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Modeling Conditional Factor Risk Premia Implied by Index Option Returns*

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Abstract

We propose a novel factor model for option returns. Option exposures are estimated nonparametrically and factor risk premia can vary nonlinearly with states. The model is estimated using regressions, with minimal assumptions on factor and option return dynamics. Using index options, we characterize the conditional risk premia for the market return, market variance, and tail and intermediary risk factors. Unconditionally, all risk premia have the expected sign and meaningful magnitudes. Market and variance risk premia display pronounced time-variation, spike during crises, and always have the expected sign. Combined, market return and variance explain more than 90% of option return variation.

JEL Classification: G10; G12; G13.

Keywords: Option Returns; Factor Models; Option-Implied Factor Risk Premia; Time-Varying Exposures; Machine Learning.

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1 Introduction

Because of the state-contingent nature of their payoffs, options are highly informative about state prices and the price of risk. Traditionally, the index and stock option literature has studied option prices. This contrasts with the literature on the underlying securities, which is almost exclusively based on returns. Moreover, while much of the study of stock returns is focused on factor models that analyze the market-wide risks determining these returns, the literature on option returns is more focused on option characteristics. While the literature on factor models of option returns has been growing, our knowledge about the properties of the implied factor risk premia and the cross-section of option return exposures is still very limited.[1]

A potential explanation for the limited progress on this topic is that existing methodologies are developed to estimate risk premia from stock returns and are not suitable for option returns. Option prices, returns, and exposures exhibit pronounced nonlinear variation across time and option characteristics, which is hard to capture using standard methods for stock returns. This paper proposes a factor model specifically designed to estimate conditional risk premia and option exposures based on large panels of option returns. The framework allows for multiple factors and is sufficiently flexible to accommodate any factor of interest. It provides reliable estimates of the relative importance of various sources of systematic risk in determining expected returns on options.

The traditional approach to estimating risk premia implied by options adopts parametric models, typically featuring jump, volatility, and tail risks.[2] These studies provide important insights about equity, variance, jump, and tail risk premia. However, their parametric structure constrains the number and type of factors that can be simultaneously accommodated. These models are therefore silent on the exposure of options beyond return, variance, and jump risks, even though recent evidence suggests the presence of other risks, such as liquidity and intermediary risks.[3] Moreover, the structure of the factor risk premia in parametric models is usually very restrictive because of the emphasis on affine models that allow for

[1] See Jones (2006), Israelov and Kelly (2017), and Horenstein, Vasquez, and Xiao (2018) for notable exceptions.

[2] For seminal contributions to this literature, see, among others, Pan (2002), Eraker, Johannes, and Polson (2003), Carr and Wu (2009), Bates (2008), Broadie, Chernov, and Johannes (2009), Todorov (2010), Bollerslev, Todorov, and Xu (2015), and Andersen, Fusari, and Todorov (2015a,b).

[3] Barras and Malkhozov (2016), Fan, Imerman, and Dai (2016a), and Gârleanu, Pedersen, and Poteshman (2009) study the impact of demand shocks on options. Christoffersen, Goyenko, Jacobs, and Karoui (2018b) show that illiquidity impacts the expected returns on delta-hedged equity options.

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closed-form solutions for option prices. Several studies demonstrate that more general non-affine option pricing models provide a better fit to the cross-section of option prices, but these models are harder to implement.[4] Israelov and Kelly (2017) document that state-of-the-art affine models produce counterfactual predictions about expected option returns.

Directly analyzing the cross-section of option returns without imposing strong parametric assumptions avoids these constraints, but it is well-known that the measurement of option exposures to risk factors over time and across moneyness and maturity is fraught with problems.[5] The development of new models that are better suited to the study of option returns is thus of paramount interest. We model option returns' factor exposures using nonparametric functions of instruments such as moneyness, maturity, and conditional volatility. This nonparametric approach allows for rich patterns in model exposures, not only across maturity and moneyness, but also over time. We model deleveraged option returns rather than raw returns to alleviate well-known problems with the estimation of expected option returns and risk premiums due to measurement errors in option prices (see e.g., Duarte, Jones, and Wang, 2019). To estimate the model, we use a two-step procedure. In a first step, we jointly estimate option return exposures to all factors. Taking these exposures as given, we then estimate the conditional risk premia associated with the factors.

We illustrate the value of our approach using a sample of more than 24 years of daily data on index option returns with various maturity and moneyness. The resulting option return panel has more than one million observations, but we find that the estimation of exposures and risk premia is fast, which proves that our framework is practical for large-scale empirical exercises. Our baseline model contains three factors: the market return and changes in its variance, as well as gamma (the squared market return). We also estimate models with additional factors: the intermediary capital factor of He, Kelly, and Manela (2017) and the tail factor in Bollerslev et al. (2015).

Consistent with the reduced-form option pricing literature, the factor risk premia depend on state variables, but unlike existing models, we are able to incorporate any set of state variables. We find that while specifications of the factor risk premia that are nonlinear in the state variables generate more variation in conditional risk premia during periods of high

[4]See for instance Jones (2003), who develops a non-affine stochastic volatility model to study index option prices, and Eraker and Wang (2015), who develop a nonlinear model of the variance premium.

[5]This issue also poses a problem when studying the cross-section of stock returns, but it is more critical in the case of option returns because of leverage and heavy tails. For discussions on the estimation of stock exposures, see Shanken (1990), Jagannathan and Wang (1996), Ghysels (1998), Lustig and van Nieuwerburgh (2005), Lewellen and Nagel (2006), Nagel and Singleton (2011), Buss and Vilkov (2012), and Bollerslev, Li, and Todorov (2016).

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uncertainty, a linear specification also generates substantial variation in risk premia and the signs of the risk premia are more frequently intuitively plausible. Our baseline model therefore uses this linear specification of the risk premia, but of course the option exposures depend nonlinearly on the state variables and the option characteristics.

Overall, the baseline model performs well and explains more than 92% of the time-series variation in option returns. This fit is impressive because it is based on daily returns, whereas most existing studies use less noisy monthly returns. One of our main empirical contributions is a characterization of the relative contribution of different economic factors in explaining the cross-section of option returns. In the baseline model, market return risk is the main driver of index option return risk and explains (depending on the option contract) between 53% and 85% of index option return total variance. Variance risk is also an important driver, with R^2s between 5% and 37%. The contribution of gamma risk to overall option return variation is small compared to these two factors. The role of the market return and market variance factors continues to be dominant when we add the intermediary and tail factors to the model.

We provide estimates of unconditional and conditional factor exposures. Unconditionally, fitted exposures to option pricing factors have the correct sign, their magnitude is plausible, and they behave as expected across moneyness. The term structure of exposures to variance risk implied by index option returns differs in some dimensions from the predictions of standard affine models. Factors have a different impact across the cross-section of options. SPX puts are much more sensitive to market variance risk than calls, and calls are more sensitive to market return risk. As expected, the tail factor is mainly important for out-of-the-money puts. The intermediary factor on the other hand is more important for calls than for puts. These results suggest that the index option market seems to be somewhat segmented, consistent with Constantinides, Jackwerth, and Perrakis (2009) and Bakshi, Crosby, and Gao (2019), who show that index call options are priced differently from index put options. To illustrate the value of the estimated conditional model exposures, we compare the hedging performance of our model with standard benchmarks. An investor who uses our model instead of the benchmark Heston (1993) model realizes considerable hedging gains. This is important for financial institutions and traders who need to manage the risk exposures of their positions and strategies.

The model also provides estimates of conditional factor risk premia. In the baseline model, the signs of the average risk premia are intuitively plausible for all three factors: positive for the market premium and negative for the variance and gamma premia. The

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average market risk premium is 9.1%, close to the sample average. The average variance risk premium is −22.1%. Moreover, the model-implied equity and variance risk premia have the expected positive and negative signs every day. Their variation over time seems plausible and they peak at the height of the financial crisis in 2008. The risk premium associated with gamma is small.

The signs and magnitudes of the market and variance risk premia are robust when including additional factors in the model. An intermediary factor based on He et al. (2017) carries a positive risk premium of 32.8%, consistent with intuition and very close to the estimate in He et al. (2017). However, the percentage of variation in option returns explained by this factor is modest compared to the role played by the market and variance factors. The same remark applies to the tail factor from Bollerslev et al. (2015). As expected, it carries a negative risk premium, but its role in explaining option returns is much smaller than that of the market return and variance risks.

We conclude that the signs, magnitudes, and time variation of the conditional risk premia are plausible. This is encouraging because compared to the existing literature, we impose relatively few restrictions. Another critical difference is that our approach is based on option returns rather than prices. Existing studies typically specify the physical and risk-neutral dynamics of the risk factors and then estimate the model by combining option prices with the underlying stock return data, because the underlying security returns are required to identify the physical dynamics and risk premium parameters. Unlike option prices, option returns embed information about both physical and risk-neutral probability measures, which allows us to identify conditional risk premia using option information only. Moreover, in the existing literature risk premia are typically estimated for conditional moments of returns only, whereas we can estimate risk premia for other factors too.

It is well known that regression models' in-sample performance does not always translate into good out-of-sample results. To assess whether the good performance of the baseline model also applies out-of-sample, we adopt a recursive estimation approach and estimate the model each year. We then construct the model's hedging and return predictions over the subsequent year holding parameters fixed but allowing conditional volatility to vary. We consider a hedging exercise that is designed to assess the robustness of factor exposures out-of-sample while the delta-hedged return prediction allows us to assess whether model-implied risk premia are stable and not overly sensitive to a particular sample period. Overall, the out-of-sample performance of the model is impressive and yields out-of-sample R^2 of 7.6%, which exceeds the out-of-sample R^2 documented in existing studies that use different

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modeling approaches.

Our model and empirical results are related to various strands of the literature on options and empirical asset pricing. Gagliardini, Ossola, and Scaillet (2016) develop a factor model with linear specification of exposures and risk premia to study the risks priced in large panels of individual stock returns. Kelly, Pruitt, and Su (2018) develop a factor model of stock returns with principal component factors and instrumented exposures. We adapt the panel framework of Gagliardini et al. (2016) to option returns while allowing for instrumented option exposures in the spirit of Kelly et al. (2018).

Our work is also closely related to a burgeoning literature that uses principal component and nonlinear factor decomposition methods for modeling option returns. Jones (2006) develops a factor model of option returns with factor innovations that are nonlinear transformations of economic variables. Israelov and Kelly (2017) model index option returns using a principal components approach applied to implied volatilities. Büchner and Kelly (2022) use the instrumented principal components analysis of Kelly et al. (2018) to model option returns. Horenstein et al. (2018) study the factor structure in equity option returns based on the principal components of delta-hedged equity option returns. These approaches provide impressive explanatory power for option returns, but they cannot directly identify the underlying determinants of option returns. We complement these studies by providing a dynamic factor model of option returns which allows us to measure the relative importance of various economic factors for the cross-section of option returns.

Our study also contributes to the rapidly growing literature that applies machine learning methods in the estimation of factor models (see Giglio, Kelly, and Xiu (2021) for an overview). More specifically, we contribute to the common theme of modeling conditional factor exposures. The first strand of this literature models factor exposures as conditionally linear in (potentially) large numbers of predictors.[6] The link between our paper and this literature is that after we specify the explanatory variables (evaluations of basis functions), our model can be estimated by linear least-squares; in other words, the evaluated basis functions could be used as inputs to the IPCA to uncover latent factors. The second strand of this literature proposes that factor exposures are more general functions of observable variables, as in e.g., Connor, Hagmann, and Linton (2012) and Gu, Kelly, and Xiu (2021). Our paper is closest to that of Gu et al. (2021). The key difference is that Gu et al. (2021) focus on the

[6] Kelly, Pruitt, and Su (2019), Kelly, Palhares, and Pruitt (2022), Büchner and Kelly (2022) use the IPCA developed by Kelly et al. (2018). Kim, Korajczyk, and Neuhierl (2020) apply the PPCA of Fan, Liao, and Wang (2016b).

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estimation of nonlinear exposures from a large number of potential predictors, whereas we use a limited number of exposure predictors suggested by option pricing theory; our focus is instead on the functional form rather than predictor selection. Our nonparametric exposure estimation is specifically tailored to the task of estimating option sensitivities to underlying risk factors, which are smooth and well-behaved functions.

The paper proceeds as follows. Section 2 provides the theoretical framework. Section 3 discusses the estimation strategy. Section 4 discusses the data, Section 5 presents the empirical results for the baseline model, and Section 6 presents robustness results. Finally, Section 7 concludes.

2 The Model

We first present the model. We then discuss the relationship between our approach and affine option pricing, using the Heston (1993) model as an example. This comparison clarifies the similarities and differences between our framework and existing approaches.

2.1 A Dynamic Factor Model of Option Returns

Our objective is to estimate option exposures to risk factors and factor risk premia while imposing as few parametric assumptions as possible. We denote by O_t the price of an option written on the underlying asset S_t observed at time t with a given exercise price K_O, time to maturity τ_O, and payoff function (i.e., call or put). Our baseline empirical analysis uses deleveraged returns on at the money (ATM) and out of the money (OTM) options.

To deleverage option returns without relying on parametric assumptions about option deltas, we scale option returns by the ratio of the option price to the price of the underlying asset. The deleveraged return on option O from t to $t + \Delta t$ is thus defined as:

$$r^O_{t+\Delta t} \equiv \left(\frac{\Delta O_{t+\Delta t}}{O_t} \right) \cdot \frac{O_t}{S_t}, \tag{1}$$

where $\Delta O_{t+\Delta t} \equiv O_{t+\Delta t} - O_t$ corresponds to the option price increment over Δt period of time.

We assume that $r^O_{t+\Delta t}$ can be characterized by a dynamic factor structure of the form

$$r^O_{t+\Delta t} = \gamma^O_t + \sum_{l=1}^{L} \beta^O_{l,t} \cdot f_{l,t+\Delta t} + \varepsilon^O_{t+\Delta t}, \tag{2}$$

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where $f_{l,t+\Delta t}$ denotes factor l's realization between t and $t+\Delta t$, L is the number of factors, and γ_t^O and $\beta_{l,t}^O$ are the option's conditional intercept and conditional loading on factor l, respectively.[7]

The model residual $\varepsilon_{t+\Delta t}^O$ satisfies $E_t^P\left[\varepsilon_{t+\Delta t}^O\right]=0$, where $E_t^P[\cdot]$ denotes the conditional expectation under the physical probability measure. We do not explicitly model heteroskedasticity and the correlation of residuals across time and options, but we account for their presence when conducting inference about the parameter estimates.

To allow for rich dynamics of the option exposures, we model the $\beta_{l,t}^O$ as nonparametric functions of option characteristics (i.e., time to maturity and moneyness) and state variables. Option exposures to a given risk factor thus vary nonlinearly with maturity and moneyness, and change over time as the state variables fluctuate. We discuss the specification of exposures in more detail in Section 2.2 below.

According to the first fundamental theorem of asset pricing, absence of arbitrage opportunities guarantees the existence of a stochastic discount factor (SDF), ξ_t, and a risk-neutral probability measure, Q, such that the price of any option, O_t, is given by

$$O_t = E_t^P\left[\frac{\xi_{t+\Delta t}}{\xi_t}O_{t+\Delta t}\right] = E_t^Q\left[\frac{O_{t+\Delta t}}{1+r_t}\right], \tag{3}$$

where r_t is the time-t conditional risk-free rate over period Δt, and $E_t^Q[\cdot]$ is the conditional expectation under the risk-neutral measure. The no-arbitrage condition is imposed in the model by setting $\gamma_t^O = \frac{O_t}{S_t}r_t - \sum_l \beta_{l,t}^O E_t^Q[f_{l,t+\Delta t}]$ in equation (2) where $\frac{O_t}{S_t}r_t$ captures $E_t^Q\left[r_{t+\Delta t}^O\right]$. Dividing equation (3) by S_t on each side, one can show (after some manipulations) that the risk premium on the deleveraged option return is given by

$$E_t^P\left[r_{t+\Delta t}^O\right] - E_t^Q\left[r_{t+\Delta t}^O\right] = -Cov_t\left(\frac{\xi_{t+\Delta t}}{\xi_t}\cdot(1+r_t)\,;r_{t+\Delta t}^O\right). \tag{4}$$

Conditioning on the factors, we assume that $\varepsilon_{t+\Delta t}^O$ in equation (2) is not priced and satisfies $E_t^Q\left[\varepsilon_{t+\Delta t}^O\right]=0$.[8] The factor structure in equation (2), equation (4), and $E_t^Q\left[\varepsilon_{t+\Delta t}^O\right]=0$

[7] To apply our framework to raw option returns, multiply the deleveraged return (1) by S_t/O_t, which changes the functional form of both the conditional intercept and the conditional factor loadings in equation (2). We use deleveraged returns because option leverage makes the first stage of the estimation procedure more vulnerable to outliers and measurement error, and this in turn affects model fit. See Duarte et al. (2019) for a discussion of the impact of measurement error on option returns.

[8] If $Cov_t\left(\frac{\xi_{t+\Delta t}}{\xi_t}(1+r_t)\,;\varepsilon_{t+\Delta t}^O\right)=0$ (i.e., $\varepsilon_{t+\Delta t}^O$ is not priced) then we have $E_t^Q\left[\varepsilon_{t+\Delta t}^O\right]=E_t^P\left[\varepsilon_{t+\Delta t}^O\right]=0$ in the absence of arbitrage opportunities.

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together imply that the conditional risk premium on the deleveraged option return is the sum of each conditional factor exposure multiplied by that factor's conditional risk premium:

$$E_t^P\left[r_{t+\Delta t}^O\right] - E_t^Q\left[r_{t+\Delta t}^O\right] = \sum_{l=1}^{L} \beta_{l,t}^O \cdot \left(E_t^P\left[f_{l,t+\Delta t}\right] - E_t^Q\left[f_{l,t+\Delta t}\right]\right). \tag{5}$$

Building on Gagliardini et al. (2016), we allow for time-variation in the physical (P) and risk-neutral (Q) conditional expectations of the factors. For a given measure m, the conditional expectation of $f_{l,t+\Delta t}$ is a linear transformation of the $q \times 1$ vector $\boldsymbol{g}_{l,t}$ of state variables.[9] This gives:

$$E_t^m\left[f_{l,t+\Delta t}\right] \equiv \boldsymbol{\mu}_l^{m'} \boldsymbol{g}_{l,t}, \tag{6}$$

for $m = P, Q$ and where $\boldsymbol{\mu}_l^m$ is a q by 1 vector of conditional expectation parameters.[10] Equation (6) in turn implies that factor risk premia are given by

$$E_t^P\left[f_{l,t+\Delta t}\right] - E_t^Q\left[f_{l,t+\Delta t}\right] = \left(\boldsymbol{\mu}_l^P - \boldsymbol{\mu}_l^Q\right)' \boldsymbol{g}_{l,t} \equiv \boldsymbol{\lambda}_l' \boldsymbol{g}_{l,t}, \tag{7}$$

where $\boldsymbol{\lambda}_l = \boldsymbol{\mu}_l^P - \boldsymbol{\mu}_l^Q$ denotes the vector of risk premium parameters. The conditional expectation and risk premium specifications (6) and (7) are quite general and nest standard asset pricing models.

In the implementation of the model, we use deleveraged excess option returns,

$$R_{t+\Delta t}^O \equiv r_{t+\Delta t}^O - E_t^Q\left[r_{t+\Delta t}^O\right], \tag{8}$$

to estimate option exposures and factor risk premium dynamics. Deleveraged excess option returns correspond to the difference between deleveraged option returns and the leverage-adjusted risk-free rate, $\frac{O_t}{S_t} r_t$, given that $E_t^Q\left[r_{t+\Delta t}^O\right] = \frac{O_t}{S_t} r_t$ under the risk-neutral measure. We thus have

$$R_{t+\Delta t}^O = r_{t+\Delta t}^O - \frac{O_t}{S_t} r_t. \tag{9}$$

To derive the model prediction for $R_{t+\Delta t}^O$, first take the risk-neutral expectation of (2) and

[9] Throughout the text, we use bold characters or letters to differentiate vectors or matrices from scalars.

[10] When required, we use the notation $\boldsymbol{x}'$ to indicate that we are taking the transpose of the vector or matrix $\boldsymbol{x}$.

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subtract the resulting expression from (2). We get

$$R^O_{t+\Delta t} = \sum_l \left[-E^Q_t \left[f_{l,t+\Delta t} \right] \cdot \beta^O_{l,t} + f_{l,t+\Delta t} \cdot \beta^O_{l,t} \right] + \varepsilon^O_{t+\Delta t} \tag{10}$$

$$= \sum_l \left[-\boldsymbol{\mu}^{Q'}_l \boldsymbol{g}_{l,t} \cdot \beta^O_{l,t} + f_{l,t+\Delta t} \cdot \beta^O_{l,t} \right] + \varepsilon^O_{t+\Delta t}, \tag{11}$$

where we use (6) to obtain the second equation by replacing $E^Q_t \left[f_{l,t+\Delta t} \right]$ with $\boldsymbol{\mu}^{Q'}_l \boldsymbol{g}_{l,t}$. Equation (11) embeds the no-arbitrage condition, which states that the conditional intercept of the excess option return is equal to the negative of the sum of the Q-expectations for all risk factors multiplied by factor sensitivities.

2.2 Option Return Exposures

Regardless of their distributional assumptions, option pricing models all specify that option exposures to risk factors are nonlinear functions of the model state variables, option characteristics such as maturity, moneyness, and the payoff structure.[11] Options thus have different exposures to systematic risks. Moreover, the relation between exposures and option characteristics is highly nonlinear even in the most parsimonious frameworks such as the Black and Scholes (1973) model. The relationship between option exposures and option characteristics may also vary with the state of the economy.

To allow for these potentially rich cross-sectional and temporal variations in risk loadings, we adopt a nonparametric approach. We model exposures to a given risk factor as nonlinear functions of a $p \times 1$ vector of option characteristics, $\boldsymbol{c}_{O,t}$, and an $s \times 1$ vector of factor-specific state variables, $\boldsymbol{i}_{l,t}$. For example, the relevant option characteristics are option type, time to maturity $\tau_{O,t}$, and moneyness, $k_{O,t} = K_O / S_t$. We thus have $\boldsymbol{c}_{O,t} \equiv [\tau_{O,t} \; k_{O,t}]'$. An option's maturity mechanically decreases with time, which introduces a source of time variation in exposures captured by $\tau_{O,t}$, while $k_{O,t}$ captures the impact of S_t on option exposure. The vector of factor-specific state variables $\boldsymbol{i}_{l,t}$ may contain additional signals (state variables) that are informative about option exposures. For instance, many option pricing models predict that option exposures are functions of the level of conditional variance. If this is the only factor-specific state variable, i.e. $i_{l,t}$ is a scalar, this corresponds to $i_{l,t} = v_t$, which is

[11] Consider for example the Black and Scholes (1973) and Heston (1993) models. In both models, the delta and vega depend nonlinearly on time to maturity and on the state variables: S_t in Black and Scholes (1973) and $\{S_t, v_t\}$ in Heston (1993), where S_t denotes the underlying asset price and v_t is the asset's stochastic variance.

