-iuk}\phi^j(iu+1)}{iue^{r\tau}}\right] du \tag{A.8}$$

$$\Pi_2^j = \frac{1}{2} + \frac{1}{\pi}\int_0^{+\infty} \mathrm{Re}\left[\frac{e^{-iuk}\phi^j(iu)}{iu}\right] du \tag{A.9}$$

Given the price of a put option with strike price K and maturity τ, we can find the price of the corresponding call option with same strike and maturity, using the put-call parity:

$$C_t^j(k,\tau) + e^{-r_t\tau}K = P_t^j(k,\tau) + S_t e^{-\delta\tau} \quad \text{(A.10)}$$

where δ indicates the dividend yield.[21]

[21]The fact that put payoff is bounded offers some numerical advantage for computing the price of put option, e.g., FCSE produces more accurate results for put option. Therefore, we compute the price of put options first and then obtain the price of call option using the put-call parity.

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Figure 1: Option-Implied Betas vs. Market Beta

This figure displays the option implied betas for individual equities versus their corresponding market betas. Option implied betas are obtained by estimating the individual equity option valuation model (3), using weekly observations of out-of-the-money options on Wednesdays, or the prior trading day if it is a holiday, from January 2006 to December 2016. β_{mkt} is the beta estimated by market model, using the monthly return of stocks over the entire sample period. The fitted line is obtained by regressing the option implied betas (negative jump beta) on the market beta (diffusive beta) in Panels A-C (D).