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what we assume in our baseline specification.

We denote by $\boldsymbol{z}_{O,l,t} \equiv \left[\boldsymbol{c}'_{O,t} \ \boldsymbol{i}'_{l,t}\right]'$ the $(p+s) \times 1$ vector of all the signals used to model exposures. The risk loading of option O on factor l then satisfies

$$\beta^O_{l,t} \equiv \beta_l \left(\boldsymbol{z}_{O,l,t}\right), \tag{12}$$

in the most general case. Equation (12) indicates that fluctuations in exposures across time and options are entirely induced by the vector of signals $\boldsymbol{z}_{O,l,t}$, while the nonlinear mapping between the signals and exposures, $\beta_l(\cdot)$, depends only on the risk factor. Note that the absence of arbitrage imposes restrictions on the exposures of put and call options with the same characteristics due to put-call parity. We impose these restrictions in estimation when applicable when defining exposures in (12).

We model $\beta_l(\cdot)$ using basis expansion theory. Basis expansions allow for transformation of a set of signals into a (smooth) nonlinear transformation, and the literature contains numerous basis expansion methods. We use the thin plate regression spline basis (TPRS basis henceforth). The TPRS basis is able to parsimoniously account for multidimensional signals, and transforms the vector $\boldsymbol{z}_{O,l,t}$ as follows

$$\boldsymbol{z}_{O,l,t} \underset{TPRS}{\longrightarrow} \left[1 \ \ \boldsymbol{z}_{O,l,t} \ \ \varphi_1\left(\boldsymbol{z}_{O,l,t}\right) \ \ \cdots \ \ \varphi_h\left(\boldsymbol{z}_{O,l,t}\right)\right]' \equiv \boldsymbol{\phi}_{O,l,t}, \tag{13}$$

where each element $\varphi_j\left(\boldsymbol{z}_{O,l,t}\right)$, $j \in \{1, ..., h\}$, is a nonlinear scalar transformation of $\boldsymbol{z}_{O,l,t}$. Internet Appendix C contains the details regarding the specification of $\varphi_j(\cdot)$ in the context of the TPRS basis transforms. From equation (13), note that $\boldsymbol{\phi}_{O,l,t}$ is a $k \times 1$ vector composed of 1, $\boldsymbol{z}_{O,l,t}$, and h nonlinear basis functions such that $k = 1 + (p+s) + h$. We further elaborate on the choice of h in the empirical analysis below.

Based on the basis expansion (13), the risk loading $\beta^O_{l,t}$ satisfies

$$\beta^O_{l,t} = \beta_l\left(\boldsymbol{z}_{O,l,t}\right) \approx \boldsymbol{\phi}'_{O,l,t}\boldsymbol{b}_l = \sum_{j=1}^{k} \phi^j_{O,l,t} b^j_l, \tag{14}$$

where $\boldsymbol{b}_l$ is a $k \times 1$ vector of factor-specific parameters capturing the basis representation of $\beta^O_{l,t}$ across O and t.[12] Thus, exposures to a given risk factor are linear ($\boldsymbol{b}_l$) in the nonlinear basis expansions ($\boldsymbol{\phi}_{O,l,t}$). However, given that $\boldsymbol{\phi}_{O,l,t}$ is fixed once the basis expansion method

[12] If the signals $\boldsymbol{z}_{O,l,t}$ are appropriately chosen, the quality of the approximation in equation (14) improves as the basis dimension k increases, i.e., $\boldsymbol{\phi}'_{O,l,t}\boldsymbol{b}_l$ converges to the true option exposure $\beta_l\left(\boldsymbol{z}_{O,l,t}\right)$.

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is chosen, it can be treated as a vector of explanatory variables.

This approach to the modeling of option exposures builds on Kelly et al. (2018), who develop a framework for modeling instrumented exposures. We extend their framework by allowing exposures to be functions of basis expansions of option characteristics. We use the TPRS basis to model the highly nonlinear nature of the option payoffs. Combining equation (11) with the definition of exposures (14) implies the following factor representation of deleveraged excess option returns

$$R^O_{t+\Delta t} = \sum_l \left[-\boldsymbol{\mu}_l^{Q'} \boldsymbol{g}_{l,t} \cdot \boldsymbol{\phi}'_{O,l,t} \boldsymbol{b}_l + f_{l,t+\Delta t} \cdot \boldsymbol{\phi}'_{O,l,t} \boldsymbol{b}_l \right] + \varepsilon^O_{t+\Delta t}, \tag{15}$$

where $\boldsymbol{\mu}_l^{Q'} \boldsymbol{g}_{l,t} \cdot \boldsymbol{\phi}'_{O,l,t} \boldsymbol{b}_l$ corresponds to $E_t^Q \left[f_{l,t+\Delta t} \right] \beta^O_{l,t}$ and $f_{l,t+\Delta t} \cdot \boldsymbol{\phi}'_{O,l,t} \boldsymbol{b}_l$ to $f_{l,t+\Delta t} \beta^O_{l,t}$.

In summary, compared to existing studies, our methodological contribution is to develop a factor model of option returns with rich option exposures and allowing for general specifications of factor risk premiums. We extend the panel approach of Gagliardini et al. (2016) to make it applicable to option returns by instrumenting option exposures in the spirit of Kelly et al. (2018).

2.3 Relation to Affine Option Pricing Models

We now discuss how the model extends standard affine option pricing frameworks by allowing for more general exposure and risk premium dynamics. To illustrate this, we consider a simple example: The stochastic volatility (SV) model of Heston (1993). Note that this example can easily be generalized to more general models considered in the literature, such as the stochastic volatility with jump case.

In the Heston (1993) model, two factors determine option prices and returns. The first factor is the underlying asset return, which follows a geometric Brownian motion. We have under the physical and risk-neutral measures P and Q respectively:

$$\begin{aligned} \tfrac{dS_{t+dt}}{S_t} &= (r + \lambda_s v_t)\, dt + \sqrt{v_t} \cdot dW^{s,P}_{t+dt} \quad \text{under} \quad P \\ \tfrac{dS_{t+dt}}{S_t} &= r dt + \sqrt{v_t} \cdot dW^{s,Q}_{t+dt} \quad \text{under} \quad Q, \end{aligned} \tag{16}$$

where λ_s denotes the return risk premium parameter, v_t the conditional variance of the underlying asset, and $dW^{s,\cdot}_{t+dt}$ denotes the Brownian motion that drives return dynamics. The second factor is the change in the underlying asset's conditional variance which fluctuates stochastically in the model. The P- and Q-dynamics for the changes in conditional variance

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are given by:

$$\begin{aligned} dv_{t+dt} &= \kappa^P \left(\theta^P - v_t\right) dt + \sigma\sqrt{v_t} \cdot dW_{t+dt}^{v,P} \quad \text{under} \quad P \\ dv_{t+dt} &= \kappa^Q \left(\theta^Q - v_t\right) dt + \sigma\sqrt{v_t} \cdot dW_{t+dt}^{v,Q} \quad \text{under} \quad Q, \end{aligned} \tag{17}$$

where κ^P and $\kappa^Q = \kappa^P + \lambda_v$ are the mean reversion speed parameters, λ_v is the variance risk premium parameter, and θ^P and $\theta^Q = \frac{\kappa^P \theta^P}{\kappa^Q}$ are the unconditional variances. Moreover, σ is the volatility of variance parameter, and $dW_{t+dt}^{v,\cdot}$ denotes the Brownian motion driving the variance dynamics such that $dW_{t+dt}^{s,\cdot} \cdot dW_{t+dt}^{v,\cdot} = \rho dt$, where ρ captures the leverage effect.

Equations (16) and (17) can be used to obtain the conditional moments of the factors. Using the notation introduced in Sections 2.1 and 2.2, we can express the physical and risk-neutral expectations of the factors in Heston (1993) as

$$\begin{aligned} E_t^m \left[\tfrac{dS_{t+dt}}{S_t}\right] &= \begin{bmatrix} r_t \\ \lambda_s \mathbf{1}_{\{m=P\}} \end{bmatrix}' \cdot \begin{bmatrix} 1 \\ v_t \end{bmatrix} dt = \boldsymbol{\mu}_s^{m,H\prime} \boldsymbol{g}_{s,t}^H \\ E_t^m \left[dv_{t+dt}\right] &= \begin{bmatrix} \kappa^m \theta^m \\ -\kappa^m \end{bmatrix}' \cdot \begin{bmatrix} 1 \\ v_t \end{bmatrix} dt = \boldsymbol{\mu}_v^{m,H\prime} \boldsymbol{g}_{v,t}^H, \end{aligned} \tag{18}$$

for $m = P, Q$ where $\mathbf{1}_{\{m=P\}}$ is the indicator function that takes the value 1 when $m = P$ and 0 otherwise, and where $\boldsymbol{\mu}_s^{m,H}$, $\boldsymbol{\mu}_v^{m,H}$, $\boldsymbol{g}_{s,t}^H$ and $\boldsymbol{g}_{v,t}^H$ are implicitly defined by equation (18).[13] Using the notation from Sections 2.1 and 2.2, the conditional factor risk premia in Heston (1993) are thus given by

$$\begin{aligned} E_t^P \left[\tfrac{dS_{t+dt}}{S_t}\right] - E_t^Q \left[\tfrac{dS_{t+dt}}{S_t}\right] &= \left(\boldsymbol{\mu}_s^{P,H} - \boldsymbol{\mu}_s^{Q,H}\right)' \boldsymbol{g}_{s,t}^H = \lambda_s v_t dt \\ E_t^P \left[dv_{t+dt}\right] - E_t^Q \left[dv_{t+dt}\right] &= \left(\boldsymbol{\mu}_v^{P,H} - \boldsymbol{\mu}_v^{Q,H}\right)' \boldsymbol{g}_{v,t}^H = \lambda_v v_t dt, \end{aligned} \tag{19}$$

respectively. Together, equations (18) and (19) provide the restrictions imposed by the Heston model on the dynamics (6) and (7) in our setup. In stark contrast with affine models which fully specify factor dynamics, our setup does not require any assumption about the factors' conditional distribution aside from the functional form of the conditional expectations of the factors.

Building on Broadie et al. (2009), one can show that the Heston (1993) model implies the following (instantaneous) dynamics of deleveraged excess option returns as defined in

[13] For instance, we have $\boldsymbol{g}_{s,t}^H = \boldsymbol{g}_{v,t}^H = [1 \quad v_t]' \, dt$.

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(10) and (11)

$$\begin{aligned} R^{O}_{t+dt} &= \sum_{l=s,v} \left[-E^{Q}_{t}\left[f_{l,t+dt} \right] \cdot \beta^{O,H}_{l,t} + f_{l,t+dt} \cdot \beta^{O,H}_{l,t} \right] \\ &= \sum_{l=s,v} \left[-\boldsymbol{\mu}^{Q,H'}_{l} \boldsymbol{g}^{H}_{l,t} \cdot \beta^{O,H}_{l,t} + f_{l,t+dt} \cdot \beta^{O,H}_{l,t} \right], \end{aligned} \tag{20}$$

where $f_{s,t+dt} = \frac{dS_{t+dt}}{S_t}$ and $f_{v,t+dt} = dv_{t+dt}$ are the factors, and where $\beta^{O,H}_{s,t} \equiv \frac{\partial O_t}{\partial S_t}$ and $\beta^{O,H}_{v,t} \equiv \frac{\partial O_t}{\partial v_t}\frac{1}{S_t}$ denote the delta and leverage-adjusted vega in the Heston (1993) model. Clearly, the return dynamics (20) are similar to equations (10) and (11) when in the limit $\Delta t \to dt$. However, consider the variance factor, $f_{v,t+\Delta t} = \Delta v_{t+\Delta t}$, for example. The affine structure implies that the factor's conditional expectation in (19) is affine in the level of variance.[14] Our framework is more general and can accommodate non-affine specifications. For instance, when $\boldsymbol{g}_{v,t}$ contains nonlinear transformations of v_t, the model's conditional P- and Q-expectations and risk premium dynamics for $f_{v,t+\Delta t} = \Delta v_{t+\Delta t}$ become nonlinear in v_t. More generally, $\boldsymbol{g}_{l,t}$ can contain any state variable (and potential nonlinear transformation of it) informative about factor l's conditional expectations and risk premium.

In summary, the existing option literature provides important insights about the pricing of volatility and jump risks. However, the nature and number of factors that existing models can accommodate is limited given the complex parametric nature of these models. In contrast, our framework can be used to study the pricing of return-based, volatility-based, as well as other factors, provided that the factors are observable or can be constructed.

3 Estimation Strategy

Our estimation strategy is designed to handle large unbalanced panels, which is critical for studying the cross-section of SPX option returns.[15] We use a multi-stage OLS estimation approach which combines features of Fama and MacBeth (1973) with standard cross-sectional panel regressions. Our approach adapts various results and methodologies originally devel-

[14] This specification is used, among others, in Heston (1993), Pan (2002), Jones (2003), Bates (2008), Egloff, Leippold, and Wu (2010), Gourier (2014), and Bégin, Dorion, and Gauthier (2020).

[15] The unbalanced nature of the option panels comes from two sources. First, options of different moneyness and maturity are introduced at different points in time. Second, the return on a given option may be missing on some days but not others.

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oped to study stock returns to our setup.[16] We first explain our strategy for estimating the exposures of option returns to factors. We then discuss the estimation of conditional risk premia.

3.1 Estimating Option Exposures

Consider equation (15), which is obtained by combining equations (6), (7), and (11) with the definition of exposures (14). Because the explanatory variables $\boldsymbol{g}_{l,t}\boldsymbol{\phi}'_{O,l,t}$ are pre- and post-multiplied by the vectors of parameters, $\boldsymbol{\mu}_l^Q$ and $\boldsymbol{b}_l$, the return dynamic (15) is nonlinear in the model parameters and cannot be estimated by OLS. The following proposition presents an alternative factor representation of deleveraged excess option returns which allows for OLS estimation.

Proposition 1 *The dynamics of deleveraged excess option return in equation (15) admit the following factor structure*

$$R^O_{t+\Delta t} = \boldsymbol{\phi}^{g'}_{O,t}\boldsymbol{a} + \boldsymbol{\phi}^{f'}_{O,t+\Delta t}\boldsymbol{b} + \varepsilon^O_{t+\Delta t}\,, \tag{21}$$

where $\boldsymbol{\phi}^{g'}_{O,t}\boldsymbol{a} = \sum_{l=1}^{L} -E^Q_t\left[f_{l,t+\Delta t}\right]\cdot\beta^O_{l,t}$ *and* $\boldsymbol{\phi}^{f'}_{O,t+\Delta t}\boldsymbol{b} = \sum_{l=1}^{L} f_{l,t+\Delta t}\cdot\beta^O_{l,t}$. *The vector* $\boldsymbol{\phi}^g_{O,t} \equiv \left[\boldsymbol{\phi}^{g'}_{O,1,t}\cdots\boldsymbol{\phi}^{g'}_{O,L,t}\right]'$ *is* $Lkq\times 1$, *with* $\boldsymbol{\phi}^g_{O,l,t} \equiv \boldsymbol{g}_{l,t}\otimes\boldsymbol{\phi}_{O,l,t}$ *and where* $\otimes$ *denotes the Kronecker product. Furthermore* $\boldsymbol{\phi}^f_{O,t+\Delta t} \equiv \left[\boldsymbol{\phi}^{f'}_{O,1,t+\Delta t}\ \cdots\ \boldsymbol{\phi}^{f'}_{O,L,t+\Delta t}\right]'$ *is an* $Lk\times 1$ *vector, with* $\boldsymbol{\phi}^f_{O,l,t+\Delta t} \equiv f_{l,t+\Delta t}\otimes\boldsymbol{\phi}_{O,l,t}$ *, and* $f_{l,t+\Delta t}$ *are the factor realizations from (2). The parameter vectors are given by*

$$\boldsymbol{a} \equiv \left[\boldsymbol{a}'_1\cdots\boldsymbol{a}'_L\right]' \quad \textit{and} \quad \boldsymbol{b} \equiv \left[\boldsymbol{b}'_1\cdots\boldsymbol{b}'_L\right]', \tag{22}$$

where $\boldsymbol{a}_l \equiv -\boldsymbol{\mu}_l^Q\otimes\boldsymbol{b}_l$ *is a vector of dimension* $qk\times 1$, *which corresponds to the product of* $\boldsymbol{b}_l$, *the* $k\times 1$ *vector of parameters defining options' exposures to the* l^{th} *factor defined in (14), and* $\boldsymbol{\mu}_l^Q$, *the* $q\times 1$ *vector of parameters defining the* l^{th} *factor's risk-neutral conditional expectation defined in (6).* $\boldsymbol{b}$ *is the collection of the vectors* $\boldsymbol{b}_l$, *and is of dimension* $Lk\times 1$.

[16] For discussions on the estimation of cross-sectional asset pricing models for stocks, see Nagel and Singleton (2011) and the surveys by Goyal (2012) and Nagel (2013).

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Proof. By the properties of the Kronecker product, equation (15) is equivalent to

$$\begin{aligned} R^O_{t+\Delta t} &= \sum_l \left[(\boldsymbol{g}_{l,t} \otimes \boldsymbol{\phi}_{O,l,t})' \left(-\boldsymbol{\mu}^Q_l \otimes \boldsymbol{b}_l \right) + (f_{l,t+\Delta t} \otimes \boldsymbol{\phi}_{O,l,t})' \boldsymbol{b}_l \right] + \varepsilon^O_{t+\Delta t} \\ &= \sum_l \left[\boldsymbol{\phi}^{g'}_{O,l,t} \boldsymbol{a}_l + \boldsymbol{\phi}^{f'}_{O,l,t+\Delta t} \boldsymbol{b}_l \right] + \varepsilon^O_{t+\Delta t} = \boldsymbol{\phi}^{g'}_{O,t} \boldsymbol{a} + \boldsymbol{\phi}^{f\,'}_{O,t+\Delta t} \boldsymbol{b} + \varepsilon^O_{t+\Delta t}. \end{aligned}$$

■

Note that $\boldsymbol{\phi}^g_{O,t} = \boldsymbol{g}_{l,t} \otimes \boldsymbol{\phi}_{O,l,t}$ and $\boldsymbol{\phi}^f_{O,t+\Delta t} = f_{l,t+\Delta t} \otimes \boldsymbol{\phi}_{O,l,t}$ are the model's explanatory variables. As illustrated by equation (21), $\boldsymbol{\phi}^{g'}_{O,t} \boldsymbol{a}$ corresponds to the sum of conditional exposures multiplied by the factors' conditional risk-neutral expectation, while $\boldsymbol{\phi}^{f'}_{O,t+\Delta t} \boldsymbol{b}$ captures the total impact of the risk factor realizations on the returns on option O, i.e. the sum of the exposures multiplied by the factor realizations.

Using data on risk factors, option characteristics, and the state variables, we first construct $\boldsymbol{g}_{l,t}$, $\boldsymbol{\phi}_{O,l,t}$, and $f_{l,t+\Delta t}$ for all factors l, options O, and dates t. The vectors $\boldsymbol{\phi}^g_{O,t}$ and $\boldsymbol{\phi}^f_{O,t+\Delta t}$ can then be obtained according to Proposition 1. Given $\boldsymbol{\phi}^g_{O,t}$ and $\boldsymbol{\phi}^f_{O,t+\Delta t}$ for all dates and options, equation (21) can then be used to derive the OLS estimators of $\boldsymbol{b}$ and $\boldsymbol{a}$. The following proposition presents this result.

Proposition 2 (First-stage estimator) *The OLS estimator of* $\boldsymbol{B} \equiv \begin{bmatrix} \boldsymbol{a}' & \boldsymbol{b}' \end{bmatrix}'$ *is given by*

$$\widehat{\boldsymbol{B}} \equiv \boldsymbol{Q}^{-1}_{\phi} \frac{1}{T} \sum_{t=1}^{T} \frac{1}{N_t} \sum_{O=1}^{N_t} \boldsymbol{\phi}^{fg}_{O,t+\Delta t} R^O_{t+\Delta t}\,, \tag{23}$$

where

$$\boldsymbol{Q}_{\phi} \equiv \frac{1}{T} \sum_{t=1}^{T} \frac{1}{N_t} \sum_{O=1}^{N_t} \boldsymbol{\phi}^{fg}_{O,t+\Delta t} \left(\boldsymbol{\phi}^{fg}_{O,t+\Delta t} \right)'\,,$$

and $\boldsymbol{\phi}^{fg}_{O,t+\Delta t} = \left[\left(\boldsymbol{\phi}^g_{O,t} \right)' \quad \left(\boldsymbol{\phi}^f_{O,t+\Delta t} \right)' \right]'$*, and where* T *is the number of days in the sample and* N_t *is the number of options on day* t*.*

Proof. The proof follows from standard OLS results applied to the factor representation in Proposition 1. ■

Under conditions stated in Assumption B.1(i) in Internet Appendix B, the estimator $\widehat{\boldsymbol{B}}$ is unbiased and exhibits, for a fixed basis dimension k, the following limiting behavior.

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Proposition 3 (Asymptotic normality of the first-stage estimator) *For fixed basis size k, and under assumption B.1(i) in Internet Appendix B,*

$$\sqrt{T}\left(\widehat{\boldsymbol{B}}-\boldsymbol{B}\right)\sim N(0,\boldsymbol{\Sigma}_{\widehat{\boldsymbol{B}}}),$$

with the true asymptotic covariance matrix of the parameter estimates defined as $\boldsymbol{\Sigma}_{\widehat{\boldsymbol{B}}}\equiv\boldsymbol{Q}_{\phi}^{-1}\boldsymbol{\Sigma}_{\phi\varepsilon}\boldsymbol{Q}_{\phi}^{-1}$ *, where* $\boldsymbol{\Sigma}_{\phi\varepsilon}\equiv Var\left[\frac{1}{\sqrt{T}}\sum_{t}\frac{1}{N_t}\sum_{O}\boldsymbol{\phi}_{O,t+\Delta t}^{fg}\varepsilon_{t+\Delta t}^{O}\right]$ *is the covariance matrix of the OLS score vector.*

Inference about model parameters requires estimating the covariance matrix of the OLS score vector $\boldsymbol{\Sigma}_{\phi\varepsilon}$. Proposition B.1 in Internet Appendix B discusses the appropriate estimator $\widehat{\boldsymbol{\Sigma}}_{\phi\varepsilon}$ which is robust to heteroskedasticity, autocorrelation, cross-correlation and cross-sectional correlation in $\varepsilon_{t+\Delta t}^{O}$.

With the estimated exposures on hand, we now turn to the estimation of the risk premia.

3.2 Estimating Factor Risk Premia

The no-arbitrage condition for traded risk factors implies that their Q-expectation is equal to the risk-free rate. One way to impose this condition is to estimate risk premia for traded and non-traded risk factors separately while expressing traded factors as excess returns (see e.g. Shanken, 1992). In our empirical set-up, the market risk factor is the only traded factor while the remaining factors are typically not tradable.[17] Separating the risk premium estimation of traded and non-traded factors also improves the precision of risk premium estimation when using option returns because the second-stage regressors – the products of the option sensitivities and risk premium predictors – are often highly correlated.

We therefore estimate the market risk premium in a first step by regressing option returns projected on the market return on market risk premium predictors. We then remove the exposures of option returns to the market factor from option returns and regress market-hedged option returns on non-traded factors to obtain the other risk premium parameters.

For ease of notation, we henceforth use the factor subscript $l=M$ for the market risk factor (i.e., $f_{M,t+\Delta t}$ denotes the excess return on the S&P 500 index) and we denote the set of non-market factors (i.e., $l\neq M$) by Ω.

[17] One of the factors is the increment in the square of the VIX index. VIX futures are tradeable, but the factor itself is not. While the Q-expected return on VIX futures thus equals the risk-free rate in the absence of arbitrage, no such restriction can be imposed on the factor and we have to estimate its conditional Q-expectation.

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3.2.1 Estimation of the Market Risk Premium (MRP)

To estimate risk premia, we first need to express deleveraged excess option returns as a linear function of risk premium parameters. We define the projection of the excess option return on the realization of the market risk factor,

$$\widehat{R}^{O,M}_{t+\Delta t} \equiv \widehat{\beta}^{O}_{M,t}\left(f_{M,t+\Delta t} - E^{Q}_{t}\left[f_{M,t+\Delta t}\right]\right), \tag{24}$$

where $E^{Q}_{t}\left[f_{M,t+\Delta t}\right] = 0$ because $f_{M,t+\Delta t}$ is the excess return on the S&P 500 index. Adding and subtracting $\beta^{O}_{M,t}E^{P}_{t}\left[f_{M,t+\Delta t}\right]$ to $\widehat{R}^{O,M}_{t+\Delta t}$ yields

$$\widehat{R}^{O,M}_{t+\Delta t} = \widehat{\beta}^{O}_{M,t}\left(E^{P}_{t}\left[f_{M,t+\Delta t}\right] - E^{Q}_{t}\left[f_{M,t+\Delta t}\right]\right) + \widehat{\beta}^{O}_{M,t}\left(f_{M,t+\Delta t} - E^{P}_{t}\left[f_{M,t+\Delta t}\right]\right) = \widehat{\beta}^{O}_{M,t}\lambda_{1,t} + \eta^{M}_{O,t+\Delta t}, \tag{25}$$

where $\lambda_{M,t} = \boldsymbol{g}'_{M,t}\boldsymbol{\lambda_M}$ is the conditional market risk premium. Note that the error term $\eta^{M}_{O,t+\Delta t} \equiv \beta^{O}_{M,t}\left(f_{M,t+\Delta t} - E^{P}_{t}\left[f_{M,t+\Delta t}\right]\right)$ has zero P-expectation by construction. Given that $\widehat{\beta}^{O}_{M,t}\lambda_{M,t} = \left(\widehat{\beta}^{O}_{M,t}\boldsymbol{g}_{M,t}\right)'\boldsymbol{\lambda}_{M}$, we obtain the following regression equation, which is linear in the market premium parameters $\boldsymbol{\lambda}_{M}$,

$$\widehat{R}^{O,M}_{t+\Delta t} = \left(\widehat{\beta}^{O}_{M,t}\boldsymbol{g}_{M,t}\right)'\boldsymbol{\lambda}_{M} + \eta^{M}_{O,t+\Delta t} = \widehat{\boldsymbol{d}}^{M'}_{O,t}\boldsymbol{\lambda}_{M} + \eta^{M}_{O,t+\Delta t}. \tag{26}$$

In equation (26), $\widehat{\boldsymbol{d}}^{M'}_{O,t} \equiv \widehat{\beta}^{O}_{M,t}\boldsymbol{g}'_{M,t}$ is a vector of dimension $s \times 1$ (where s is the number of MRP predictors) which contains the product of the estimated option loadings on the market excess return and the market risk premium predictors. It is worth noting that $\eta^{M}_{O,t+\Delta t}$ is orthogonal to $\varepsilon^{O}_{t+\Delta t}$, the regression residual from the estimation of the exposures in equation (21) and to $\widehat{\boldsymbol{d}}^{M}_{O,t}$, which is important for the purpose of inference.

In principle, the regression equation (26) can be estimated by OLS. However, as pointed out by Shanken (1992), this leads to inference problems, because the measurement error in $\widehat{\beta}^{O}_{M,t}$ impacts both $\widehat{R}^{O,M}_{t+\Delta t}$ and $\widehat{\boldsymbol{d}}^{M}_{O,t}$, i.e. it is present on both sides of equation (26). While the presence of measurement error on the right-hand side of equation (26) only requires corrections to the asymptotic variance of the OLS estimator, its presence on the left-hand side results in asymptotic bias in the standard OLS estimator.[18] With this in mind, the following proposition characterizes the asymptotic properties of the bias-corrected OLS estimator of the vector of market risk premium parameters.

[18] Similar to Shanken (1992), we do not explicitly account for the errors-in-variables bias by assuming that the matrix $\boldsymbol{Q}_{\widehat{\boldsymbol{d}}^{M}}$ defined in Proposition 4 converges to its probability limit.

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Proposition 4 (Limiting behavior of the bias-corrected estimator of the MRP $\boldsymbol{\lambda}_M$) *Under Assumptions B.2(iii)-B.2(iv) in Internet Appendix B, with the unbiased estimator of the market premium parameters $\widehat{\boldsymbol{\lambda}}_M^{\Psi}$ given in Proposition B.4 in Internet Appendix B, $\boldsymbol{Q}_{\widehat{\boldsymbol{d}}^M} \equiv \frac{1}{T}\sum_t \frac{1}{N_t}\sum_O \widehat{\boldsymbol{d}}_{O,t}^M \left(\widehat{\boldsymbol{d}}_{O,t}^M\right)'$, the unbiased estimator of the market premium parameters has the asymptotic distribution*

$$\sqrt{T}\left(\widehat{\boldsymbol{\lambda}}_M^{\Psi} - \boldsymbol{\lambda}_M\right) \sim N(0, \boldsymbol{\Sigma}_{\widehat{\boldsymbol{\lambda}}_M^{\Psi}}),$$

with $\boldsymbol{\Sigma}_{\widehat{\boldsymbol{\lambda}}_M^{\Psi}}$ the covariance matrix of the OLS score vector of regression (26), which accounts for the presence of measurement error in the regression, and is discussed in Proposition B.5 in Internet Appendix B. The estimator of $\boldsymbol{\Sigma}_{\widehat{\boldsymbol{\lambda}}_M^{\Psi}}$ is provided in Proposition B.6 in Internet Appendix B.

3.2.2 Estimation of the Risk Premia for Non-Market Factors

We now outline our approach to estimating the risk premium parameters associated with the remaining factors, which closely follows the estimation strategy defined for the market risk premium. First, define the regressand used to estimate the remaining risk premia which corresponds to the projection of option returns on non-market risk factors and their Q-expectations. We have

$$\widehat{R}_{t+\Delta t}^{O,\Omega} \equiv \sum_{l\in\Omega} -\widehat{\beta}_{l,t}^O E_t^Q\left[f_{l,t+\Delta t}\right] + \sum_{l\in\Omega} \widehat{\beta}_{l,t}^O f_{l,t+\Delta t}\,. \tag{27}$$

Adding and subtracting $\sum_{l\in\Omega}\widehat{\beta}_{l,t}^O E_t^P\left[f_{l,t+\Delta t}\right]$ in (27), we can express the non-market component of the option return as a linear combination of risk premium parameters

$$\begin{aligned}\widehat{R}_{t+\Delta t}^{O,\Omega} &= \sum_{l\in\Omega}\widehat{\beta}_{l,t}^O\left(E_t^P\left[f_{l,t+\Delta t}\right] - E_t^Q\left[f_{l,t+\Delta t}\right]\right) + \eta_{O,t+\Delta t}^{\Omega} \\ &= \sum_{l\in\Omega}\widehat{\beta}_{l,t}^O\left(\boldsymbol{g}_{l,t}'\boldsymbol{\lambda}_l\right) + \eta_{O,t+\Delta t}^{\Omega} = \widehat{\boldsymbol{d}}_{O,t}^{\Omega\prime}\boldsymbol{\lambda}_{\Omega} + \eta_{O,t+\Delta t}^{\Omega}\end{aligned} \tag{28}$$

where $\eta_{O,t+\Delta t}^{\Omega} \equiv \sum_{l\in\Omega}\left(f_{l,t+\Delta t} - E_t^P\left[f_{l,t+\Delta t}\right]\right)\beta_{l,t}^O$ with $E_t^P\left[\eta_{O,t+\Delta t}^{\Omega}\right] = 0$. Note that we use the definition of the factor risk premium (7) to obtain the second equality, and we define $\widehat{\boldsymbol{d}}_{O,t}^{\Omega} \equiv [\ \widehat{\beta}_{2,t}^O\boldsymbol{g}_{2,t}'\cdots\widehat{\beta}_{L,t}^O\boldsymbol{g}_{L,t}'\ ]'$ and $\boldsymbol{\lambda}_{\Omega} \equiv [\boldsymbol{\lambda}_2'\cdots\boldsymbol{\lambda}_L']'$ to obtain the third equality.[19] In (28),

[19]Note that to define $\widehat{\boldsymbol{d}}_{O,t}^{\Omega}$ and $\boldsymbol{\lambda}_{\Omega}$, we implicitly assume that non-market factors are ordered from 2 to L.

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$\widehat{\boldsymbol{d}}_{O,t}^{\Omega}$ corresponds to the $(L-1)\,s \times 1$ vector of non-market exposures times non-market return risk premium state variables, and $\boldsymbol{\lambda}_{\Omega}$ is the vector of risk premium parameters.

Note that the return in equation (28) does not have an intercept because it is derived under the assumption of absence of arbitrage opportunities, i.e. it assumes the model prices all assets perfectly. Similarly to testing the CAPM for example, allowing for an intercept could provide insights about model misspecification. This can be done by augmenting $\widehat{\boldsymbol{d}}_{O,t}^{\Omega}$ with a vector of ones and by adding the respective intercept parameter to $\boldsymbol{\lambda}_{\Omega}$. The intercept then absorbs the component of expected option returns that is not explained by the factors.

Similarly to the case of the market risk premium parameter, the standard OLS estimator of $\boldsymbol{\lambda}_{\Omega}$ is biased due to measurement error in exposures, which propagates to both sides of regression (28). Results for obtaining the bias-corrected estimator of the risk premia for non-market factors, $\widehat{\boldsymbol{\lambda}}_{\Omega}^{\Psi}$, and for statistical inference are analogous to Proposition 4 and are collected in Section B.2.2 of Internet Appendix B.

4 Data

Our empirical analysis relies on five publicly available datasets.

The first factor is the market return in excess of the risk-free rate. We use the S&P 500 index (SPX), which we obtain from OptionMetrics, to proxy for the market portfolio. Data on risk-free rates are also obtained from OptionMetrics. We omit dividend payments from the excess returns because the options are written on the ex-dividend index value. We use daily deleveraged excess index option returns as defined in equations (1) and (9), computed from end-of-day SPX option mid-quotes. We retain options with maturities ranging from one to six months. We further restrict the sample to put options with moneyness (K_O/S_t) between 0.80 and 1.025 and call options with moneyness between 0.975 and 1.15. This allows us to span almost the entire moneyness spectrum without relying on in-the-money (ITM) options, which are less liquid. From the resulting sample, we filter out options with zero bid prices and ask prices five times larger than the bid price.

We use the square of the VIX index, which we obtain from the CBOE website, to proxy for the index variance.[20] The systematic variance factor Δv_t is thus proxied by the daily increment in the square of the VIX index. Henceforth we denote this factor by VAR. One of the models we analyze in the robustness section uses a left-tail jump factor, proxied by the daily increment in the tail variance TVAR, the square of the Tail Volatility index of

[20] See https://www.cboe.com/tradable_products/vix/vix_historical_data/

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Bollerslev et al. (2015). We obtain this variable from the Tail Index website of Andersen and Todorov.[21] We denote this factor by LTJ.

Another model we analyze in the robustness section includes the intermediary factor of He et al. (2017) and the intermediary capital ratio, which we obtain from Manela's website.[22] Other factors and state variables we use in the empirical analysis are simple functions of the data discussed above. For instance, the gamma factor is simply the squared market return. We use the level and the square of VIX^2 and the level of the tail variance TVAR to predict the risk premia.

We report results for puts and calls by moneyness and maturity buckets throughout our empirical analysis. For calls, we use the K_O/S_t cutoffs of (0.975;1.02], (1.02;1.07] and (1.07;1.15] for moneyness buckets of ATM, OTM, and deep out of the money (DOTM) options respectively. For ATM puts, we use (0.975;1.02], while for OTM and DOTM puts we use (0.9;0.975] and (0.8;0.9] respectively. We use a wider range of moneyness for SPX puts because of the higher liquidity of deep OTM puts relative to calls. For each option type and moneyness bucket, we report results for options with maturity of 1–2, 2–3, and 3–6 months. This provides insights into the term structure of expected option returns and option sensitivities.

The left sub-panels of Panels A and B of of Table 1 present descriptive statistics for puts (Panel A) and calls (Panel B), respectively. The average return, standard deviation, and Sharpe ratio are annualized assuming 252 trading days in a year. On average, deleveraged call returns are positive across maturity and moneyness buckets, while put returns are negative. Average put returns are substantially larger in absolute value than call returns. This is due to calls' positive exposure to two factors with risk premia of opposite sign. They are positively exposed to market return and variance risks. Puts load negatively on the market return premium, which is positive, and positively on the variance risk premium, which is negative. The market return and variance risk premia thus do not cancel out for puts.

To provide more intuition, the right sub-panels of Panels A and B of Table 1 report on delta-hedged deleveraged option returns, because the properties of delta-hedged returns are better known. We delta-hedge one long position in each deleveraged option using the estimated deltas on a daily basis. Unlike deleveraged returns, both put and call delta-hedged excess returns are negative on average. This finding is consistent with the existing literature, see for instance Bakshi, Kapadia, and Madan (2003).[23] Table 1 also indicates

[21] See https://tailindex.com/volatilitymethodology.html

[22] See http://apps.olin.wustl.edu/faculty/manela/

[23] This finding is supported by a related literature on negative variance risk premia, see e.g. Coval and

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that both unhedged and delta-hedged deleveraged put returns have a higher Sharpe ratio (in absolute value), are less skewed, and have lower kurtosis compared to call returns. The magnitude of the Sharpe ratio of both calls and puts increases (in absolute value) and their standard deviation decreases after delta-hedging. The increase in the Sharpe ratio is much greater for put options than for call options.

Table 1 also suggests several patterns in returns across maturity and moneyness buckets. First, put options with short maturities (1–2 months) earn more negative returns (before and after delta-hedging) than put options with longer maturities. For call options, we only observe this pattern in delta-hedged returns, and the pattern is weaker compared to puts. Second, like average returns, return volatility differs across option type, moneyness and maturity. Third, the Sharpe ratios of delta-hedged short-maturity options are significantly lower than those of options with longer maturities, in all moneyness buckets except for DOTM calls. Finally, the returns on OTM options are smaller in magnitude than returns on ATM options. This due to the deleveraging of returns.

[Table 1 about here.]

5 Empirical Results

Our main objectives are twofold. First, we aim to quantify option return exposures to various systematic risk factors. Second, taking these estimated exposures as given, we aim to estimate the factors' conditional risk premiums.

We first discuss our baseline model specification. We then present results on model fit and unconditional exposures and discuss the decomposition of index option returns according to risk factors. We discuss the cross-section of the option exposures to the factors and assess the contribution of each factor to index option expected returns. We also investigate the implications for the model's hedging performance. Finally, we present our estimates of factor risk premiums and discuss the model's out-of-sample performance.

5.1 The Baseline Model Specification

Our baseline model features three risk factors that are deeply rooted in option pricing theory. We denote the first ($l = 1$) factor by MKT. It corresponds to the market return risk factor

Shumway (2001), Buraschi and Jackwerth (2001) and Carr and Wu (2009) .

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and is measured using daily SPX returns in excess of the risk-free rate and net of dividends. The second ($l = 2$) factor captures aggregate variance risk and is denoted by VAR. It is measured using daily changes in SPX conditional variance, proxied by changes in the square of the VIX index. The third ($l = 3$) factor, GAM, is the gamma risk factor, which is proxied by daily squared SPX returns. We show below that the GAM factor accounts for a relatively small percentage of the variation in option returns, but omitting it from the model leads to omitted variable bias and its inclusion improves the model's performance in predicting option returns. GAM controls for the fact that the return on an option is a nonlinear function of the return on the market factor.[24]

Given the choice of model factors, we need to specify the P- and Q-conditional expectations of each factor l by choosing the vector of conditional expectation parameters $\boldsymbol{\mu}_l^m$ in equation (6) for $m = P, Q$ along with the vector of risk premium predictors $\boldsymbol{g}_{l,t}$. Recall that $\boldsymbol{\mu}_l^m$ for $m = P, Q$ combined with $\boldsymbol{g}_{l,t}$ define $E_t^m\left[f_{l,t+\Delta t}\right] = \boldsymbol{\mu}_l^{m'}\boldsymbol{g}_{l,t}$ and therefore the dynamics of the conditional risk premium, $E_t^P\left[f_{l,t+\Delta t}\right] - E_t^Q\left[f_{l,t+\Delta t}\right]$.

Even though the model allows for the presence of nonlinear risk premia, our baseline analysis focuses on a specification with a linear risk premium.[25] For the MKT factor, we consider the vector of physical expectation parameters $\boldsymbol{\mu}_1^P = [\ \mu_{0,1}^P\ \mu_{1,1}^P\ \mu_{2,1}^P\ ]'$ and set $\boldsymbol{g}_{1,t}$ equal to $[1 \quad \text{VIX}_t^2 \quad \text{TVAR}_t]$, where TVAR_t is the square of the level of tail volatility from Bollerslev et al. (2015). Note that the no-arbitrage condition requires $E_t^Q\left[\text{MKT}\right] = 0$, as the market return is expressed in excess of the risk-free rate. This choice of state variables for the market risk premium builds on the existing literature. A large class of equilibrium models predicts that the expected return on the market is related to the level of market variance (see, e.g., Merton, 1980, and extensions thereof). Furthermore, the literature on nonparametric equity premium bounds suggests that the market risk premium and the VIX_t^2 should be closely related (see Martin, 2017). Finally, the disaster risk/jump literature argues that the evolution of the expected magnitude and intensity of jumps impacts risk premia (see e.g., Gabaix, 2012, Wachter, 2013, Barro and Liao, 2021), which we account for in the model by including TVAR_t.

[24] It is important to distinguish between the GAM and VAR factors. While VAR is the change in the level of aggregate variance risk, GAM is the squared SPX return, so that $E_t[\text{GAM}] = V_t[\text{MKT}] + (E_t[\text{MKT}])^2$, where $V_t[\cdot]$ is the conditional variance operator. The realization of GAM should thus on average be close to the level of aggregate variance at time t. In our sample, the correlation of GAM and our proxy for aggregate variance, VIX_t^2, is 51.2%, while the correlation between GAM and VAR is close to zero. The correlation between GAM and MKT is also close to zero.

[25] For discussions of nonlinear pricing kernels and risk premia see, among others, Dittmar (2002), Jones (2003), Chabi-Yo (2011), Christoffersen, Heston, and Jacobs (2013), and Schneider and Trojani (2019).

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For VAR, we set $\boldsymbol{\mu}_2^m = [\ \mu_{0,2}^m\ \mu_{1,2}^m\ ]'$ for $m = P, Q$ and $\boldsymbol{g}_{\text{VAR},t} = [1 \quad \text{VIX}_t^2]$. For GAM, for simplicity we adopt a constant risk premium specification and set $\boldsymbol{\mu}_3^m = \mu_{0,3}^m$ and $\boldsymbol{g}_{3,t} = 1$. As discussed above, the GAM factor does not have an economically large impact on expected option returns, and time-varying risk specifications do not change the results.

Finally, we need to specify option exposures. Recall from Section 2.2 that the exposure of option O to a given risk factor corresponds to the TPRS-basis expansion of the vector $\boldsymbol{z}_{O,l,t} = \left[\boldsymbol{c}_{O,t}'\ \boldsymbol{i}_{l,t}'\right]'$, where $\boldsymbol{c}_{O,t} \equiv [\tau_{O,t}\ k_{O,t}]'$ is the vector consisting of option O's moneyness and time to maturity, and $\boldsymbol{i}_{l,t}$ is the vector of instruments used to model option exposures to factor l. We incorporate the intuition underlying standard reduced-form no-arbitrage models (see e.g., Heston, 1993), which imply that option exposures are functions of the conditional variance, and specify $i_{l,t} = \text{VIX}_t^2$ for all factors. Furthermore, we impose the no-arbitrage conditions on the exposures and impose put-call parity on the exposures of puts and calls with the same strike price and maturity.[26] Finally, we set the TPRS basis dimension k in equation (13) equal to twenty for all factors. Henceforth, consistent with the literature, we refer to market return risk exposures as deltas, to market variance exposures as vegas, and to gamma risk exposures as gammas.