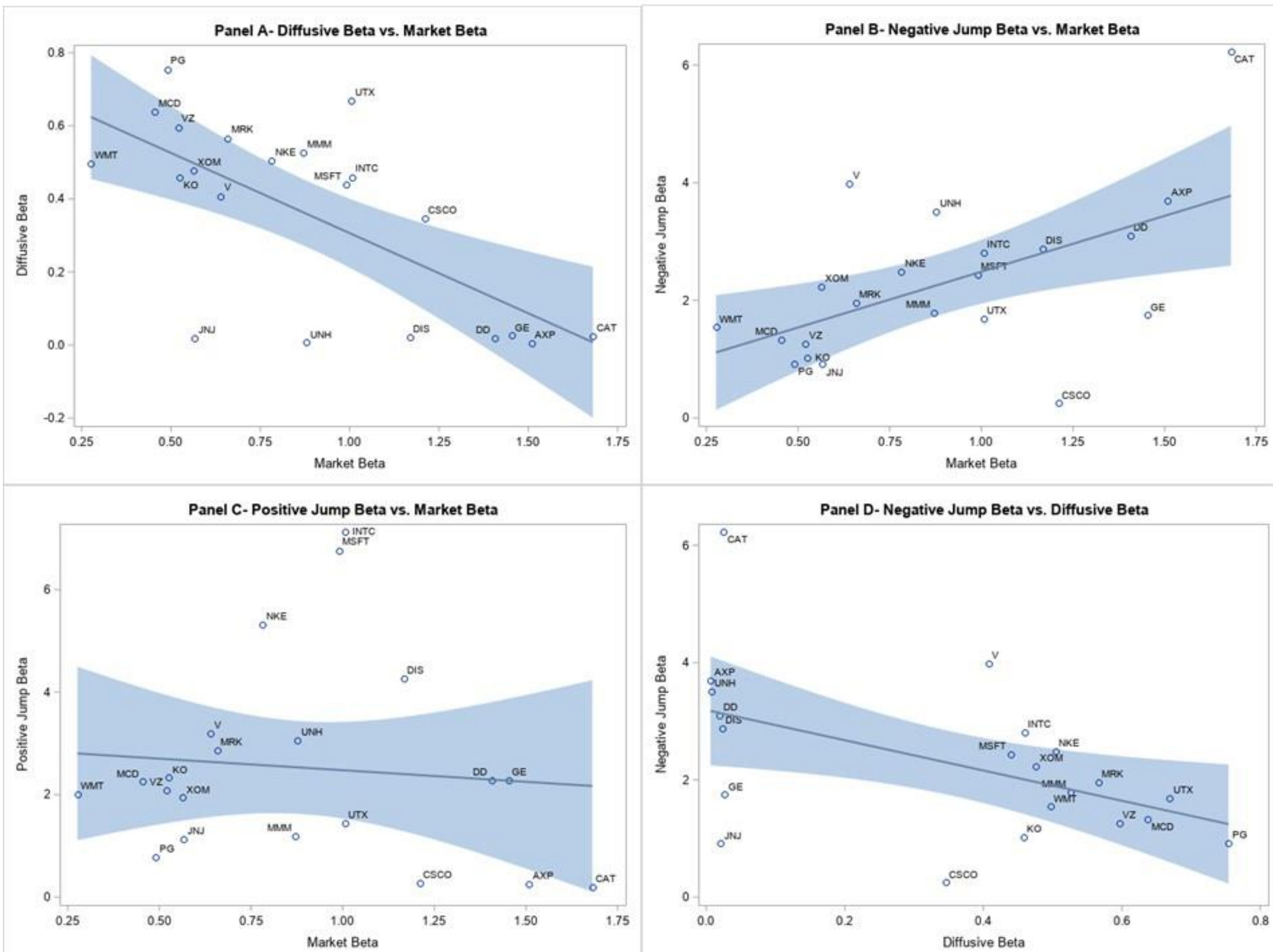

Table 1: Summary Statistics for the Option Panels

The table reports the number of call and put option contracts for the index and each individual equity over the in-sample period 2006-2016, along with their average moneyness and maturity. Moneyness is measured as the ratio of stock price over strike price (S/K). DTM refers to the average of the remaining days to maturity across the options. This sample is used in the first step of our optimization procedure, when we estimate the state vector (V_t, U_t).

	Call Options			Put Options		
Ticker	**N**	**S/K**	**DTM**	**N**	**S/K**	**DTM**
SPX	135,623	0.94	93.90	259,083	1.12	70.60
AXP	17,550	0.92	95.78	20,254	1.10	79.36
CAT	24,519	0.90	95.62	26,754	1.11	78.57
CSCO	18,078	0.89	113.59	16,365	1.12	94.47
DD	14,794	0.92	96.95	18,450	1.10	83.83
DIS	18,528	0.92	92.17	23,981	1.10	73.76
GE	17,184	0.90	121.79	17,983	1.11	101.97
INTC	20,439	0.89	112.17	19,429	1.12	90.33
JNJ	11,876	0.94	106.21	18,985	1.10	85.04
KO	15,501	0.93	111.51	19,742	1.10	98.66
MCD	17,595	0.93	98.37	24,262	1.10	85.38
MMM	13,349	0.92	115.45	17,670	1.10	87.22
MRK	16,152	0.92	96.26	19,950	1.10	82.52
MSFT	24,156	0.90	104.15	26,396	1.11	87.48
NKE	17,001	0.91	98.11	20,036	1.10	79.30
PG	14,531	0.94	97.48	19,873	1.09	81.63
UNH	18,193	0.91	103.00	20,503	1.10	84.39
UTX	15,152	0.92	109.98	18,763	1.10	87.89
V	21,029	0.90	107.43	22,730	1.10	83.45
VZ	17,163	0.93	110.10	26,105	1.11	93.68
WMT	18,003	0.92	104.06	21,689	1.10	84.21
XOM	19,126	0.92	95.08	24,015	1.10	76.89

Table 2- Model Parameters for Market Index and Individual Equity Options

The parameters obtained by estimating the market model (1) for the market index (SPX) and equity model (3) for individual equities, using weekly observations of out-of-the-money options on Wednesdays, from January 2006 to December 2016. If Wednesday is a holiday, we use the observations on the prior trading day. We obtain the models parameters by optimizing the objective function (6) over the entire sample period. To ease the computational load at this step, we shrink the sample size by choosing the most liquid options from all over the volatility surface. See section 4 for further discussion. Tables A1 report the summary statistics of the sample used here. Parameters are estimated nonlinear optimizations (fmincon in Matlab).11