We estimate our baseline model on individual (i.e., not portfolios of) SPX option delevered returns, following the estimation methodology outlined in Section 3. We apply the results in Propositions 1 and 2 to estimate option exposures and use the results in Proposition 3 for first-stage inference. We use the results in Proposition B.4 in Internet Appendix B to obtain unbiased market risk premium parameters and conduct inference on these estimates using Proposition 4. Finally, we apply the results of Section B.2.2 in Internet Appendix B for unbiased estimation of and inference on the remaining factors' unbiased risk premium parameters.

5.2 Unconditional Factor Exposures

The existing literature on the relative contribution of economic factors to option returns is still limited. Option valuation models are usually estimated using prices, and the return implications of these models have received less attention. Moreover, parametric models are usually silent about the exposures of options to factors other than return, volatility and jump risks.

[26] For instance, the mapping between exposures of puts and calls with the same characteristics to the MKT factor is given by put-call parity. Similarly, the exposures of puts and calls with the same characteristics to the remaining factors are also constrained to be equal.

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[Table 2 about here.]

[Figure 1 about here.]

We start by inspecting the unconditional exposures and factor decomposition of option returns through the lens of the model. Table 2 presents the results from the first stage estimation. In Panel A we report the sample average (unconditional) exposures of options by moneyness and maturity buckets. To obtain the unconditional exposures, we first average the daily exposures of all options in a given bucket to obtain one observation on each day and then average the daily exposures of each bucket over the full sample. The results in Table 2 indicate that the unconditional exposures have the sign predicted by no-arbitrage option pricing models and are consistent with economic intuition, for both calls and puts. The deltas, vegas, and gammas of calls decrease with increasing moneyness (K_O/S_t) for all maturities. For puts, the deltas increase towards zero as moneyness decreases, and the vegas and gammas decrease. Averages of ATM call deltas range from 0.41 to 0.47 across maturity buckets while averages of ATM put deltas range from -0.43 to -0.40.

Figure 1 complements Table 2 by plotting the exposures to each factor, computed at the sample median values of the state variables. We compute the cross-section of exposures for calls and puts with maturities of 1, 3, and 6 months by varying moneyness over the respective ranges (i.e., $K_O/S_t \in [0.8, 1.0]$ for puts and $K_O/S_t \in [1.0, 1.15]$ for calls). Model-implied exposures display complex patterns across moneyness depending on the factor, the type of option (i.e., puts or calls), and maturity. The delta of ATM calls (top panels) is around 0.5 and decreases monotonically as moneyness increases. The delta of DOTM puts is close to zero and decreases to -0.5 as K_O/S_t increases. Model deltas thus behave similarly to deltas implied by standard reduced-form models.

In contrast, the relation between model exposures and market variance risk (VAR) as a function of maturity (middle panels) is more complex. For ATM puts, vega decreases with maturity. This pattern contrasts with the predictions of standard affine models such as Black and Scholes (1973) and Heston (1993), which predict that vega increases monotonically with maturity for ATM options. Model prediction for the vega of ATM calls is broadly consistent with standard affine models. Finally, the term structure of gammas for ATM calls and puts is downward sloping when $K_O/S_t = 1$. This is also consistent with standard option pricing models.

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5.3 A Variance Decomposition of Option Returns

Panel B of Table 2 presents the R^2 contribution of each factor to the total variance of option returns. To compute the contribution of each factor to the R^2, we first calculate the average of daily exposures and the average return based on all options in a given bucket B on a given day t:

$$\widehat{\beta}^B_{l,t} = \frac{1}{N_{B,t}} \sum_{O \in B} \widehat{\beta}^O_{l,t}, \qquad R^B_{t+\Delta t} = \frac{1}{N_{B,t}} \sum_{O \in B} R^O_{t+\Delta t}\,. \tag{29}$$

We then multiply the average exposure by the daily factor realizations to get the total impact of a given factor on option returns for each bucket on each day. R^2 is then defined as

$$R^2 = 1 - \frac{\frac{1}{T}\sum_t \left(R^B_{t+\Delta t} - \sum_l \widehat{\beta}^B_{l,t} f_{l,t+\Delta t}\right)^2}{\frac{1}{T}\sum_t \left(R^B_{t+\Delta t}\right)^2} \tag{30}$$

The rightmost column of the Put and Call option sub-panels reports the total R^2 for each option bucket. The fit from the baseline model is impressive. The R^2 ranges between 84% and 95% across buckets for calls, and between 91% to 96% for puts. Long-maturity DOTM calls give the lowest R^2.

We also compute the Shapley-Owen values of R^2 for each bucket. The Shapley-Owen value of R^2 conveniently isolates the contribution of each factor in explaining deleveraged option returns, while accounting for potential co-movements between factors.[27]

The R^2 contributions of market return and variance risk far exceed that of gamma risk for both calls and puts. Two factors (MKT and VAR) explain more than 85% of the variation in index option returns for calls and puts, and across maturity and moneyness buckets. Overall, market risk is more important than variance risk, but the relative contribution of market risk is larger for calls than for puts. For DOTM puts, the contributions of the two factors are relatively close. The contribution of market risk to option return R^2 ranges from 60% to 85% for calls, and from 53% to 83% for puts. For variance risk, the contribution ranges from 5% to 23% for calls and from 12% to 37% for puts. The contribution of gamma risk to option return R^2 is small, and ranges from 1% to 3% for puts and from 0% to 1% for calls.

[27] The Shapley-Owen value R^2 is defined as the average marginal contribution of a variable towards R^2 across all possible model specifications and orders of variable inclusion. This measure does not depend on the order in which the regressors are included in the model, and the total contribution of the regressors in the full sample is equal to the R^2 of the complete regression model. For more information see e.g. Huettner and Sunder (2012).

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5.4 Conditional Exposures and Hedging Performance

Banks, pension funds, and hedge funds actively trade index options and need to quantify the conditional exposures of their positions to relevant risks for the purpose of asset allocation and risk management. We therefore assess the hedging performance of the model based on conditional factor exposures. We take the view of an investor who wants to hedge as much risk as possible.

Panel A of Table 3 presents the hedging performance of the model relative to the Heston (1993) model.[28] Each day, we hedge one long position in each deleveraged option by subtracting the products of the conditional exposures and the factor realizations from deleveraged option returns. Conditional exposures are computed based on the baseline model specification or the Heston model. For each maturity/moneyness bucket and each day, we first calculate the average hedged return for all options in that bucket, denoted $R^B_{t+\Delta t}$ and defined in equation (29). Using the time series of daily average hedged returns for each bucket, we then compute the R^2 coefficient for each bucket.

The model consistently exhibits an advantage over the Heston model across option type, maturity and moneyness. Overall, the model manages to hedge away 94.7% of risk in deleveraged option returns, whereas the Heston model removes 87.6% of the risk. More specifically, the improvements provided by the model are (a) consistently higher for call options than for put options; (b) increasing as the options become further OTM; and (c) decreasing with option maturity. Thus, the performance gap between our model and the Heston model is highest for DOTM, short-maturity call options (where our model's R^2 is 32.0 percentage points higher), and lowest for long-maturity ATM put and call options (where our model's R^2 is 5.1 percentage points higher).

Overall, we conclude that the economic magnitude of the gains from hedging with the new model instead of the Heston (1993) model are considerable.

[Table 3 about here.]

5.5 Factor Contributions to Expected Returns

To provide additional insights into expected option returns, Panel A of Table 4 presents results on a factor decomposition of expected deleveraged option returns. In columns 1-

[28] The Heston model is estimated using particle filtering on the same sample; the increment in the filtered variance replaces the VAR factor in the hedging exercise and is filtered from option prices in order to ensure the best possible model performance.

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3 of each sub-panel, we report the contribution of each factor to expected put and call option returns. Column 4 presents the model prediction for total expected returns, which corresponds to the sum of columns 1 through 3. Consistent with the results on factor exposures in Table 2, market return risk is the main determinant of expected option returns for both calls and puts, followed by market variance risk, while gamma risk plays a relatively minor role.[29]

To further assess the model's ability to adequately capture option return dynamics (conditional expected option returns), we next regress daily deleveraged option returns against model predictions using the full-sample time series of daily option returns. Panel B of Table 4 reports the results, which provide strong evidence that the model correctly captures the conditional dynamics of option returns.[30] Impressively, when considering all put options (all maturities and moneyness levels together), the point estimates of the intercept and slope of the regression are 0.05 and 1.00, respectively. For put options, the slope coefficients are also all positive and close to one across buckets. The results for calls are slightly worse, but the intercepts and slopes are not statistically significantly different from zero and one respectively. The model struggles for ATM and OTM (but not DOTM) calls with medium to long maturities, but none of the estimated loadings are statistically different from one at standard significance levels. These findings further support the model's ability to reliably extract the information embedded in option return dynamics.

[Table 4 about here.]

5.6 Implied Factor Risk Premia

Index options are highly informative about the dynamics of risk premia because they embody market expectations for different states of nature. However, the existing literature has mainly focused on the study of variance and jump risk premia implied by index options through the lens of no-arbitrage affine pricing models. Our approach allows us to back out risk premium dynamics associated with all relevant factors. We may also be able to improve on the estimation of the conditional premium for market return risk compared to existing

[29]Note that the sum of the factor contributions to option returns in column 4 of Table 4 are slightly different from the average option returns reported in Table 1. In Table 1 each option has the same weight in estimation. In Table 4, returns are first averaged within each option type, moneyness and maturity group on each day and subsequently averaged over time.

[30]The expected returns in Panel B of Table 4 include an intercept term which is identical for all options, and thus not reported in Panel A. The inclusion of this intercept term in the estimation of risk premia is discussed in Section 3.2.

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approaches, because we use option returns and exploit the tight no-arbitrage relationships between option and index returns.

Table 5 illustrates the model's implications for factor risk premia embedded in index option returns. Panel A reports the risk premium parameters and corresponding p-values, as well as the F-statistics for the significance of the premia on each factor. Panel B presents various summary statistics on risk premia, including the (annualized) average, median, standard deviation, skewness, and kurtosis of the time series of daily conditional risk premium estimates for each factor. Because the risk premium on the GAM factor is constant in our baseline model, we do not report higher-order statistics for this factor.

[Table 5 about here.]

The parameter estimates in Panel A of Table 5 are economically plausible. The market risk premium loads positively on both state variables.[31] This is consistent with the idea that as the state of the economy deteriorates and systematic volatility and tail risks increase, the expected market return increases. The loading of the VAR risk premium on VIX^2 is negative, indicating that the conditional variance risk premium decreases and becomes more negative as volatility increases. This is consistent with a wealth of empirical evidence and it is a model feature in many reduced-form option pricing models as well as more structural models.

Panel B of Table 5 characterizes the distribution of daily conditional risk premia. The average market risk premium implied by the model is 9.1% and the variance risk premium is -22.1%. The risk premium associated with gamma risk is negligible. Impressively, the annualized market and variance risk premium have the anticipated sign and their magnitudes are reasonable, despite the fact that the sample includes both the collapse of the dot-com bubble and the financial crisis. Daily risk premium estimates are highly leptokurtic.

Note that the model-implied variance risk premium captures the impact of changes in the level of aggregate variance on deleveraged option returns and one needs to be careful when comparing it with the existing literature. It differs from the often-used definition of the variance risk premium as the difference between the P- and Q-expectations of the variance. For example, in the no-arbitrage reduced-form Heston model discussed in Section 2.3, the counterpart of our definition of the variance risk premium is given by

[31] The market risk premium's F-statistic is not statistically significant. We therefore also estimated a specification with a constant market risk premium. This specification is statistically significant, but it strongly hampers the model's ability to fit the dynamics of option returns. We report on this specification in Section 6 below.

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$\frac{1}{dt}\left(E^P[dv_t] - E^P[E_t^Q[dv_t]]\right) = -\kappa^Q(\theta^Q - \theta^P)$, whereas the standard definition is $E^P[v_t] - E^Q[v_t] = \theta^P - \theta^Q$. Estimates in the existing literature, such as Christoffersen, Fournier, and Jacobs (2018a) for example, yield a risk premium of approximately -2.5% per year, much smaller than our estimates in Table 5. One potential explanation for this difference is misspecification in these affine models. A more closely related estimate is provided by Cheng (2018), who reports an average VIX return premium of about -2.51% per month using VIX futures data, which translates into a -30% annual premium, similar to our estimate.

[Figure 2 about here.]

Figure 2 complements the results in Table 5 by plotting the time-series of annualized conditional risk premia, computed by multiplying the daily estimates by 252. We split the sample in three sub-periods and use different scaling on the Y-axes because of the very high premia during the financial crisis. The signs of the estimated market and variance risk premiums are not only plausible on average, they have the expected sign on each day. This is particularly impressive because our estimation uses potentially noisy returns on individual option contracts in estimation, rather than portfolio returns. Figure 2 indicates that the risk premia are substantially higher during the financial crisis period and the sovereign debt crisis.

We conclude that the cross-section of index option returns contains valuable information about market return risk premium levels and dynamics.

5.7 Out-of-Sample Performance

We examine two dimensions of the model's out-of-sample (OOS) performance: its hedging ability, i.e. the accuracy of its OOS prediction of option sensitivities to systematic risk factors; and its predictive ability, i.e. the joint accuracy of its predicted option sensitivities and risk premia. We estimate the model recursively on an expanding sample, starting with the 1996-2005 sample period. After each estimation, we obtain predictions of option sensitivities and risk premia based on observed option characteristics and risk premia predictors for a period of one year, i.e. using the 1996–2005 estimates, we obtain predictions for 2006. We subsequently expand the estimation sample by one year and repeat the exercise; the final out-of-sample period is January–June 2019.

Panel B of Table 3 presents the model's OOS hedging performance relative to the Heston model, which was also estimated recursively using the same implementation. Hedging

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performance is evaluated as in Section 5.4. Overall, our model continues to outperform the Heston model in OOS hedging, but the differences are smaller compared to the in-sample results in Panel A. Similar to the in-sample results, the model's advantage is larger for call options and for shorter-maturity options. However, in contrast to the in-sample results, the model underperforms the Heston model when hedging DOTM options, both calls and puts.[32]

Panel C of Table 3 presents the results for in- and out-of-sample delta-hedged and deleveraged option return prediction. We focus on monthly returns to facilitate comparisons with results in the existing literature. We calculate monthly delta-hedged and deleveraged option returns as

$$DR^{O}_{t+1M} = \frac{O_{t+1M} - O_t - \widehat{\beta}^{O}_{1,t}\left(S_{t+1M} - S_t\right)}{S_t}\,, \tag{31}$$

where $\widehat{\beta}^{O}_{l,t}$ is calculated based on observed option characteristics and relevant instruments at time t and estimated model coefficients. The holding period is from the first day of each month to the first day of the following month.

Expected delta-hedged deleveraged returns are calculated as

$$E_t\left[DR^{O}_{t+1M}\right] = N_{1M} \times \sum_{l=2}^{3} \widehat{\beta}^{O}_{l,t}\widehat{\lambda}_{l,t}\,, \tag{32}$$

where $\widehat{\lambda}^{O}_{1,t}$ is computed based on risk premia predictors observed at time t and estimated model coefficients, and N_{1M} is the number of business days in the holding period (i.e, holding the position from Friday to Monday counts as one day). We then aggregate the individual option returns into the moneyness- and maturity-sorted buckets (denoted DR^{B}_{t+1M}, as in equation (29)). As is customary in the literature, we measure the model's predictive performance with the predictive R^2, defined as

$$R^2_P = 1 - \frac{\frac{1}{T}\sum_t \left(DR^{B}_{t+\Delta t} - E_t\left[DR^{B}_{t+1M}\right]\right)^2}{\frac{1}{T}\sum_t \left(DR^{B}_{t+\Delta t}\right)^2}\,. \tag{33}$$

We denote the in-sample (out-of-sample) predictive R^2 with an I (O) subscript. The in-sample results indicate that the degree of option return predictability is high (an overall $R^2_{I,P}$ of 15.6%) but it varies across option type and moneyness, and somewhat across maturities. Put option returns are much more predictable than call option returns (the $R^{2,P}_{I}$ is 27.3%

[32]While we can incorporate sign constraints on the estimated factor sensitivities, we do not impose them because it facilitates the asymptotics. Such constraints are naturally embedded in the Heston model.

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for puts and 5.6% for calls). The level of put option return predictability is similar across all moneyness and maturity levels, except for DOTM, long-maturity options. Call options exhibit high predictability for ATM and OTM options (without a clear pattern in moneyness or maturity), and large prediction errors for deep OTM options. Predictive power remains remarkably high OOS, with an average $R^2_{O,P}$ of 7.6%, approximately double the average OOS predictive power reported in Büchner and Kelly (2022). The model accurately predicts the most actively traded options, such as short-maturity OTM and DOTM puts. It delivers high-quality predictions for for ATM call options and for all puts except for those that are DOTM and long maturity.

6 Additional Results and Robustness Analysis

Tables 6 through 7 report several additional results and robustness exercises. We first report on additional factors. Our baseline model includes three factors: the market return MKT, the market variance innovation VAR, and the squared market return GAM. Here we also report on two additional factors. Subsequently we investigate the robustness of our results to changes in the specification of the risk premia. The third robustness exercise reports on different sample periods, the fourth on option buckets based on adjusted moneyness, the fifth on the choice of basis dimension, and the sixth on the choice of proxy for the variance factor.

6.1 Alternative Factors

Prior intuition and the empirical results for the baseline model suggest that the MKT and VAR factors ought to be included in any model of expected option returns. The third factor, the squared market return (GAM), does not account for a large share of expected option returns and it has little impact on the estimation of risk premia for the other two factors. However, its inclusion improves the model's explanatory power for option returns. To demonstrate this, column (2) in Panel A of Table 6 reports results for the two-factor model with only MKT and VAR as factors. The sample period and other aspects of model specification are identical to those of the baseline model. A comparison with the results in Panel B of Table 5, which are repeated in column (1) of Table 6, shows that the results are very similar. The risk premium on the MKT factor is slightly larger (11.2% vs. 9.1%), while the risk premium on the VAR factor is slightly smaller in absolute value (−20.0% vs.

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−22.1%). However, Internet Appendix A shows that the three-factor specification is much more accurate when predicting conditional expected returns.

[Table 6 about here.]

Next, we report on additional factors. We focus on two interesting factors studied in the recent literature: the HKM factor from He et al. (2017) and the downside LTJ factor. The HKM factor captures the importance of intermediaries in asset markets in general and derivatives markets in particular, see for instance Gârleanu et al. (2009), He and Krishnamurthy (2013), He et al. (2017). The LTJ factor builds on an extensive literature showing that volatility factors are not sufficient to characterize the cross-section of option prices, and that jumps and tail risk explain a substantial amount of the variation in option prices. For HKM we follow equation (12) in He et al. (2017) and specify the premium as an affine function of the inverse capital ratio of financial intermediaries. For LTJ we mimic our specification for other factors and specify the premium as an affine function of the level of the tail variance.

Columns (3) and (4) of Panel A of Table 6 report on the risk premia associated with these two factors. In both cases we include HKM and LTJ as the fourth factor in a model with MKT, VAR, and GAM as the first three factors. Both LTJ and HKM have the expected sign on average, negative and positive in the case of LTJ and HKM respectively. The average estimate of the HKM risk premium is 32.8%, similar to the estimate in He et al. (2017). Our estimate of the premium on LTJ is −7.1%. We are not aware of existing studies that provide benchmark estimates of risk premiums for this factor.

Figure 3 plots the variation in LTJ (HKM) over time for the 1996–2019 and 2000–2019 sample periods. As in Figure 2, we break the sample in three sub-periods and use different scaling on the Y-axes because of the very high premia during the financial crisis. For both factors, the absolute value of the risk premium is by far the largest in the financial crisis, but the time-series behavior of the risk premia is very different outside of the 2008–2009 period. The time series of the HKM risk premium is much more persistent and it has relatively few outliers. Not surprisingly, the risk premium associated with the LTJ factor is transitory and contains many large outliers.

[Figure 3 about here.]

Table IA.3 in the Internet Appendix A reports on option loadings, the Shapley-Owen R^2 values and the contribution of the HKM and LTJ factors to expected returns for options with

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different moneyness and maturity. The contributions of both factors to expected returns are an order of magnitude smaller than the contributions of MKT and VAR, and HKM and LTJ impact different parts of the cross-section of options.

6.2 Alternative Model Specifications

While the baseline model specifies factor risk premiums that are linear in the state variables, the proposed methodology allows for alternative assumptions on the functional form of the risk premia. In Panel B of Table 6, columns (2)-(4) report on the robustness of the risk premium estimates to their specification and functional form. We consider the following alternatives to the baseline specification: column (2) reports on constant premia, column (3) reports on premia that are affine in the positive and negative risk-neutral semi-variance as defined in Feunou, Jahan-Parvar, and Okou (2017), and column (4) on premia that are quadratic in the level of the risk-neutral variance.[33] All three specifications use the three-factor model with the MKT, VAR, and GAM factors, but we only report on MKT and VAR to save space.[34]

The results in column (2) of Panel B suggest that the constant risk premium specification is misspecified. This is confirmed by its performance in predicting option returns, which are reported in Internet Appendix A. The average risk premium associated with the MKT factor is relatively similar to the baseline result, but the risk premium associated with the VAR factor is much smaller in absolute value.

The semi-variance specification reported in column (3) of Panel B is motivated by the role of downside risk in determining expected index returns, which is widely reported in the literature. Our results confirm that the lower and upper semi-variance components of VIX^2 have distinct implications for the market and variance risk premia.[35] However, our main conclusion is that the average levels of the premia are close to the results from the baseline model.

[33] Lower (upper) risk-neutral semi-variance is measured as the price of a portfolio of OTM put (call) options with the weights used for the calculation of the VIX index for the put (call) options, and zero for the call (put) options. The correlation between the two semi-variance series in our sample is 84%. By construction, the VIX^2 index is equal to the sum of the lower and upper semi-variances.

[34] As discussed above, the risk premia associated with the GAM factor are relatively small regardless of the specification. We therefore specify a constant risk premium on GAM throughout for simplicity. Allowing for a time-varying GAM premium does not change the results.

[35] The premium on MKT loads positively on both semi-variance components, but the coefficient on the lower semi-variance is approximately five times larger than the one on the upper semi-variance. Consistent with economic intuition, the premium on VAR loads negatively on the lower semi-variance, but positively on the upper semi-variance, and the latter coefficient is larger in magnitude.

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Finally, the only change resulting from the quadratic specification in column (4) of Panel B is that the time series of the MKT and VAR premia are far more skewed and leptokurtic compared to the other models. Moreover, the MKT premium spikes much more from its baseline levels than the VAR premium in periods of market stress, which implies higher kurtosis.

We conclude that the model delivers average risk premium estimates that are robust to the specification of the risk premia.

6.3 The Sample Period

Figures 2 and 3 clearly indicate the highly time-varying nature of the risk premia, and specifically the enormous increase in risk premia during the financial crisis. This raises the question whether the financial crisis and the sample period more in general affect model estimates. Panel C of Table 6 therefore reports on three subsamples. The 1996–2007 sample does not include the crisis and is dominated by the period of great moderation after the collapse of the dot-com bubble. The 2008–2019 sample contains the key events of the Great Financial Crisis. The sample denoted "Apr 2009–2019" starts in April 2009, the first month after the stock market reached its lowest post-crisis level. All results are based on the three-factor model with MKT, VAR, and GAM and the baseline risk premium specification.

The estimates of the MKT premium for the 2008–2019 sample are similar to those of the baseline sample. However, both risk premiums are larger in absolute value for the post-crisis Apr 2009–2019 sample, and the VAR premium is significantly more negative in columns (3) and (4), for both samples that do not include the period prior to 2008. The pre-crisis 1996–2007 sample yields estimates of the MKT and VAR premia that are much smaller in absolute value. Also, the risk premium estimates have the anticipated sign at every point in time in the samples that contain the financial crisis, but not in the samples that exclude the crisis period. Note that we do not expect the risk premiums to be constant across samples. Perhaps more importantly, in all samples except for the post-crisis sample, the average MKT premium is close to the average realized excess return on the S&P 500 index for the relevant period.

6.4 Estimation with Adjusted Moneyness

While our baseline model uses moneyness $k_O = K_O/S$ as an option characteristic, the literature often uses moneyness adjusted for volatility and maturity. We therefore repeat

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our analysis with adjusted moneyness $k_{O,\text{adj.}} = \frac{\log(K_O/S)}{\text{VIX}\sqrt{\tau}}$ replacing moneyness k as an option characteristic in the estimation, where VIX is used as the volatility measure.

Panel D(2) of Table 6 shows that the resulting estimates of risk premia are very similar to the baseline case. However, one notable difference with the baseline specification is that call options deltas are high at all moneyness levels. This raises the estimates of call option vegas, which in turn affects the fit of the expected returns on call options. These findings are due to the facts that it is more difficult to disentangle the effects of moneyness and the VIX^2 state variable.

6.5 The Dimension of the Basis Expansion

Panel E of Table 6 investigates the sensitivity of the results to the dimension of the TRPS basis expansion used to measure exposures. In the baseline specification, we use a basis dimension k of 20. Here we also report on two other dimensions, 10 and 30. Clearly, the risk premium estimates are very similar. However, in unreported results we find that a dimension that is too small is detrimental to the quality of conditional return prediction.

6.6 Alternative Proxies for the Variance Factor

While the measurement of the MKT factor is straightforward, this is not the case for the VAR factor. Our choice of the squared VIX as a variance factor avoids critiques of data-mining, but it is useful to consider alternative proxies for the VAR factor. Existing variance measures differ in at least two respects. First, the squared VIX is a measure of the risk-neutral variance and therefore includes a risk premium. It is worth investigating if measures of physical variance lead to similar results. Second, most variance estimates include a diffusive as well as a jump component, and it is useful to verify if removing the jump component impacts inference.

We re-estimate our baseline three-factor model using alternative proxies for the variance factor. We consider four proxies that are constructed from daily increments in measures of the physical variance of index returns. The first proxy, labeled SPOT, is based on the nonparametric volatility estimate of Todorov (2019), which is option-based but measures the physical volatility of the Brownian component of index returns. The second proxy, labeled GARCH, is based on the filtered conditional variance series from a GJR-GARCH(1,1) model (Glosten, Jagannathan, and Runkle, 1993) estimated on index returns. The remaining two proxies are realized variance (RV) and integrated quadratic variation (IQV), which are

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obtained from intraday high-frequency return data. While RV is an estimate of the total instantaneous variance of index returns, IQV is an estimate of the Brownian component of return variance, i.e. it estimates the same object as the SPOT proxy.[36] Thus, GARCH and RV contain both continuous and jump variance components, while SPOT and IQV exclusively contain the continuous component.

We report on four sets of results that may be impacted by the choice of variance proxy: (a) the time-series properties of the proxies; (b) the estimates of the market return and variance risk premia; (c) their explanatory power for option returns; and (d) the accuracy of the resulting expected option returns. Table 7 summarizes our findings. Panel A reports on the time-series properties of the variance proxies. SPOT and GARCH are highly correlated with the baseline VAR factor and have similar volatility-of-volatility as well as kurtosis. RV and IQV are markedly different from VAR and SPOT, and the higher kurtosis suggests these measures are more noisy. GARCH is highly correlated with VAR and SPOT, but its high positive skewness is more similar to that of the RV and IQV proxies.

Panel B of Table 7 reports summary statistics of the risk premium estimates for the MKT factor and the various proxies for the variance factor. The time-series properties of the MKT premium are virtually unchanged when VAR is replaced by SPOT or GARCH. When RV and IQV are used, the estimate of the MKT premium is much lower on average, and is positive only 68% (86%) of the time with RV (IQV). The magnitudes of the risk premium on the variance factor are similar for VAR, SPOT and GARCH but larger (in absolute value) for IQV and RV.

Panel C of Table 7 reports the variance proxies' explanatory power for option returns. GARCH, RV and IQV only explain between 2% and 3% of option return variation, while the VAR factor in the baseline specification explains 23% (36%) of call (put) return variation. SPOT accounts for 10% (24%) of call (put) return variation. While the average sensitivities of the option returns to VAR and SPOT are similar, the estimated sensitivities to the remaining proxies are an order of magnitude higher. This fact, combined with the low explanatory power and high vol-of-vol suggests that these factors may be less precise estimates of aggregate market variance.

Finally, Panel D of Table 7 uses Mincer-Zarnowitz regressions to examine how well the alternative variance proxies fit the conditional dynamics of option returns. The results

[36]The data for SPOT were obtained from www.tailindex.com. The estimates of realized variance and integrated quadratic variation used for RV and IQV were obtained from https://realized.oxford-man.ox.ac.uk. The chosen estimator for IQV is the MedRV of Andersen, Dobrev, and Schaumburg (2012).

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suggest that SPOT captures the dynamics of put option returns rather well. IQV captures the conditional dynamics of both puts and calls, but fails to match the average return level. GARCH and RV fail on both measures of return fit.

We conclude that the choice of VAR proxy is important. Alternatives to our proxy for VAR in our baseline model, the daily increment in VIX^2, lead to similar conclusions about the dynamics of risk premia, but VIX^2 delivers results that are most consistent with economic priors. Variance factors recovered from option prices contain information that is not incorporated in variance factors exclusively based on low-frequency (GARCH) or high-frequency (IQV and RV) return data.

[Table 7 about here.]

7 Conclusion

We propose and implement a novel dynamic factor model for option returns. Our framework is very general. Options' factor exposures are modeled nonparameterically using parsimonious basis expansions of moneyness, maturity, and conditional volatility. The model provides estimates of the conditional risk premiums of any economic factor of interest and the time-varying exposure to those factors for options with different moneyness and maturity.

We illustrate the value of our approach with an implementation on a 24-year sample of daily returns on index option calls and puts with various moneyness and maturity. Our baseline model contains a market factor, a factor capturing variance risk, and a gamma factor. We also investigate richer models with the intermediary risk factor of He et al. (2017) and a tail risk factor (Bollerslev et al., 2015). The signs of the factors' average risk premia are consistent with economic intuition, the magnitudes are plausible, and, as expected, the premia spike during crises. The risk premia on the market and variance factors are by far the largest, and they have the expected sign on every day in the sample. The model explains expected option returns well across moneyness and maturities, and its hedging performance is impressive.

Several issues are left for future research. It may be useful to extend the model, for instance by allowing for a more refined aggregate jump factor (see e.g., Bates, 2008, Bollerslev and Todorov, 2011, Kelly, Lustig, and Van Nieuwerburgh, 2016) or to account for frictions in option markets (see e.g., Bollen and Whaley, 2004, Bates, 2003, Gârleanu et al., 2009). Combining index option and equity option returns may provide additional insights as well as

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improved conditional risk premium estimates (see e.g. Bégin et al., 2020, Duarte et al., 2019). More generally, the model setup can be used to study the cross-section of returns on other securities such as corporate bonds (see e.g., Bai, Bali, and Wen, 2019) and derivatives such as VIX futures, VVIX options, or CDX swaptions, because exposures of traded securities to risk factors typically display pronounced nonlinearities that differ over time and across contract characteristics. The model is also well-suited to explore the nonlinear and time-varying exposure of equity returns to factors such as asset volatility (Schneider, Wagner, and Zechner, 2020).

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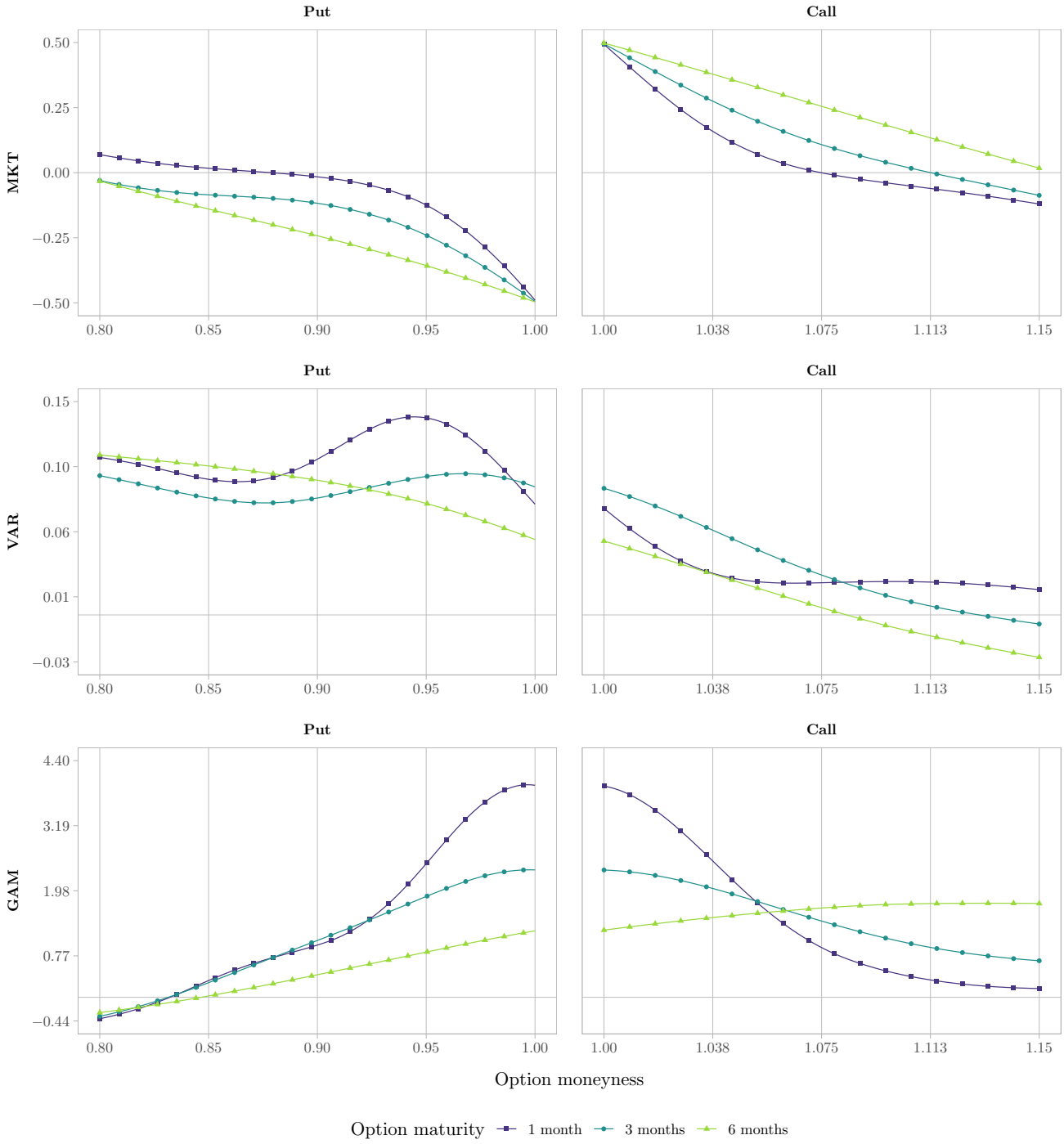

Figure 1: Option Exposures

We plot model exposures for each factor against moneyness for out-of-the-money calls and puts. Exposures are calculated by setting the state variables to their sample medians and varying moneyness and maturity. In each panel, we consider three maturities: 1, 3, and 6 months.

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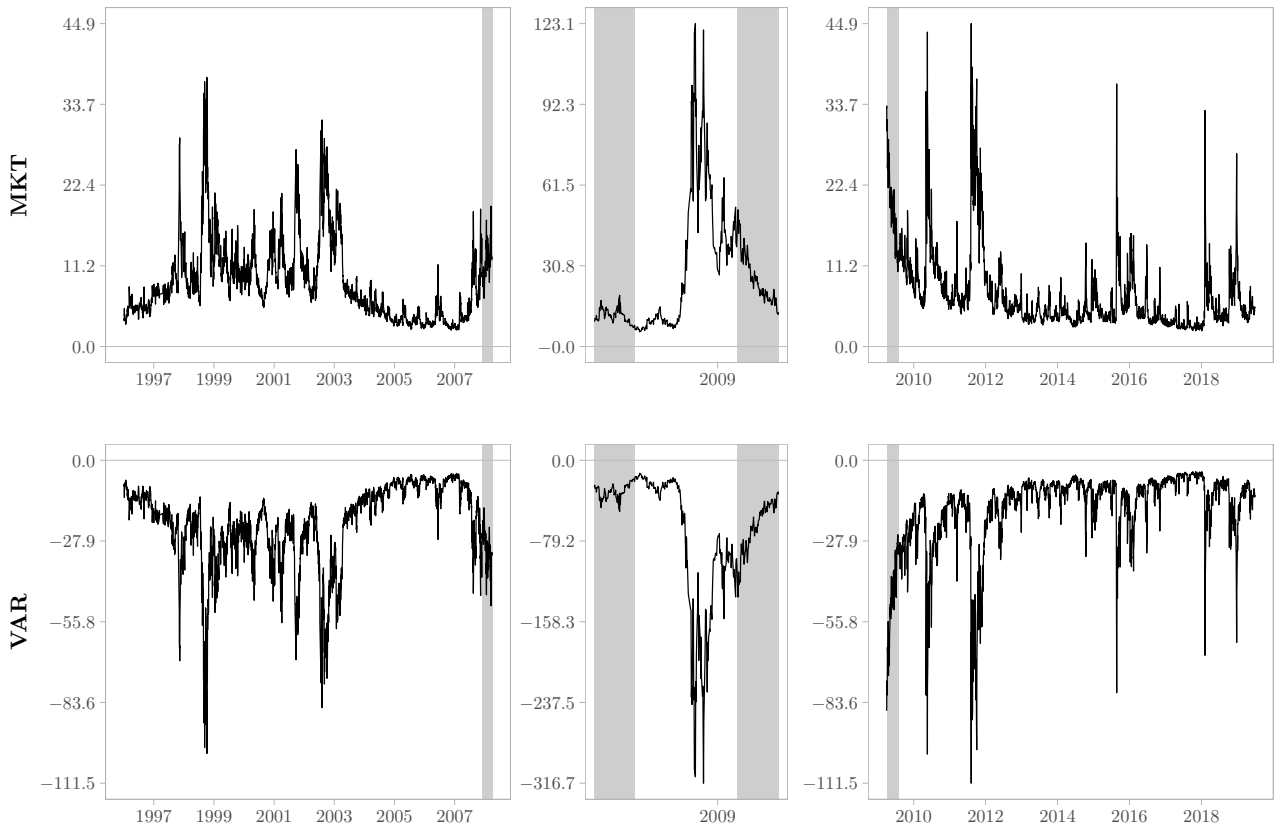

Figure 2: Time Series of Risk Premia on MKT and VAR

We plot daily time series of annualized (assuming 252 days per year) risk premia implied by the baseline model specification. The sample period is from January 3$^{\text{rd}}$, 1996 to June 27$^{\text{th}}$, 2019.

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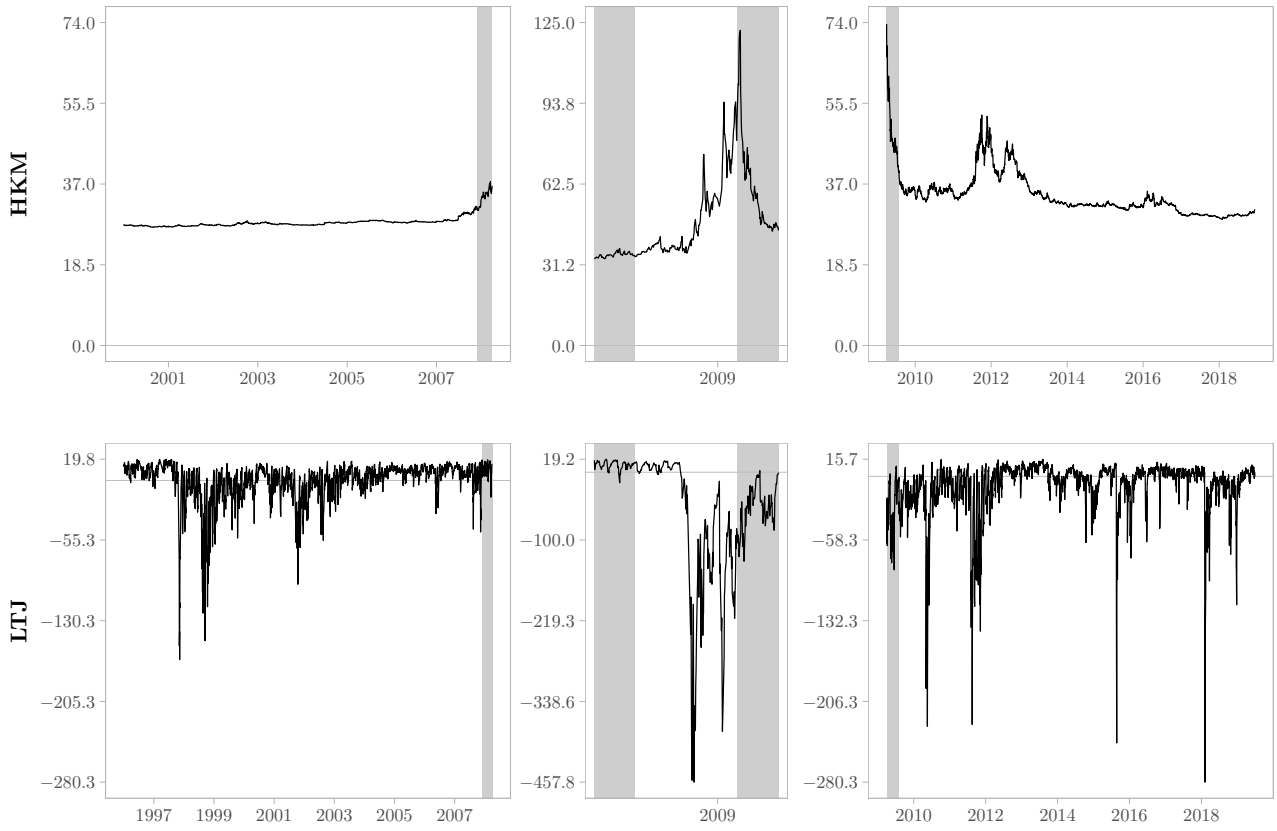

Figure 3: Time Series of Risk Premia on HKM and LTJ

We plot the daily time series of annualized risk premia on the HKM and LTJ factors implied by the model. The risk premium on HKM is an affine function of the squared inverse capital ratio at the end of each day, as in equation (12) in He et al. (2017). The risk premium on LTJ is an affine function of the level of TVAR_t at the end of each day. The premia are annualized assuming 252 trading days. For LTJ, the sample period is from January 3^{rd}, 1996 to June 27^{th}, 2019. For HKM, the sample period starts on January 2^{nd}, 2000.

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Table 1: Option Returns: Descriptive Statistics

We present summary statistics for daily deleveraged excess returns on S&P 500 index options for the January 1996 and June 2019 period. Expected returns (ER), standard deviations (SD) and Sharpe Ratios (SR) are expressed in percent per annum, with 252 trading days per year. Option returns are averaged within each day and bucket before calculating the descriptive statistics. We also report Skewness (SK) and Kurtosis (K). Delta-hedged returns are calculated with deltas obtained from our baseline factor model specification described in Section 5. For ATM options, $K/S_t \in (0.975, 1.02]$. For OTM calls (puts), $K/S_t \in (1.02, 1.07]$ ($K/S_t \in (0.9, 0.975]$). For DOTM calls (puts), $K/S_t \in (1.07, 1.15]$ ($K/S_t \in (0.8, 0.9]$).