Ticker	ρ^M	υ^M	κ_υ^M	σ_υ^M	κ_U^M	C_0^{+M}	λ_-^M	λ_+^M	μ_υ^M	μ_U^M
SPX	-0.89	0.01	9.36	0.60	0.76	7.48	23.35	78.25	20.83	18.67
	ρ^S	υ^S	κ_υ^S	σ_υ^S	κ_U^S	C_0^{+S}	λ_-^S	λ_+^S	μ_υ^S	μ_U^S
AXP	-0.36	0.63	0.00	0.99	1.95	11.54	21.16	75.95	8.48	17.98
CAT	-0.90	0.00	46.28	0.89	1.98	72.96	28.90	35.11	18.41	22.51
CSCO	-0.86	0.01	5.99	0.38	0.49	7.39	13.54	78.27	22.62	18.66
DD	-0.24	0.00	4.51	0.97	1.96	10.20	29.89	77.70	22.93	20.32
DIS	-0.24	0.01	0.17	0.97	1.94	10.12	41.32	77.31	20.46	19.69
GE	-0.47	0.00	7.71	0.98	1.89	7.67	18.08	78.21	22.03	18.72
INTC	-0.27	0.00	13.17	0.98	1.92	8.52	33.20	77.95	19.20	18.83
JNJ	-0.40	0.00	10.91	0.84	0.94	7.39	27.95	78.25	20.12	18.71
KO	-0.44	0.00	12.76	0.89	1.50	7.50	32.45	78.23	19.18	18.67
MCD	-0.28	0.00	16.63	0.84	1.27	8.01	44.77	78.05	17.16	18.95
MMM	-0.15	0.00	7.29	0.72	1.51	8.23	37.29	78.43	20.50	20.29
MRK	-0.33	0.00	11.93	0.95	1.50	7.98	32.00	78.08	19.62	18.90
MSFT	-0.33	0.00	15.62	0.96	1.67	8.23	39.59	78.09	17.94	18.82
NKE	-0.56	0.00	15.31	0.95	1.76	8.01	38.68	78.10	18.00	18.66
PG	-0.15	0.00	14.51	0.73	1.54	7.33	39.50	78.24	18.05	18.74
UNH	-0.32	0.00	2.00	0.97	1.98	9.93	24.49	77.24	23.19	18.49
UTX	-0.23	0.00	11.94	0.96	1.80	7.79	29.31	78.11	19.94	18.91
V	-0.64	0.01	13.21	0.95	1.56	8.26	39.03	77.97	18.17	18.98
VZ	-0.55	0.00	11.71	0.91	1.69	7.63	30.20	78.17	19.83	18.79
WMT	-0.83	0.00	16.58	0.89	1.28	7.63	43.82	78.16	17.11	18.73
XOM	-0.93	0.00	14.21	0.95	1.59	7.71	38.90	78.13	18.22	18.83

Table 3- Summary Statistics of the Variance factors, V^M and V^S

This table reports the summary statistics of the time series of the variance factor for the market index (SPX), V^M and for individual equities V^S. We obtain the weekly variance factors by estimating models (1) and (3) on every Wednesday, from 2006 to 2016. If Wednesday is a holiday, we use the observations on the prior trading day. The summary statistics of the option sample used to derive the variance factors are provided in Table 1. The column Kurt reports the excess kurtosis. AC indicates autocorrelation and Q denotes quintile.

Ticker	Mean	Std Dev	Skew	Kurt	Q(0.25)	Q(0.50)	Q(0.75)	AC(1)	AC(15)	AC(30)
SPX	0.006	0.009	6.530	59.249	0.002	0.004	0.008	0.247	0.099	0.034
AXP	0.015	0.038	7.171	58.745	0.004	0.007	0.011	0.404	0.295	0.138
CAT	0.009	0.011	4.940	32.014	0.004	0.007	0.011	0.196	0.036	0.073
CSCO	0.009	0.012	4.187	21.803	0.003	0.006	0.010	0.339	0.119	-0.070
DD	0.007	0.011	6.376	48.416	0.003	0.005	0.009	0.390	0.076	0.009
DIS	0.009	0.012	7.136	59.971	0.004	0.007	0.010	0.522	0.135	0.036
GE	0.008	0.016	16.377	321.963	0.004	0.006	0.009	0.126	0.008	0.070
INTC	0.007	0.007	6.207	61.162	0.004	0.006	0.009	0.129	0.121	-0.021
JNJ	0.004	0.003	0.730	0.484	0.002	0.004	0.006	0.235	0.102	-0.010
KO	0.005	0.005	8.479	127.996	0.003	0.005	0.007	0.243	0.052	0.067
MCD	0.006	0.007	11.631	198.901	0.003	0.005	0.007	0.147	0.067	0.055
MMM	0.006	0.005	3.822	32.132	0.003	0.006	0.009	0.176	0.009	-0.053
MRK	0.006	0.007	7.337	80.858	0.003	0.005	0.008	0.206	0.086	0.021
MSFT	0.008	0.010	6.994	66.512	0.004	0.006	0.010	0.240	-0.003	-0.015
NKE	0.008	0.010	7.999	93.235	0.004	0.006	0.010	0.112	0.014	0.002
PG	0.005	0.003	1.973	10.585	0.003	0.004	0.007	0.071	0.108	0.122
UNH	0.011	0.020	8.094	87.539	0.004	0.007	0.011	0.321	0.199	0.099
UTX	0.007	0.007	7.738	101.202	0.004	0.006	0.009	0.207	-0.048	0.050
V	0.010	0.015	5.346	33.273	0.005	0.007	0.010	0.227	0.087	0.037
VZ	0.005	0.004	2.071	10.927	0.002	0.004	0.007	0.102	0.042	0.030
WMT	0.006	0.005	4.306	42.246	0.003	0.005	0.008	0.297	0.056	0.092
XOM	0.006	0.004	1.286	3.864	0.003	0.006	0.009	0.274	0.149	0.146