Panel A: Put Options

	Unlevered Excess Returns					Unlevered and Delta-Hedged Excess Returns				
mat. grp.	ER	SD	SR	SK	K	ER	SD	SR	SK	K
Moneyness: All										
All	−2.89	6.51	−0.44	1.33	20.51	−1.89	2.10	−0.90	1.80	33.99
1-2M	−3.14	6.18	−0.51	1.63	25.28	−2.08	1.99	−1.05	2.35	35.99
2-3M	−2.71	6.50	−0.42	1.29	19.57	−1.83	2.06	−0.89	1.89	32.32
3-6M	−2.81	6.85	−0.41	1.13	17.72	−1.74	2.25	−0.77	1.34	32.98
Moneyness: ATM										
All	−3.91	8.82	−0.44	1.06	12.65	−2.19	2.33	−0.94	1.68	18.13
1-2M	−4.64	8.76	−0.53	1.26	14.01	−2.69	2.43	−1.11	2.03	22.51
2-3M	−3.35	8.79	−0.38	0.97	11.65	−2.03	2.30	−0.88	1.59	15.42
3-6M	−3.69	8.91	−0.41	0.96	12.27	−1.85	2.26	−0.82	1.33	14.90
Moneyness: OTM										
All	−3.02	6.15	−0.49	1.35	20.07	−2.04	2.10	−0.97	2.26	26.53
1-2M	−3.22	5.53	−0.58	1.68	26.80	−2.25	1.96	−1.15	2.74	30.39
2-3M	−2.96	6.17	−0.48	1.41	20.42	−2.00	2.10	−0.95	2.42	29.10
3-6M	−2.88	6.69	−0.43	1.08	15.72	−1.89	2.22	−0.85	1.78	21.60
Moneyness: DOTM										
All	−1.74	3.42	−0.51	1.78	42.07	−1.42	1.86	−0.77	1.29	80.23
1-2M	−1.54	2.69	−0.57	2.24	64.79	−1.31	1.45	−0.90	2.01	102.52
2-3M	−1.80	3.33	−0.54	1.95	44.46	−1.47	1.72	−0.85	1.44	84.17
3-6M	−1.87	4.09	−0.46	1.43	30.05	−1.49	2.28	−0.65	0.93	60.36

Panel B: Call Options

	Unlevered Excess Returns					Unlevered and Delta-Hedged Excess Returns				
mat. grp.	ER	SD	SR	SK	K	ER	SD	SR	SK	K
Moneyness: All										
All	0.67	5.76	0.12	1.07	23.48	−0.59	1.96	−0.30	2.10	33.82
1-2M	0.57	5.33	0.11	1.60	30.99	−0.75	2.00	−0.38	2.67	41.66
2-3M	0.56	5.81	0.10	1.27	25.25	−0.67	1.94	−0.34	2.09	34.82
3-6M	0.86	6.11	0.14	0.55	17.13	−0.35	1.94	−0.18	1.51	24.15
Moneyness: ATM										
All	0.78	7.74	0.10	0.55	12.31	−0.76	2.35	−0.32	1.83	26.54
1-2M	0.96	7.54	0.13	0.91	14.75	−1.07	2.40	−0.44	2.25	30.02
2-3M	0.81	7.84	0.10	0.74	13.50	−0.85	2.35	−0.36	1.77	27.45
3-6M	0.57	7.86	0.07	0.03	8.93	−0.34	2.29	−0.15	1.39	21.03
Moneyness: OTM										
All	1.02	5.37	0.19	1.59	29.53	−0.59	1.95	−0.30	2.08	31.63
1-2M	0.62	4.56	0.14	2.43	45.69	−0.78	1.94	−0.40	3.01	46.77
2-3M	0.54	5.38	0.10	1.69	30.99	−0.65	1.94	−0.33	2.10	31.67
3-6M	1.88	6.06	0.31	1.09	20.65	−0.34	1.96	−0.17	1.16	17.01
Moneyness: DOTM										
All	0.15	3.04	0.05	3.67	88.56	−0.41	1.45	−0.28	2.63	49.53
1-2M	0.07	2.21	0.03	7.71	218.80	−0.37	1.50	−0.24	2.82	57.33
2-3M	0.32	2.95	0.11	4.56	100.40	−0.50	1.35	−0.37	2.71	49.07
3-6M	0.09	3.71	0.02	2.14	51.18	−0.36	1.50	−0.24	2.38	41.55

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Table 2: Option Exposures

Panel A reports the sample averages of option exposures to the factors. Exposures are averaged within each day and bucket before calculating the time-series averages. Panel B reports Shapley-Owen values of R^2 (Huettner and Sunder, 2012). The R^2 is calculated as in equation (30). Note that the R^2 calculated in this manner can be negative on a subsample of the original dataset.

Panel A: Average Option Exposures

	Put Options			Call Options		
maturity group	MKT	VAR	GAM	MKT	VAR	GAM
Moneyness: All						
All	−0.24	0.09	1.49	0.25	0.05	1.67
1-2M	−0.20	0.10	1.99	0.21	0.05	1.92
2-3M	−0.24	0.09	1.42	0.26	0.05	1.73
3-6M	−0.28	0.09	1.06	0.30	0.05	1.35
Moneyness: ATM						
All	−0.42	0.09	2.37	0.44	0.08	2.38
1-2M	−0.40	0.10	3.30	0.41	0.07	3.20
2-3M	−0.42	0.10	2.26	0.44	0.08	2.37
3-6M	−0.43	0.09	1.53	0.47	0.08	1.52
Moneyness: OTM						
All	−0.22	0.10	1.55	0.24	0.05	1.71
1-2M	−0.16	0.12	1.93	0.17	0.04	1.91
2-3M	−0.22	0.10	1.45	0.24	0.05	1.85
3-6M	−0.27	0.09	1.26	0.31	0.05	1.36
Moneyness: DOTM						
All	−0.08	0.08	0.55	0.07	0.02	0.86
1-2M	−0.03	0.08	0.74	0.02	0.03	0.49
2-3M	−0.08	0.08	0.53	0.07	0.03	0.90
3-6M	−0.12	0.09	0.38	0.11	0.01	1.17

Panel B: Factor Explanatory Power for Realized Option Return

	Put options				Call options			
maturity group	MKT	VAR	GAM	Tot R^2	MKT	VAR	GAM	Tot R^2
Moneyness: All								
All	0.77	0.17	0.02	0.96	0.83	0.11	0.00	0.94
1-2M	0.76	0.17	0.03	0.96	0.81	0.13	0.00	0.94
2-3M	0.77	0.17	0.02	0.96	0.82	0.12	0.00	0.94
3-6M	0.77	0.17	0.01	0.95	0.84	0.10	0.00	0.93
Moneyness: ATM								
All	0.82	0.13	0.02	0.96	0.84	0.11	0.00	0.95
1-2M	0.80	0.13	0.02	0.96	0.85	0.11	−0.01	0.95
2-3M	0.82	0.13	0.02	0.96	0.84	0.11	0.00	0.95
3-6M	0.83	0.12	0.01	0.96	0.84	0.10	0.00	0.95
Moneyness: OTM								
All	0.74	0.19	0.02	0.96	0.81	0.12	0.00	0.93
1-2M	0.72	0.21	0.03	0.95	0.77	0.16	0.00	0.93
2-3M	0.74	0.19	0.02	0.96	0.80	0.13	0.00	0.93
3-6M	0.75	0.19	0.01	0.96	0.83	0.10	0.00	0.93
Moneyness: DOTM								
All	0.54	0.36	0.02	0.93	0.75	0.11	0.00	0.86
1-2M	0.53	0.37	0.03	0.94	0.60	0.23	0.01	0.85
2-3M	0.57	0.34	0.03	0.94	0.74	0.14	0.01	0.90
3-6M	0.54	0.37	0.01	0.91	0.80	0.05	0.00	0.84

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Table 3: Option Hedging and Predictive Performance

Panel A reports on the relative in-sample hedging performance of our model vs. the Heston model in the full 1996–2019 sample. Hedging performance is measured as the R^2 arising from removing exposures to the three factors and is calculated as in equation (30) in Section 5.3. R^2_I is the in-sample R^2 of our model, R^2_{IH} is the in-sample R^2 of the Heston model, and "diff." denotes the difference between the former two quantities. Panel B reports on the relative out-of-sample hedging performance of our model vs. the Heston model in the sub-period 2006–2019. Panel C reports the on predictive performance of the model for monthly delta-hedged deleveraged option returns in the sub-period 2006–2019. $R^2_{I/O,P}$ is the predictive R^2 calculated as in equation (33) in Section 5.7, in-sample and out-of-sample. % Smpl denotes the fraction of options in the evaluation sample that belong to a given moneyness/maturity bucket.

Panel A: In-sample Hedging: Model vs Heston

	Option Moneyness											
	All			ATM			OTM			DOTM		
Maturity	R^2_I	R^2_{IH}	diff.	R^2_I	R^2_{IH}	diff.	R^2_I	R^2_{IH}	diff.	R^2_I	R^2_{IH}	diff.
All Options												
All	94.7	87.6	7.2	95.6	89.7	5.9	93.3	86.4	8.0	89.8	77.1	12.7
Put Options												
All	95.6	89.3	6.3	96.1	90.5	5.6	95.5	89.0	6.6	92.6	82.6	10.0
1M-2M	95.7	87.9	7.7	96.0	89.5	6.5	95.4	87.5	7.9	93.5	73.6	20.0
2M-3M	95.8	89.5	6.3	96.2	90.6	5.6	95.6	89.2	6.4	93.7	83.0	10.8
3M-6M	95.4	90.4	5.1	96.2	91.6	4.6	95.6	89.8	5.8	91.4	86.2	5.2
Call Options												
All	93.5	85.2	8.3	94.9	88.5	6.4	93.0	83.0	10.0	86.0	69.5	16.5
1M-2M	93.8	82.4	11.4	94.8	86.6	8.2	92.8	76.9	15.9	84.7	52.7	32.0
2M-3M	94.1	85.5	8.6	95.0	88.9	6.1	93.2	82.9	10.3	89.6	68.2	21.5
3M-6M	92.9	87.1	5.8	94.9	90.0	4.8	92.9	86.5	6.3	83.3	75.7	8.7

Panel B: Out-of-sample Hedging: Model vs Heston

	Option Moneyness											
	All			ATM			OTM			DOTM		
Maturity	R^2_O	R^2_{OH}	diff.	R^2_O	R^2_{OH}	diff.	R^2_O	R^2_{OH}	diff.	R^2_O	R^2_{OH}	diff.
All Options												
All	91.3	87.2	4.1	94.9	89.1	5.8	90.6	85.9	4.7	74.1	80.1	−6.0
Put Options												
All	93.3	90.0	3.3	95.2	90.0	5.2	93.3	90.0	3.3	83.4	90.1	−6.7
1M-2M	93.1	89.7	3.5	95.2	89.5	5.6	92.5	90.5	2.1	79.0	87.8	−8.8
2M-3M	94.0	90.0	4.0	95.3	89.9	5.4	94.0	89.9	4.1	86.2	90.5	−4.3
3M-6M	93.0	90.3	2.7	95.1	90.6	4.6	93.2	89.6	3.5	83.7	91.0	−7.2
Call Options												
All	88.5	83.2	5.3	93.3	87.8	6.6	86.9	80.0	6.9	61.3	65.1	−3.8
1M-2M	88.2	80.8	7.4	93.4	85.6	7.8	85.0	74.9	10.1	55.5	57.5	−2.0
2M-3M	89.2	83.4	5.8	93.8	88.2	5.6	88.1	79.7	8.4	65.4	63.2	2.1
3M-6M	88.1	84.8	3.3	95.9	89.4	6.5	87.2	83.4	3.8	61.1	69.4	−8.3

Panel C: In- and Out-of-sample Predictive R^2, Delta-hedged Returns

	Option Moneyness															
	All				ATM				OTM				DOTM			
Maturity	$R^2_{I,P}$	$R^2_{O,P}$	diff.	% Smpl	$R^2_{I,P}$	$R^2_{O,P}$	diff.	% Smpl	$R^2_{I,P}$	$R^2_{O,P}$	diff.	% Smpl	$R^2_{I,P}$	$R^2_{O,P}$	diff.	% Smpl
All Options																
All	15.6	7.6	8.0	100.0	19.5	12.8	6.7	22.3	15.9	6.7	9.2	47.3	6.7	0.0	6.8	30.4
Put Options																
All	27.3	16.0	11.3	63.2	33.0	25.5	7.5	11.3	29.9	18.7	11.3	29.4	14.4	1.5	12.9	22.6
1M-2M	30.7	21.5	9.2	39.0	30.2	24.4	5.8	7.0	31.8	20.0	11.8	18.8	29.4	13.6	15.9	13.1
2M-3M	31.6	18.2	13.4	14.0	36.6	26.9	9.7	2.6	31.3	18.9	12.4	6.3	23.3	5.4	17.9	5.1
3M-6M	20.4	10.2	10.2	10.3	33.3	25.4	7.9	1.7	26.6	17.3	9.3	4.3	6.2	−2.7	8.9	4.3
Call Options																
All	5.6	0.5	5.2	36.8	9.8	4.1	5.7	11.0	4.1	−2.9	7.0	18.0	−2.5	−2.0	−0.5	7.8
1M-2M	6.2	0.2	6.0	21.6	10.0	4.8	5.2	7.0	3.6	−3.9	7.5	11.1	−0.1	−3.8	3.7	3.5
2M-3M	4.2	−0.5	4.7	8.6	6.7	1.0	5.7	2.4	5.1	−2.7	7.8	4.2	−5.2	0.0	−5.2	2.1
3M-6M	6.5	2.3	4.3	6.5	14.1	7.9	6.3	1.6	3.6	−1.7	5.3	2.7	−3.2	−1.5	−1.8	2.2

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Table 4: Expected Return Decomposition and Goodness of Fit

We report expected deleveraged excess option returns in percent per annum. Panel A indicates the expected returns due to each factor, i.e. the sample average of $\beta_{lt}^{O} \cdot \lambda_{lt}$. Panel B presents the results of Mincer-Zarnowitz regressions of realized returns on expected returns,

$$R_{O,t+\Delta t} = \alpha + \gamma E_t[R_{O,t+\Delta t}] + \nu_{O,t}\,.$$

The αs are reported in % per annum. The column labeled "p-val α" reports the p-value for the hypothesis that $\alpha = 0$. The column labeled "p-val γ" reports the p-value for the hypothesis that $\gamma = 1$.

Panel A: Decomposition of Expected Returns (% per annum)

	Put Options				Call Options			
τ group	(1) MKT	(2) VAR	(3) GAM	(4) Tot	(1) MKT	(2) VAR	(3) GAM	(4) Tot
Moneyness: All								
All	−2.28	−1.88	−0.09	−4.25	2.66	−1.23	−0.10	1.33
1-2M	−2.05	−1.87	−0.12	−4.03	2.40	−1.29	−0.11	1.00
2-3M	−2.29	−1.82	−0.08	−4.19	2.68	−1.33	−0.10	1.26
3-6M	−2.50	−1.95	−0.06	−4.51	2.88	−1.07	−0.08	1.73
Moneyness: ATM								
All	−3.77	−1.96	−0.14	−5.87	4.15	−1.68	−0.14	2.33
1-2M	−3.69	−2.03	−0.19	−5.92	4.03	−1.62	−0.19	2.22
2-3M	−3.75	−1.99	−0.13	−5.88	4.16	−1.77	−0.14	2.25
3-6M	−3.85	−1.86	−0.09	−5.80	4.27	−1.65	−0.09	2.53
Moneyness: OTM								
All	−2.21	−2.01	−0.09	−4.32	2.61	−1.23	−0.10	1.28
1-2M	−1.88	−2.11	−0.11	−4.10	2.17	−1.19	−0.11	0.87
2-3M	−2.22	−1.91	−0.08	−4.21	2.61	−1.32	−0.11	1.18
3-6M	−2.54	−2.01	−0.07	−4.63	3.05	−1.18	−0.08	1.79
Moneyness: DOTM								
All	−0.86	−1.67	−0.03	−2.55	1.11	−0.75	−0.05	0.31
1-2M	−0.58	−1.45	−0.04	−2.07	0.84	−1.04	−0.03	−0.23
2-3M	−0.87	−1.55	−0.03	−2.45	1.14	−0.85	−0.05	0.24
3-6M	−1.12	−1.99	−0.02	−3.13	1.33	−0.39	−0.07	0.87

Panel B: Mincer-Zarnowitz Regressions

	Put Options				Call Options			
τ group	$\widehat{\alpha}$	p-val $\widehat{\alpha} = 0$	$\widehat{\gamma}$	p-val $\widehat{\gamma} = 1$	$\widehat{\alpha}$	p-val $\widehat{\alpha} = 0$	$\widehat{\gamma}$	p-val $\widehat{\gamma} = 1$
Moneyness: All								
All	0.05	1.00	1.00	1.00	0.42	0.92	1.13	1.00
1-2M	0.17	0.99	1.02	1.00	0.77	0.71	1.09	1.00
2-3M	−0.52	0.95	0.81	0.96	−0.86	0.81	1.30	0.98
3-6M	0.34	0.98	1.13	0.98	0.92	0.82	1.07	1.00
Moneyness: ATM								
All	−0.17	1.00	0.97	1.00	1.55	0.79	0.35	0.91
1-2M	0.28	0.99	1.08	1.00	1.22	0.84	0.78	0.99
2-3M	−2.44	0.77	0.39	0.79	0.05	1.00	−0.01	0.77
3-6M	1.32	0.88	1.37	0.88	5.42	0.50	−0.68	0.75
Moneyness: OTM								
All	−0.01	1.00	0.99	1.00	0.04	1.00	1.68	0.93
1-2M	0.14	1.00	1.03	1.00	0.53	0.85	1.46	0.97
2-3M	−0.35	0.98	0.91	0.99	−1.28	0.75	1.97	0.91
3-6M	−0.14	1.00	0.98	1.00	−0.31	0.99	2.10	0.88
Moneyness: DOTM								
All	0.17	0.98	0.99	1.00	0.46	0.52	1.49	0.94
1-2M	−0.11	0.99	0.72	0.93	0.90	0.24	0.95	1.00
2-3M	0.60	0.89	1.32	0.93	−0.10	0.99	3.70	0.27
3-6M	0.37	0.96	1.14	0.98	−0.07	1.00	1.20	0.99

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Table 5: Estimates of Factor Premia

We report estimates of risk premium parameters (Panel A) and summary statistics (Panel B) for the time-series of risk premia for the baseline model specification. The sample period is from January 1996 until June 2019. p-values are reported in parentheses. Coefficient standard errors and F-statistics are robust to general autocorrelation, cross-correlation and cross-sectional correlation patterns between options and account for the measurement error in option betas (see Internet Appendix B for details).

Panel A: Estimates of λ

$E_t[\text{MKT}] = \lambda_0^{(1)} + \lambda_1^{(1)} \cdot VIX_t^2 + \lambda_2^{(1)} \cdot TVAR_t$

λ_0	λ_1	λ_2	F-stat
1.62e-05	0.00692	0.00423	2.67
(0.966)	(0.735)	(0.953)	(0.446)

$E_t[\text{VAR}] = \lambda_0^{(2)} + \lambda_1^{(2)} \cdot VIX_t^2$

λ_0	λ_1	λ_2	F-stat
5.36e-06	−0.0192	–	6.02
(0.993)	(0.549)	–	(0.049)

$E_t[\text{GAM}] = \lambda_0^{(3)}$

λ_0	λ_1	λ_2	F-stat
−2.3e-06	–	–	0.19
(0.666)	–	–	(0.666)

Panel B: Summary Statistics of Risk Premia

Factor	Avg (%)	Med (%)	Std. dev. (%)	Skewness	Kurtosis	Exp. sign (%)
MKT	9.1	6.8	0.57	5.0	40.8	100.0
VAR	−22.1	−16.0	1.45	−4.9	40.5	100.0
GAM	−0.1					

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Table 6: Robustness of Risk Premia Estimates

We report summary statistics of factor risk premia with additional factors (Panel A), alternative premia specifications (Panel B), for subsamples (Panel C), with raw returns and volatility-adjusted moneyness (Panel D), and for different basis dimension choices (Panel E). The results for our baseline model are repeated in the first column of each panel. The average premium and its standard deviation are reported in percent per annum. The row denoted "Exp. sign" reports the fraction, in percent, of the sample period in which the factor premium is of the expected sign, i.e., positive in the case of MKT and HKM, and negative in the case of VAR and LTJ. The sample is from 1996 until 2019, except for the model with the HKM factor, which starts in 2000.

	Panel A: Additional Factors									
	(1) Baseline		(2) 2F		(3) 3F + HKM			(4) 3F + LTJ		
	MKT	VAR	MKT	VAR	MKT	VAR	HKM	MKT	VAR	LTJ
Average (%)	9.1	−22.1	11.2	−20.0	6.2	−22.2	32.8	12.2	−20.5	−7.1
Std. dev. (%)	0.6	1.5	0.9	1.3	0.4	1.9	0.5	0.9	1.1	2.1
Skewness	5.0	−4.9	4.7	−4.7	5.1	−4.9	4.1	4.9	−5.0	−5.4
Kurtosis	40.8	40.5	38.0	38.0	40.1	38.1	29.4	40.8	41.1	47.4
Exp. sign (%)	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	44.0

	Panel B: Alternative Premia Specifications							
	(1) Baseline		(2) 3F Constant		(3) 3F Semi-VIX		(4) 3F Quadratic	
	MKT	VAR	MKT	VAR	MKT	VAR	MKT	VAR
Average (%)	9.1	−22.1	7.9	−8.0	10.5	−17.3	7.9	−23.1
Std. dev. (%)	0.6	1.5			0.8	1.8	0.7	1.9
Skewness	5.0	−4.9			4.8	−4.8	11.7	−7.5
Kurtosis	40.8	40.5			38.4	47.5	183.6	86.8
Exp. sign (%)	100.0	100.0	100.0	100.0	100.0	89.4	100.0	100.0

	Panel C: Subsamples							
	(1) Baseline		(2) 1996–2007		(3) 2008–2019		(4) Apr 2009–2019	
	MKT	VAR	MKT	VAR	MKT	VAR	MKT	VAR
Average (%)	9.1	−22.1	3.4	−4.4	10.7	−36.4	20.1	−31.6
Std. dev. (%)	0.6	1.5	0.7	0.6	0.8	3.3	2.2	2.3
Skewness	5.0	−4.9	2.2	−1.5	4.5	−4.5	2.7	−2.6
Kurtosis	40.8	40.5	10.1	6.4	30.1	29.9	12.4	11.4
Exp. sign (%)	100.0	100.0	52.3	60.7	100.0	100.0	74.0	98.8

	Panel D: Standardised Moneyness / Raw Returns					
	(1) Baseline		(2) Vol-adj. Strike		(3) Raw Returns	
	MKT	VAR	MKT	VAR	MKT	VAR
Average (%)	9.1	−22.1	10.5	−17.6	12.3	−20.7
Std. dev. (%)	0.6	1.5	0.9	1.5	0.8	4.5
Skewness	5.0	−4.9	4.7	−4.7	−1.7	−4.9
Kurtosis	40.8	40.5	38.0	38.0	25.4	40.5
Exp. sign (%)	100.0	100.0	98.9	98.0	92.8	51.6

	Panel E: Robustness to Basis Dimension					
	(1) Baseline ($k = 20$)		(2) $k = 10$		(3) $k = 30$	
	MKT	VAR	MKT	VAR	MKT	VAR
Average (%)	9.1	−22.1	9.3	−24.4	9.2	−23.0
Std. dev. (%)	0.6	1.5	0.6	1.6	0.6	1.7
Skewness	5.0	−4.9	4.9	−4.9	5.0	−4.9
Kurtosis	40.8	40.5	40.2	40.5	40.8	40.5
Exp. sign (%)	100.0	100.0	100.0	100.0	100.0	100.0

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Table 7: Alternative Variance Proxies

We report results for alternative variance factor proxies. Panel A reports summary statistics for the alternative variance proxies. Panel B reports the summary statistics of the premia on the MKT factor and each variance proxy. The standard deviation is reported on an annualized basis and can be understood as a volatility-of-volatility. ρ_{VAR} denotes the correlation of the proxy with the VAR factor, and ρ_{SPOT} denotes the correlation of a proxy with the SPOT proxy. Panel C reports the option sensitivities with respect to the alternative variance proxies and the proxies' explanatory power, measured as the Shapley-Owen value of R^2. Panel D reports the coefficients of Mincer-Zarnowitz regressions of realized option returns on expected option returns, again for implementations with different variance proxies. The proxies are: SPOT, i.e. the daily increment in the square of the spot volatility of Todorov (2019) obtained from www.tailindex.com, GARCH, i.e. the daily increment in the annualized daily filtered variance from a GJR-GARCH(1,1) (Glosten et al., 1993) model with Student-t innovations estimated on the underlying SPX excess returns, RV and IQV, i.e., realized variance and integrated quadratic variation (estimated using the MedRV measure of Andersen et al. (2012)) of intraday S&P 500 index returns, obtained from realized.oxford-man.ox.ac.uk.

	Panel A: Summary Statistics of Alternative Variance Factor Proxies				
	(1) VAR	(2) SPOT	(3) GARCH	(4) RV	(5) IQV
Std. dev.	0.19	0.19	0.19	0.68	0.36
Skewness	0.35	−0.34	5.89	13.09	6.13
Kurtosis	94.2	70.8	104.3	452.2	217.7
ρ_{VAR}		0.67	0.76	0.22	0.32
ρ_{SPOT}			0.61	0.17	0.17

	Panel B: Premia with Alternative Variance Factor Proxies									
	(1) Baseline		(2) SPOT		(3) GARCH		(4) RV		(5) IQV	
	MKT	VAR	MKT	SPOT	MKT	GARCH	MKT	RV	MKT	IQV
Average (%)	9.1	−22.1	9.5	−51.6	8.0	−36.8	1.3	−664.0	5.3	−62.8
Std. dev. (%)	0.6	1.5	0.6	4.6	0.5	4.2	1.5	116.9	0.8	16.4
Skewness	5.0	−4.9	5.0	−4.7	5.9	−5.9	−5.1	−7.6	−6.1	−11.1
Kurtosis	40.8	40.5	39.1	33.4	53.3	53.3	59.3	83.8	110.4	186.8
Exp. sign (%)	100.0	100.0	99.7	100.0	100.0	86.4	68.1	68.2	85.6	49.1
Sample	1996–2019		2008–2019		1996–2019		2000–2019		2000–2019	

	Panel C: Explanatory power of variance proxies									
	(1) Baseline		(2) Spot vol.		(3) GARCH		(4) RV		(5) IQV	
	Puts	Calls	Puts	Calls	Puts	Calls	Puts	Calls	Puts	Calls
Sens.	0.09	0.05	0.07	0.04	1.05	2.72	2.54	2.56	2.24	2.48
R^2	0.36	0.23	0.24	0.10	0.02	0.03	0.03	0.03	0.03	0.03

	Panel D: Mincer-Zarnowitz Regressions with Variance Proxies									
	(1) Baseline		(2) SPOT		(3) GARCH		(4) RV		(5) IQV	
	Puts	Calls	Puts	Calls	Puts	Calls	Puts	Calls	Puts	Calls
$\widehat{\alpha}$	0.05	0.42	1.64	1.34	−1.45	0.69	−2.18	0.81	−1.53	−0.50
$\widehat{\gamma}$	1.00	1.13	1.32	0.08	0.36	0.62	0.94	0.47	0.98	1.10

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Internet Appendix

to

Modeling Conditional Factor Risk Premia Implied by Index Option Returns

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Internet Appendix A Additional Tables

Table IA.1: Conditional Expected Return in the Two-Factor Linear Specification

The table presents the results of Mincer-Zarnowitz regressions of realized returns on expected returns,

$$R_{O,t,t+\Delta_t} = \alpha + \gamma E_t[R_{O,t,t+\Delta_t}] + \nu_{O,t}\,,$$

for the two-factor specification with linear risk premia discussed in Section 6.1. The αs are reported in % per annum. The column labeled "p-val α" reports the p-value for the hypothesis that $\alpha = 0$. The column labeled "p-val γ" reports the p-value for the hypothesis that $\gamma = 1$.

	Put options				Call options			
τ group	$\widehat{\alpha}$	p-val $\widehat{\alpha}=0$	$\widehat{\gamma}$	p-val $\widehat{\gamma}=1$	$\widehat{\alpha}$	p-val $\widehat{\alpha}=0$	$\widehat{\gamma}$	p-val $\widehat{\gamma}=1$
Moneyness: All								
All	−0.14	0.99	0.91	0.99	0.36	0.93	1.37	0.86
1-2M	−0.09	1.00	0.89	0.98	0.84	0.63	1.26	0.93
2-3M	−0.37	0.96	0.88	0.98	−1.14	0.67	1.79	0.45
3-6M	−0.08	1.00	0.99	1.00	0.72	0.85	1.21	0.96
Moneyness: ATM								
All	−0.55	0.97	0.87	0.98	1.24	0.75	0.70	0.96
1-2M	−0.53	0.97	0.85	0.98	1.28	0.67	0.99	1.00
2-3M	−1.36	0.80	0.69	0.85	−1.11	0.86	0.90	0.99
3-6M	0.36	0.98	1.13	0.97	5.14	0.29	−0.50	0.64
Moneyness: OTM								
All	−0.18	0.99	0.91	0.99	0.21	0.98	1.74	0.72
1-2M	−0.10	1.00	0.91	0.99	0.77	0.72	1.42	0.90
2-3M	−0.23	0.99	0.96	1.00	−1.17	0.73	2.18	0.24
3-6M	−0.51	0.96	0.87	0.98	−0.54	0.93	2.21	0.38
Moneyness: DOTM								
All	0.02	1.00	0.89	0.98	−0.04	1.00	1.91	0.33
1-2M	−0.07	0.99	0.73	0.91	0.56	0.43	1.44	0.80
2-3M	0.16	0.99	1.07	0.99	−0.87	0.69	3.34	0.00
3-6M	0.04	1.00	0.99	1.00	−0.61	0.82	1.84	0.60

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Table IA.2: Conditional Expected Return in the Three-Factor Constant Specification

The table presents the results of Mincer-Zarnowitz regressions of realized returns on expected returns,

$$R_{O,t,t+\Delta_t} = \alpha + \gamma E_t[R_{O,t,t+\Delta_t}] + \nu_{O,t}\,,$$

for the three-factor specification with constant risk premia discussed in Section 6.2. The αs are reported in % per annum. The column labeled "p-val α" reports the p-value for the hypothesis that $\alpha = 0$. The column labeled "p-val γ" reports the p-value for the hypothesis that $\gamma = 1$.

	Put options				Call options			
τ group	$\widehat{\alpha}$	p-val $\widehat{\alpha}=0$	$\widehat{\gamma}$	p-val $\widehat{\gamma}=1$	$\widehat{\alpha}$	p-val $\widehat{\alpha}=0$	$\widehat{\gamma}$	p-val $\widehat{\gamma}=1$
Moneyness: All								
All	−0.36	0.51	1.05	0.99	−0.28	0.69	1.24	0.90
1-2M	−0.20	0.82	1.08	0.98	−0.09	0.95	1.36	0.80
2-3M	−0.80	0.31	0.90	0.98	−0.43	0.73	0.62	0.83
3-6M	−0.73	0.57	0.98	1.00	−0.86	0.42	1.74	0.45
Moneyness: ATM								
All	−4.38	0.50	−0.03	0.52	−2.91	0.69	1.94	0.78
1-2M	0.70	0.97	1.21	0.96	−3.94	0.59	2.52	0.65
2-3M	−27.01	0.03	−5.50	0.02	−3.89	0.78	1.53	0.94
3-6M	−12.44	0.53	−1.79	0.54	8.16	0.61	−1.36	0.69
Moneyness: OTM								
All	0.82	0.79	1.57	0.74	−1.24	0.48	2.29	0.57
1-2M	1.32	0.66	1.81	0.66	−1.35	0.50	3.33	0.47
2-3M	0.04	1.00	1.28	0.93	−3.28	0.48	2.82	0.66
3-6M	−0.67	0.91	1.01	1.00	−3.93	0.42	3.26	0.44
Moneyness: DOTM								
All	0.30	0.90	1.80	0.73	−0.60	0.34	3.44	0.21
1-2M	2.10	0.56	4.37	0.53	0.21	0.61	5.42	0.15
2-3M	1.87	0.85	3.27	0.80	−3.57	0.12	9.34	0.13
3-6M	−2.58	0.48	−0.21	0.62	−1.25	0.44	2.49	0.52

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Table IA.3: Option Exposures and Expected Returns with Additional Factors

We report scaled option exposures ($\beta \times 10^2$), Shapley-Owen R^2s (Huettner and Sunder, 2012) and the contribution to expected returns ("Exp. ret.") for additional factors in alternative model specifications. The estimates are based on four-factor models that add the HKM or LTJ factor to the baseline specification. The contribution to expected return (in percent per annum) is calculated as the sample average of the products of the exposures and premia. Exposures and returns are averaged within each day (and bucket) before calculating the time-series averages. HKM is the intermediary risk factor of He et al. (2017), with the risk premium specified as in equation (12) therein, i.e. an affine function of the square of the intermediary leverage ratio; daily data on the HKM factor was obtained from the website of Asaf Manela. LTJ is the daily increment in the square of the Tail Volatility factor of Bollerslev et al. (2015); daily data on Tail Volatility was obtained from the TailIndex website.

	Put options						Call options					
	HKM			LTJ			HKM			LTJ		
Maturity	$\beta \times 10^2$	R^2	Exp. ret.	$\beta \times 10^2$	R^2	Exp. ret.	$\beta \times 10^2$	R^2	Exp. ret.	$\beta \times 10^2$	R^2	Exp. ret.
Moneyness: All												
All	−0.14	0.000	−0.04	−0.514	0.000	−0.05	0.47	0.008	0.16	2.100	0.001	−0.23
1-2M	0.07	0.000	0.03	0.231	0.001	−0.11	0.27	0.007	0.09	0.944	0.001	−0.16
2-3M	−0.15	0.000	−0.04	0.012	0.000	−0.07	0.48	0.009	0.16	1.055	0.000	−0.18
3-6M	−0.33	0.000	−0.10	−1.757	0.000	0.01	0.67	0.009	0.22	4.230	0.003	−0.34
Moneyness: ATM												
All	0.08	0.000	0.03	−1.740	0.000	0.00	0.34	0.005	0.12	−0.550	0.000	−0.06
1-2M	0.24	0.000	0.09	−1.519	0.000	−0.03	0.47	0.006	0.16	−0.547	0.000	−0.07
2-3M	0.02	0.000	0.02	−2.979	0.000	0.06	0.28	0.004	0.10	−1.871	0.000	0.01
3-6M	−0.03	0.000	0.00	−0.776	0.000	−0.03	0.26	0.002	0.09	0.734	0.001	−0.10
Moneyness: OTM												
All	−0.24	0.007	−0.07	0.428	0.002	−0.09	0.50	0.021	0.17	4.688	0.005	−0.43
1-2M	0.00	0.002	0.00	2.507	0.019	−0.23	−0.12	0.006	−0.03	2.136	0.004	−0.25
2-3M	−0.20	0.005	−0.06	1.575	0.010	−0.11	0.54	0.023	0.19	3.526	0.004	−0.38
3-6M	−0.50	0.010	−0.16	−2.708	0.001	0.07	1.03	0.043	0.34	8.082	−0.003	−0.63
Moneyness: DOTM												
All	−0.25	0.000	−0.07	−0.236	0.001	−0.07	0.58	0.010	0.19	2.334	0.003	−0.21
1-2M	−0.04	0.000	−0.01	−0.292	0.002	−0.07	0.42	0.009	0.14	1.376	0.003	−0.16
2-3M	−0.25	0.000	−0.07	1.434	0.005	−0.15	0.61	0.012	0.21	1.728	0.002	−0.18
3-6M	−0.45	0.002	−0.14	−1.774	0.000	0.00	0.71	0.010	0.24	3.879	0.005	−0.29

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Internet Appendix B Estimation and Inference

B.1 Estimation of factor loadings

Recall, with the definitions from Proposition 1 in the main text, we can write

$$R^O_{t+\Delta t} = \left(\boldsymbol{\phi}^g_{O,t}\right)' \boldsymbol{a} + \left(\boldsymbol{\phi}^f_{O,t+\Delta t}\right)' \boldsymbol{b} + \varepsilon^O_{t+\Delta t},$$

where $\boldsymbol{\phi}^{g'}_{O,t}\boldsymbol{a} = \sum_{l=1}^{L} -E^Q_t\left[f_{l,t+\Delta t}\right] \cdot \beta^O_{l,t}$ and $\boldsymbol{\phi}^{f'}_{O,t+\Delta t}\boldsymbol{b} = \sum_{l=1}^{L} f_{l,t+\Delta t} \cdot \beta^O_{l,t}$. The vector $\boldsymbol{\phi}^g_{O,t} \equiv \left[\boldsymbol{\phi}^{g'}_{O,1,t} \cdots \boldsymbol{\phi}^{g'}_{O,L,t}\right]'$ is $Lkq \times 1$, with $\boldsymbol{\phi}^g_{O,l,t} \equiv \boldsymbol{g}_{l,t} \otimes \boldsymbol{\phi}_{O,l,t}$ and where $\otimes$ denotes the Kronecker product. Furthermore $\boldsymbol{\phi}^f_{O,t+\Delta t} \equiv \left[\boldsymbol{\phi}^{f'}_{O,1,t+\Delta t} \cdots \boldsymbol{\phi}^{f'}_{O,L,t+\Delta t}\right]'$ is an $Lk \times 1$ vector, with $\boldsymbol{\phi}^f_{O,l,t+\Delta t} \equiv f_{l,t+\Delta t} \otimes \boldsymbol{\phi}_{O,l,t}$, and $f_{l,t+\Delta t}$ are the factor realizations from equation (2) in the main text. We further define $\boldsymbol{\phi}^{fg}_{O,t+\Delta t} \equiv \left[\boldsymbol{\phi}^{g'}_{O,t} \quad \boldsymbol{\phi}^{f'}_{O,t+\Delta t}\right]'$

In this section we present the results for statistical inference in the first-stage estimation. The results are given for a fixed dimension k of the TPRS basis expansion. We account for time-series and cross-sectional correlation in the residuals $\varepsilon^O_{t+\Delta t}$.

Assumptions B.1 *In the remainder of this Section, we assume the following:*

(i) $E\left[\varepsilon^O_{t+\Delta t} | \boldsymbol{\phi}^{fg}_{O,t+\Delta t}\right] = 0$

(ii) $\varepsilon^O_{t+\Delta t}$ *is spatially strong mixing, as in Driscoll and Kraay (1998).*

Assumption B.1(i) means that the option strikes and maturities contained in the data panel on a given day do not depend on instruments nor factor realizations.[37]

Proposition B.1 (Estimator of $\boldsymbol{\Sigma}_{\phi\varepsilon}$) *Define* $\boldsymbol{h}_t \equiv \frac{1}{N_t}\sum_{O=1}^{N_t} \boldsymbol{\phi}^{fg}_{O,t+\Delta t}\varepsilon^O_{t+\Delta t}$ *and set* S*, a positive integer lag order. Under assumption B.1(ii),* $\boldsymbol{\Sigma}_{\phi\varepsilon}$ *can be estimated with*

$$\widehat{\boldsymbol{\Sigma}}_{\phi\varepsilon} \equiv \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{h}_t\boldsymbol{h}_t' + \frac{1}{T}\sum_{s=1}^{S}\sum_{t=s+1}^{T} w_s\left(\boldsymbol{h}_t\boldsymbol{h}_{t-l}' + \boldsymbol{h}_{t-l}\boldsymbol{h}_t'\right) ,$$

where $w_s = 1 - \frac{s}{S+1}$*, which is the Newey and West (1987) estimator applied to the time-series of* $\boldsymbol{h}_t$*.*

Because option exposures $\widehat{\beta}^O_{l,t}$ are linear transformations of the estimated vector of parameters $\widehat{\boldsymbol{b}}_l$, the asymptotic distribution of $\widehat{\beta}^O_{l,t}$ inherits the distributional properties of $\widehat{\boldsymbol{b}}_l$.

[37] Arguably, this assumption would be tenuous if we were to use the whole option surface in our analysis. Because we limit the strike ranges and option maturities considered, it eliminates this possible source of bias.

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Proposition B.2 *With deleveraged excess option returns satisfying (21) and option exposures defined by (14), the asymptotic distribution of option O's exposure to factor l on day t satisfies*

$$\widehat{\beta}^{O}_{l,t} = \boldsymbol{\phi}'_{O,l,t}\widehat{\boldsymbol{b}}_l \sim N\left(\boldsymbol{\phi}'_{O,l,t}\boldsymbol{b}_l; \boldsymbol{\phi}'_{O,l,t}\boldsymbol{\Sigma}_{\widehat{\boldsymbol{b}}_l}\boldsymbol{\phi}_{O,l,t}\right), \tag{B.1}$$

where $\boldsymbol{b}_l$ and $\boldsymbol{\Sigma}_{\widehat{\boldsymbol{b}}_l}$ are the appropriate subvector and submatrix of $\boldsymbol{B}$ and $\boldsymbol{\Sigma}_{\widehat{\boldsymbol{B}}}$ from Proposition 3, respectively. Moreover, the joint hypothesis test that the parameters in $\boldsymbol{b}_l$ are jointly equal to zero (i.e., factor l does not influence the dynamics of option returns) can be tested by the following feasible Wald statistic:

$$\widehat{\boldsymbol{b}}'_l\widehat{\boldsymbol{\Sigma}}^{-1}_{\widehat{\boldsymbol{b}}_l}\widehat{\boldsymbol{b}}_l \sim \boldsymbol{\chi}^2_{\boldsymbol{k}}. \tag{B.2}$$

B.2 Estimation of risk premia

In this section we present the results allowing for inference about risk premia parameters. We split the standard second-stage estimation into two independent sub-stages: the premium for tradable factors (MKT in the main text), and the premium for non-tradable factors. Inference in the two sub-stages is identical after accounting for the different regressors carried over from the first stage. We first lay out the complete set of results for the estimation of premia on tradable factors in Section B.2.1, which is a multivariate extension of the market risk premium estimation presented in the core of the paper. Then, in Section B.2.2, we indicate the required changes to apply to the methodology presented in Section B.2.1 to estimate non-tradable factors' risk premia parameters.

B.2.1 Premia on excess return factors

In this sub-stage we first project the realized option returns on the tradable factors which are excess returns, and then regress these projections on the products of the basis expansions of option exposure instruments with the predictors of risk premia for tradable factors. Without loss of generality assume that factors 1 through $D \leq L$ are tradable. We denote the projection as

$$R^{O,M}_{t+\Delta t} \equiv \sum_{k=1}^{D} \beta^{O}_{k,t} f_{k,t+\Delta t}\,.$$

Let $\boldsymbol{d}^{M}_{O,t} \equiv \left[\beta^{O}_{1t}\boldsymbol{g}'_{1t} \quad \dots \quad \beta^{O}_{Dt}\boldsymbol{g}'_{Dt}\right]$, and let $\widehat{\boldsymbol{d}}^{M}_{O,t}$ be its sample analogue, i.e., with $\widehat{\beta}^{O}_{k,t}$ replacing $\beta^{O}_{k,t}$. Then the following relationship holds between the projection of option returns on the

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tradable factors and the risk premia parameters,

$$R_{t+\Delta t}^{O,M} = \left(\boldsymbol{d}_{O,t}^{M}\right)' \boldsymbol{\lambda}_M + \eta_{O,t+\Delta t}^{M}\,.$$

Equipped with this result, we estimate $\boldsymbol{\lambda}_M$ based on the feasible version of the above equation, that is, using estimated exposures. We get

$$\widehat{R}_{t+\Delta t}^{O,M} \equiv \sum_{k=1}^{D} \widehat{\beta}_{k,t}^{O} f_{k,t+\Delta t} = \left(\widehat{\boldsymbol{d}}_{O,t}^{M}\right)' \boldsymbol{\lambda}_M + \widetilde{\eta}_{O,t+\Delta t}^{M}\,. \tag{B.3}$$

In the above regression equation, the quantities on both sides contain measurement error. There are two consequences of the measurement error: asymptotic bias, and increases in the variance of the estimated coefficients.

Throughout this section, we rely on the following assumptions.

Assumptions B.2 *(iii)* $E\left[\eta_{O,t+\Delta t}^{M} | \boldsymbol{\phi}_{O,t+\Delta t}^{fg}\right] = 0$

(iv) $\eta_{O,t+\Delta t}^{M}$ *is spatially strong mixing, as in Driscoll and Kraay (1998).*

Proposition B.3 (Estimator of $\boldsymbol{\lambda}_M$) *The OLS estimator of* $\boldsymbol{\lambda}_M$ *is given by*

$$\widehat{\boldsymbol{\lambda}}_M \equiv \boldsymbol{Q}_{\widehat{\boldsymbol{d}}^M}^{-1} \frac{1}{T} \sum_{t=1}^{T} \frac{1}{N_t} \sum_{O=1}^{N_t} \widehat{\boldsymbol{d}}_{O,t}^{M} \cdot \left(\boldsymbol{\phi}_{O,t+\Delta t}^{f_M}\right)' \widehat{\boldsymbol{b}}_M\,,$$

where $\widehat{\boldsymbol{b}}_M$ *is the subvector of* $\widehat{\boldsymbol{b}}$ *which contains the loadings on the tradable factors,* $\boldsymbol{\phi}_{O,t+\Delta t}^{f_M}$ *is the subvector of the vector* $\boldsymbol{\phi}_{O,t+\Delta t}^{f}$ *with entries related to tradable factors, and* $\boldsymbol{Q}_{\widehat{\boldsymbol{d}}^M} \equiv \frac{1}{T}\sum_{t=1}^{T} \frac{1}{N_t} \sum_{O=1}^{N_t} \widehat{\boldsymbol{d}}_{O,t}^{M} \left(\widehat{\boldsymbol{d}}_{O,t}^{M}\right)'$.