Table 4- Option Implied Betas

Option implied betas are obtained by estimating the individual equity option valuation model (3), using weekly observations of out-of-the-money options on Wednesdays, or the prior trading day if it is a holiday, from January 2006 to December 2016. β_{mkt} is the beta estimated by market model, using the monthly returns of stocks over the entire sample period. We obtain the option implied betas by optimizing the objective function (6) over the entire sample period. To ease the computational load at this step, we shrink the sample size by choosing the most liquid options from all over the volatility surface. See section 4 for further discussion. Tables A1 report the summary statistics of the sample used here. Parameters are estimated nonlinear optimizations (fmincon in Matlab). SSR is the systematic risk ratio of individual equities, which is equal to $(1/t)\sum_{t=1}^{T}(\beta_{diff}^2 V^M)/(V^S + \beta_{diff}^2 V^M)$.

Ticker	β_{diff}	β_{jump}^{+}	β_{jump}^{-}	β_{mkt}	SR
AXP	0.006	0.254	3.697	1.509	0.007
CAT	0.025	0.197	6.231	1.681	0.019
CSCO	0.346	0.282	0.252	1.210	0.156
DD	0.019	2.289	3.098	1.407	0.028
DIS	0.023	4.262	2.877	1.168	0.012
GE	0.026	2.290	1.747	1.454	0.011
INTC	0.460	7.131	2.817	1.007	0.203
JNJ	0.020	1.129	0.916	0.567	0.018
KO	0.458	2.332	1.026	0.525	0.210
MCD	0.638	2.254	1.331	0.455	0.326
MMM	0.526	1.189	1.789	0.870	0.229
MRK	0.566	2.875	1.957	0.657	0.276
MSFT	0.440	6.766	2.429	0.990	0.185
NKE	0.505	5.311	2.482	0.780	0.213
PG	0.754	0.778	0.918	0.490	0.396
UNH	0.007	3.064	3.504	0.878	0.007
UTX	0.669	1.451	1.692	1.006	0.291
V	0.408	3.205	3.980	0.639	0.145
VZ	0.596	2.083	1.264	0.520	0.338
WMT	0.497	2.008	1.544	0.276	0.241
XOM	0.476	1.942	2.224	0.563	0.205

Table 5- Summary Statistics of the Variance factors, U^M and U^S

This table reports the summary statistics of the time series of the tail factor for the market index (SPX), U^M and for individual equities U^S. We obtain the weekly variance factors by estimating models (1) and (3) on every Wednesday, from 2006 to 2016. If Wednesday is a holiday, we use the observations on the prior trading day. The summary statistics of the option sample used to derive the variance factors are provided in Table 1. The column Kurt reports the excess kurtosis. AC indicates autocorrelation and Q denotes quintile.

Ticker	Mean	Std Dev	Skew	Kurt	Q(0.25)	Q(0.50)	Q(0.75)	AC(1)	AC(15)	AC(30)
SPX	4.064	2.293	1.092	0.167	2.383	3.021	5.389	0.607	0.307	0.220
AXP	4.684	3.862	2.737	8.745	2.673	3.157	4.461	0.529	0.292	0.124
CAT	4.892	4.345	3.915	20.994	2.839	3.312	4.972	0.433	0.313	0.043
CSCO	5.331	6.473	4.214	20.866	2.739	3.134	4.441	0.294	0.089	-0.101
DD	4.063	2.376	3.430	16.637	2.756	3.325	4.465	0.323	0.112	0.022
DIS	3.962	2.962	3.859	21.242	2.632	2.969	3.711	0.440	0.105	-0.036
GE	3.826	2.847	3.978	20.130	2.541	3.033	3.848	0.453	0.186	0.097
INTC	4.507	4.093	4.323	23.377	2.772	3.128	4.233	0.233	0.026	0.002
JNJ	2.857	1.494	3.230	15.865	2.036	2.470	3.183	0.285	0.056	-0.011
KO	3.104	1.807	2.838	9.841	2.129	2.600	3.292	0.276	0.059	0.020
MCD	3.565	2.800	5.830	49.821	2.466	2.837	3.478	0.329	0.112	0.024
MMM	3.655	2.333	3.642	17.336	2.492	2.954	3.876	0.475	0.163	-0.034
MRK	3.822	2.389	3.730	17.987	2.713	3.151	3.999	0.296	0.125	0.030
MSFT	4.009	3.640	3.919	17.836	2.548	2.885	3.611	0.389	0.085	-0.064
NKE	4.534	4.221	3.598	15.850	2.620	3.013	4.216	0.310	0.102	-0.082
PG	3.355	2.191	3.430	16.136	2.215	2.727	3.618	0.431	0.193	0.052
UNH	4.259	3.058	3.087	11.897	2.717	3.095	4.254	0.364	0.193	0.034
UTX	3.660	2.768	7.619	94.697	2.550	2.998	3.709	0.297	0.089	0.001
V	4.401	3.571	3.281	13.169	2.690	3.086	4.293	0.338	0.097	-0.043
VZ	3.494	2.368	4.793	33.482	2.477	2.950	3.614	0.322	0.064	0.033
WMT	3.698	3.291	4.458	25.949	2.361	2.725	3.469	0.323	0.100	0.011
XOM	3.863	2.981	4.134	22.323	2.577	2.899	3.811	0.322	0.048	0.068

Table 6- Correlation between Option Implied Betas and Market Beta

The table reports the correlations between option implied betas for individual equities and their corresponding market betas. Table 4 provides the option betas obtained by estimating the individual equity model (3) and market betas by estimating the market model. P-values are reported in the parentheses.

	β_{diff}	β_{jump}^{+}	β_{jump}^{-}	β_{mkt}
β_{diff}	1.000	0.118	-0.499	-0.669
		(0.609)	(0.021)	(0.001)
β_{jump}^{+}		1.000	0.103	-0.092
			(0.656)	(0.691)
β_{jump}^{-}			1.000	0.559
				(0.008)
β_{mkt}				1.000

Table A1- Summary Statistics for the Option Panels

The table reports the number of call and put option contracts for the index and each individual equity over the in-sample period 2006-2016, along with their average moneyness and maturity. Moneyness is measured as the ratio of stock price over strike price (S/K). DTM refers to the average of the remaining days to maturity across the options. This sample is used in the first step of our optimization procedure, when we estimate the parameter vector $\Psi_0^M \equiv \left\{\kappa_\upsilon^M, \bar{\upsilon}^M, \sigma_\upsilon^M, \rho^M, \kappa_U^M, \mu_\upsilon^M, \mu_U^M, c_0^{+M}, \lambda_-^M, \lambda_+^M\right\}$ for market index and $\Psi^S \equiv \left\{\beta_{diff}^S, \beta_{jump}^{+S}, \beta_{jump}^{-S}, \kappa_\upsilon^S, \bar{\upsilon}^S, \sigma_\upsilon^S, \rho^S, \kappa_U^S, \mu_\upsilon^S, \mu_U^S, c_0^{+S}, \lambda_-^S, \lambda_+^S\right\}$ for individual equities, over the entire sample period. To ease the computational burden, on each Wednesday and for each maturity, we retain the most liquid option that falls in one of the moneyness buckets, which range from 0.7 to 1.3 with steps of 0.05.

	Call Options			**Put Options**		
Ticker	**N**	**S/K**	**DTM**	**N**	**S/K**	**DTM**
SPX	23,325	0.91	149.44	41,338	1.14	121.91
AXP	9,731	0.90	119.67	12,124	1.12	100.95
CAT	13,356	0.88	117.61	15,311	1.13	101.96
CSC	12,951	0.89	121.25	13,732	1.13	104.53
DD	8,684	0.91	121.53	11,324	1.12	103.96
DIS	10,043	0.90	117.29	12,587	1.12	97.89
GE	12,740	0.89	129.34	15,033	1.12	112.54
INT	12,946	0.89	121.54	14,846	1.13	103.34
JNJ	7,433	0.92	130.47	11,573	1.12	109.32
KO	9,116	0.92	125.26	13,410	1.12	112.28
MCD	10,309	0.91	124.68	14,334	1.12	108.60
MMM	8,595	0.91	130.21	11,702	1.12	106.59
MRK	9,243	0.91	119.68	12,501	1.12	101.36
MSF	13,511	0.89	123.72	16,395	1.13	106.27
NKE	9,973	0.89	122.30	11,877	1.12	103.63
PG	7,852	0.92	123.44	12,465	1.12	105.37
TRV	4,857	0.91	145.32	6,513	1.12	130.01
UNH	10,624	0.89	124.46	12,774	1.12	105.83
UTX	9,649	0.90	132.44	12,101	1.12	109.40
V	10,443	0.89	124.19	12,672	1.13	105.61
VZ	10,219	0.91	129.30	15,194	1.13	108.03
WMT	10,462	0.91	127.51	14,182	1.12	109.03
XOM	10,200	0.90	122.68	13,350	1.12	102.68