Proof. The proposition follows immediately from writing the OLS estimator of $\boldsymbol{\lambda}_M$ from equation (B.3) and recognising that $\widehat{R}_{t+\Delta t}^{O,M} = \sum_{k=1}^{D} \widehat{\beta}_{k,t}^{O} f_{k,t+\Delta t} = \left(\boldsymbol{\phi}_{O,t+\Delta t}^{f_M}\right)' \widehat{\boldsymbol{b}}_M$. ■

Corollary B.1 (Estimation error in $\widehat{\boldsymbol{\lambda}}_M$) *The following decomposition of estimation error in* $\widehat{\boldsymbol{\lambda}}_M$ *holds:*

$$\begin{aligned}
\sqrt{T}\left(\widehat{\boldsymbol{\lambda}}_M - \boldsymbol{\lambda}_M\right) = \boldsymbol{Q}_{\widehat{\boldsymbol{d}}^M}^{-1} \frac{1}{\sqrt{T}} \sum_{t=1}^{T} \frac{1}{N_t} \sum_{O=1}^{N_t} \Big[& \boldsymbol{d}_{O,t}^{M} \eta_{O,t+\Delta t}^{M} + \boldsymbol{d}_{O,t}^{M} \left(\boldsymbol{\phi}_{O,t+\Delta t}^{f_M}\right)' \left(\widehat{\boldsymbol{b}}_M - \boldsymbol{b}_M\right) \\
& + \left(\widehat{\boldsymbol{d}}_{O,t}^{M} - \boldsymbol{d}_{O,t}^{M}\right) \left(\boldsymbol{d}_{O,t}^{M}\right)' \boldsymbol{\lambda}_M + \left(\widehat{\boldsymbol{d}}_{O,t}^{M} - \boldsymbol{d}_{O,t}^{M}\right) \eta_{O,t+\Delta t}^{M} \\
& + \underbrace{\left(\widehat{\boldsymbol{d}}_{O,t}^{M} - \boldsymbol{d}_{O,t}^{M}\right) \left(\boldsymbol{\phi}_{O,t+\Delta t}^{f_M}\right)' \left(\widehat{\boldsymbol{b}}_M - \boldsymbol{b}_M\right)}_{\textit{asymptotic bias term}} \Big]\,.
\end{aligned}$$

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Furthermore, all terms in the decomposition, except for the last one, have zero-expectation.

Proof. The decomposition follows from observing that $\widehat{\boldsymbol{d}}_{O,t}^{M} = \boldsymbol{d}_{O,t}^{M} + \left(\widehat{\boldsymbol{d}}_{O,t}^{M} - \boldsymbol{d}_{O,t}^{M}\right)$ and $\widehat{\boldsymbol{b}}_{M} = \boldsymbol{b}_{M} + \left(\widehat{\boldsymbol{b}}_{M} - \boldsymbol{b}_{M}\right)$ in the definition of $\widehat{\boldsymbol{\lambda}}_{M}$. The result that the first four terms have zero expectation stems from the fact that (a) the first stage estimates are unbiased, and (b) the first-stage estimation error is independent of factor realizations (by Assumption B.1(i)). ■

To derive further results, it is useful to define the matrix $\widetilde{\boldsymbol{D}}_{O,t}$ which allows to express the vector $\boldsymbol{d}_{O,t}^{M}$ as $\boldsymbol{d}_{O,t}^{M} = \widetilde{\boldsymbol{D}}_{O,t}\boldsymbol{b}_{M}$ (as a matrix product of explanatory variables and the vector of first-stage parameters).

Definition 1 *Let the matrix $\widetilde{\boldsymbol{D}}_{O,t}$ of size $DP \times Dk$ be of the form:*

$$\widetilde{\boldsymbol{D}}_{O,t} \equiv \begin{bmatrix} \boldsymbol{\phi}'_{1,O,t} \otimes \boldsymbol{g}'_{1,1,t} & 0 & 0 & \dots & 0 \\ \vdots & & & & \\ \boldsymbol{\phi}'_{1,O,t} \otimes \boldsymbol{g}'_{1,p,t} & 0 & 0 & \dots & 0 \\ \vdots & & & & \\ 0 & \dots & \boldsymbol{\phi}'_{d,O,t} \otimes \boldsymbol{g}'_{d,p,t} & \dots & 0 \\ \vdots & & & & \\ 0 & \dots & 0 & \dots & \boldsymbol{\phi}'_{D,O,t} \otimes \boldsymbol{g}'_{D,P,t} \end{bmatrix},$$

so that $\boldsymbol{d}_{O,t}^{M}$ as $\boldsymbol{d}_{O,t}^{M} = \widetilde{\boldsymbol{D}}_{O,t}\boldsymbol{b}_{M}$, where $\boldsymbol{b}_{M}$ is the $Dk \times 1$ vector of the parameters of option sensitivities to traded factors.[38]

Corollary B.1 implies that the estimator is asymptotically biased due to the fact that measurement error is present on both sides in the regression equation. Below we demonstrate how to remove the bias.

Proposition B.4 (Unbiased estimator of $\boldsymbol{\lambda}_M$)

$$\widehat{\boldsymbol{\lambda}}_{M}^{\Psi} \equiv \widehat{\boldsymbol{\lambda}}_{M} - \boldsymbol{\Psi}_{M}$$

is an unbiased estimator of the vector of risk premia parameters. The asymptotic bias term

[38] In the interest of simplicity we assume we have D factors and a k-dimensional basis expansion, and P risk premia instruments per factor. It is straightforward to generalize the result to a setting with an arbitrary number of instruments per factor.

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is given by

$$\Psi_M \equiv \left[plim\ \boldsymbol{Q}_{\widehat{\boldsymbol{d}}^M}\right]^{-1}\ plim\ \left[\frac{1}{T}\sum_{t=1}^{T}\frac{1}{N_t}\sum_{O=1}^{N_t}\widetilde{\boldsymbol{D}}_{O,t}\left(\widehat{\boldsymbol{b}}_M-\boldsymbol{b}_M\right)\left(\widehat{\boldsymbol{b}}_M-\boldsymbol{b}_M\right)'\boldsymbol{\phi}_{O,t+\Delta t}^{f_M}\right],$$

and it is estimated with

$$\widehat{\Psi}_M=\boldsymbol{Q}_{\widehat{\boldsymbol{d}}^M}^{-1}\frac{1}{T}\sum_{t=1}^{T}\frac{1}{N_t}\sum_{O=1}^{N_t}\widetilde{\boldsymbol{D}}_{O,t}\widehat{\Sigma}_{\widehat{\boldsymbol{b}}}\boldsymbol{\phi}_{O,t+\Delta t}^{f_M},$$

where $\widehat{\Sigma}_{\widehat{\boldsymbol{b}}}$ *is the submatrix of* $\widehat{\Sigma}_{\widehat{\boldsymbol{B}}}$ *from Proposition 3 that contains the covariance matrix of* $\widehat{\boldsymbol{b}}_M$.

Proof. The proof follows directly from Corollary B.1 and the observation that the final term in the decomposition of estimation noise is only a function of the estimation error in the factor loading parameters $\widehat{\boldsymbol{b}}_M$:

$$\left(\widehat{\boldsymbol{d}}_{O,t}^M-\boldsymbol{d}_{O,t}^M\right)\left(\boldsymbol{\phi}_{O,t+\Delta t}^{f_M}\right)'\left(\widehat{\boldsymbol{b}}_M-\boldsymbol{b}_M\right)=\widetilde{\boldsymbol{D}}_{O,t}\left(\widehat{\boldsymbol{b}}_M-\boldsymbol{b}_M\right)\left(\widehat{\boldsymbol{b}}_M-\boldsymbol{b}_M\right)'\boldsymbol{\phi}_{O,t+\Delta t}^{f_M}.$$

■

Using the results from Corollary B.1, we now state a result about the variance of the estimation error.

Proposition B.5 (Variance of estimation error) *The variance of estimation error consists of five components, four of which are due to measurement error arising from first-stage estimation.*

$$Var\left[\boldsymbol{Q}_{\widehat{\boldsymbol{d}}^M}\sqrt{T}\left(\widehat{\boldsymbol{\lambda}}_M-\boldsymbol{\lambda}_M-\boldsymbol{\Psi}_M\right)\right]=\boldsymbol{\Sigma}_{\boldsymbol{d}^M\boldsymbol{\eta}^M}+\boldsymbol{\Sigma}_{\boldsymbol{d}^M\widehat{\boldsymbol{b}}^M}+\boldsymbol{\Sigma}_{\widehat{\boldsymbol{d}}^M}+\boldsymbol{C}_{\widehat{\boldsymbol{b}}^M\widehat{\boldsymbol{d}}^M}+\boldsymbol{C}'_{\widehat{\boldsymbol{b}}^M\widehat{\boldsymbol{d}}^M}+\boldsymbol{\Sigma}_{\widehat{\boldsymbol{d}}^M\eta^M}.$$

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The components are

$$\begin{aligned}
\boldsymbol{\Sigma}_{\boldsymbol{d}^M\boldsymbol{\eta}^M} &\equiv Var\left[\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\frac{1}{N_t}\sum_{O=1}^{N_t}\boldsymbol{d}_{O,t}^M\eta_{O,t+\Delta t}^M\right], \\
\boldsymbol{\Sigma}_{\boldsymbol{d}^M\widehat{\boldsymbol{b}}^M} &\equiv Var\left[\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\frac{1}{N_t}\sum_{O=1}^{N_t}\boldsymbol{d}_{O,t}^M\left(\boldsymbol{\phi}_{O,t+\Delta t}^{f_M}\right)'\left(\widehat{\boldsymbol{b}}_M-\boldsymbol{b}_M\right)\right], \\
\boldsymbol{\Sigma}_{\widehat{\boldsymbol{d}}^M} &\equiv Var\left[\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\frac{1}{N_t}\sum_{O=1}^{N_t}\left(\widehat{\boldsymbol{d}}_{O,t}^M-\boldsymbol{d}_{O,t}^M\right)\left(\boldsymbol{d}_{O,t}^M\right)'\boldsymbol{\lambda}_{\boldsymbol{M}}\right], \\
\boldsymbol{C}_{\widehat{\boldsymbol{b}}^M\widehat{\boldsymbol{d}}^M} &\equiv E\left[\frac{1}{T}\sum_{t=1}^{T}\frac{1}{N_t}\sum_{O=1}^{N_t}\left(\boldsymbol{d}_{O,t}^M\right)'\widehat{\boldsymbol{\lambda}}_M\cdot\boldsymbol{d}_{O,t}^M\left(\boldsymbol{\phi}_{O,t+\Delta t}^{f_M}\right)'\left(\widehat{\boldsymbol{b}}_M-\boldsymbol{b}_M\right)\left(\widehat{\boldsymbol{d}}_{O,t}^M-\boldsymbol{d}_{O,t}^M\right)'\right], \\
\boldsymbol{\Sigma}_{\widehat{\boldsymbol{d}}^M\eta^M} &\equiv Var\left[\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\frac{1}{N_t}\sum_{O=1}^{N_t}\left(\widehat{\boldsymbol{d}}_{O,t}^M-\boldsymbol{d}_{O,t}^M\right)\eta_{O,t+\Delta t}^M\right].
\end{aligned}$$

Proof. Note that

$$\begin{aligned}
Var\left[\boldsymbol{Q}_{\widehat{\boldsymbol{d}}^M}\sqrt{T}\left(\widehat{\boldsymbol{\lambda}}_M-\boldsymbol{\lambda}_M-\boldsymbol{\Psi}_M\right)\right] = Var\Bigg[&\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\frac{1}{N_t}\sum_{O=1}^{N_t}\Big[\boldsymbol{d}_{O,t}^M\eta_{O,t+\Delta t}^M+\boldsymbol{d}_{O,t}^M\left(\boldsymbol{\phi}_{O,t+\Delta t}^{f_M}\right)'\left(\widehat{\boldsymbol{b}}_M-\boldsymbol{b}_M\right) \\
&+\left(\widehat{\boldsymbol{d}}_{O,t}^M-\boldsymbol{d}_{O,t}^M\right)\left(\boldsymbol{d}_{O,t}^M\right)'\boldsymbol{\lambda}_{\boldsymbol{M}}+\left(\widehat{\boldsymbol{d}}_{O,t}^M-\boldsymbol{d}_{O,t}^M\right)\eta_{O,t+\Delta t}^M \\
&+\left(\widehat{\boldsymbol{d}}_{O,t}^M-\boldsymbol{d}_{O,t}^M\right)\left(\boldsymbol{\phi}_{O,t+\Delta t}^{f_M}\right)'\left(\widehat{\boldsymbol{b}}_M-\boldsymbol{b}_M\right)\Big]\Bigg].
\end{aligned}$$

Then recall that $\boldsymbol{\Sigma}_{\boldsymbol{d}^M\boldsymbol{\eta}^M}$ is the variance of the standard OLS score, which is conditionally independent from the other components. $\boldsymbol{\Sigma}_{\boldsymbol{d}^M\widehat{\boldsymbol{b}}^M}$ and $\boldsymbol{\Sigma}_{\widehat{\boldsymbol{d}}^M}$ both arise from estimation error in $\widehat{\boldsymbol{b}}_M$, and thus the two underlying error components are correlated, with their covariance contributing $\boldsymbol{C}_{\widehat{\boldsymbol{b}}^M\widehat{\boldsymbol{d}}^M}$. $\boldsymbol{\Sigma}_{\widehat{\boldsymbol{d}}^M\eta^M}$ arises from the multiplicative contribution of the estimation error in $\widehat{\boldsymbol{b}}_M$ and the regression error $\eta_{t+\Delta t}^O$; these terms have zero covariance with the preceding terms due to the conditional independence of the first-stage estimation noise and second-stage regression error $\eta_{t+\Delta t}^O$. ■

Proposition B.6 (Estimators of estimation noise variance) *The consistency of $\widehat{\boldsymbol{b}}^M$ (from Proposition 3) implies that in what follows, when estimating the components of estimation noise variance listed in Proposition B.5 we can replace $\boldsymbol{d}^M=\widetilde{\boldsymbol{D}}_{\boldsymbol{O},\boldsymbol{t}}\boldsymbol{b}_M$ with $\widehat{\boldsymbol{d}}^M=\widetilde{\boldsymbol{D}}_{\boldsymbol{O},\boldsymbol{t}}\widehat{\boldsymbol{b}}_M$ where applicable. Furthermore, Proposition B.4 implies that we can also replace $\boldsymbol{\lambda}_M$ with $\widehat{\boldsymbol{\lambda}}_M^{\Psi}$. Under assumption (iv), the following are consistent estimators of estimation noise variance.*

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(a) $\boldsymbol{\Sigma}_{\boldsymbol{d}^M\boldsymbol{\eta}^M}$. *Define* $\boldsymbol{h}_t \equiv \frac{1}{N_t}\sum_{O=1}^{N_t}\widehat{\boldsymbol{d}}_{O,t}^M\eta_{O,t+\Delta t}^M$ *and set* S*, a positive integer lag order.* $\boldsymbol{\Sigma}_{\boldsymbol{d}^M\boldsymbol{\eta}^M}$ *can be estimated with*

$$\widehat{\boldsymbol{\Sigma}}_{\boldsymbol{d}^M\boldsymbol{\eta}^M} \equiv \frac{1}{T}\sum_{t=1}^{T}\boldsymbol{h}_t\boldsymbol{h}_t' + \frac{1}{T}\sum_{s=1}^{S}\sum_{t=s+1}^{T} w_s\left(\boldsymbol{h}_t\boldsymbol{h}_{t-l}' + \boldsymbol{h}_{t-l}\boldsymbol{h}_t'\right),$$

where $w_s = 1 - \frac{s}{S+1}$*, which is the Newey and West (1987) estimator applied to the time series of* $\boldsymbol{h}_t$*.*

(b) $\boldsymbol{\Sigma}_{\boldsymbol{d}^M\widehat{\boldsymbol{b}}^M}$. *The estimator of the variance component is given by*

$$\widehat{\boldsymbol{\Sigma}}_{\boldsymbol{d}^M\widehat{\boldsymbol{b}}^M} \equiv \left[\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\frac{1}{N_t}\sum_{O=1}^{N_t}\widehat{\boldsymbol{d}}_{O,t}^M\left(\boldsymbol{\phi}_{O,t+\Delta t}^{f_M}\right)'\right]\widehat{\boldsymbol{\Sigma}}_{\widehat{\boldsymbol{b}}_M}\left[\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\frac{1}{N_t}\sum_{O=1}^{N_t}\boldsymbol{\phi}_{O,t+\Delta t}^{f_M}\left(\widehat{\boldsymbol{d}}_{O,t}^M\right)'\right],$$

where $\widehat{\boldsymbol{\Sigma}}_{\widehat{\boldsymbol{b}}_M}$ *is the submatrix of* $\widehat{\boldsymbol{\Sigma}}_{\widehat{\boldsymbol{B}}}$ *defined in Proposition 3 that corresponds to the loadings on tradable factors.*

(c) $\boldsymbol{\Sigma}_{\widehat{\boldsymbol{d}}^M}$. *Replace* $\left(\boldsymbol{d}_{O,t}^M\right)'\boldsymbol{\lambda}_M$ *with* $\left(\widehat{\boldsymbol{d}}_{O,t}^M\right)'\widehat{\boldsymbol{\lambda}}_M^{\Psi}$*. Then use the fact that* $\widehat{\boldsymbol{d}}_{O,t}^M - \boldsymbol{d}_{O,t}^M = \widetilde{\boldsymbol{D}}_{O,t}\left(\widehat{\boldsymbol{b}}_M - \boldsymbol{b}_M\right)$*. The estimator is given by*

$$\widehat{\boldsymbol{\Sigma}}_{\widehat{\boldsymbol{d}}^M} = \left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\frac{1}{N_t}\sum_{O=1}^{N_t}\left(\left(\widehat{\boldsymbol{d}}_{O,t}^M\right)'\widehat{\boldsymbol{\lambda}}_M^{\Psi}\right)\cdot\widetilde{\boldsymbol{D}}_{O,t}\right)\widehat{\boldsymbol{\Sigma}}_{\widehat{\boldsymbol{b}}_M}\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\frac{1}{N_t}\sum_{O=1}^{N_t}\left(\left(\widehat{\boldsymbol{d}}_{O,t}^M\right)'\widehat{\boldsymbol{\lambda}}_M^{\Psi}\right)\cdot\widetilde{\boldsymbol{D}}_{O,t}'\right).$$

(d) $\boldsymbol{C}_{\widehat{\boldsymbol{b}}^M\widehat{\boldsymbol{d}}^M}$. *Replace* $\left(\boldsymbol{d}_{O,t}^M\right)'\boldsymbol{\lambda}_M$ *with* $\left(\widehat{\boldsymbol{d}}_{O,t}^M\right)'\widehat{\boldsymbol{\lambda}}_M^{\Psi}$*. The estimator is given by*

$$\widehat{\boldsymbol{C}}_{\widehat{\boldsymbol{b}}^M\widehat{\boldsymbol{d}}^M} \equiv \frac{1}{T}\left(\sum_{t=1}^{T}\frac{1}{N_t}\sum_{O=1}^{N_t}\left(\widehat{\boldsymbol{d}}_{O,t}^M\right)'\widehat{\boldsymbol{\lambda}}_M^{\Psi}\cdot\widehat{\boldsymbol{d}}_{O,t}^M\left(\boldsymbol{\phi}_{O,t+\Delta t}^{f_M}\right)'\right)\widehat{\boldsymbol{\Sigma}}_{\widehat{\boldsymbol{b}}_M}\left(\sum_{t=1}^{T}\frac{1}{N_t}\sum_{O=1}^{N_t}\widetilde{\boldsymbol{D}}_{O,t}\right).$$

(e) $\boldsymbol{\Sigma}_{\widehat{\boldsymbol{d}}^M\boldsymbol{\eta}^M}$. *With* $\widehat{\boldsymbol{d}}_{O,t}^M - \boldsymbol{d}_{O,t}^M = \widetilde{\boldsymbol{D}}_{O,t}\left(\widehat{\boldsymbol{b}}_M - \boldsymbol{b}_M\right)$*, define* $\widehat{\boldsymbol{\Sigma}}_{\widehat{\boldsymbol{b}}_M}^{1/2}$ *as the principal matrix square root of* $\widehat{\boldsymbol{\Sigma}}_{\widehat{\boldsymbol{b}}_M}$*, and define* $\boldsymbol{h_t} \equiv \frac{1}{N_t}\sum_{O=1}^{N_t}\eta_{O,t+\Delta t}^M\widetilde{\boldsymbol{D}}_{O,t}\widehat{\boldsymbol{\Sigma}}_{\widehat{\boldsymbol{b}}_M}^{1/2}$*. Set* S*, a positive integer lag order.* $\boldsymbol{\Sigma}_{\widehat{\boldsymbol{d}}^M\boldsymbol{\eta}^M}$ *can be estimated with*

$$\widehat{\boldsymbol{\Sigma}}_{\widehat{\boldsymbol{d}}^M\boldsymbol{\eta}^M} \equiv \frac{1}{T}\sum_{t=1}^{T}\boldsymbol{h}_t\boldsymbol{h}_t' + \frac{1}{T}\sum_{s=1}^{S}\sum_{t=s+1}^{T} w_s\left(\boldsymbol{h}_t\boldsymbol{h}_{t-l}' + \boldsymbol{h}_{t-l}\boldsymbol{h}_t'\right),$$

where $w_s = 1 - \frac{s}{S+1}$*, which is the Newey and West (1987) estimator applied to the time series of* $\boldsymbol{h}_t$*.*

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B.2.2 Premia on non-tradable factors

The results in this section closely follow the preceding section, with the exception that the projection of option returns on first-stage variables contains the $-E^Q[f_{k,t+\Delta t}]$ component:

$$R^{O,\Omega}_{t+\Delta t} \equiv \sum_{k=D+1}^{L} \left(f_{k,t+\Delta t} - E^Q[f_{k,t+\Delta t}]\right) \beta^O_{k,t} = \left(\boldsymbol{\phi}^{gf_\Omega}_{O,t+\Delta t}\right)' \boldsymbol{B}_\Omega \,,$$

where Ω denotes the set of the non-tradable factors, $\boldsymbol{\phi}^{gf_\Omega}_{O,t+\Delta t}$ is the subvector of the vector $\boldsymbol{\phi}^{gf}_{O,t+\Delta t}$ associated with the non-tradable factors and the corresponding risk-premia predictors, and $\boldsymbol{B}_\Omega$ is the subvector of $\boldsymbol{B}$ which contains $\boldsymbol{a}$ and the subvector of $\boldsymbol{b}$ associated with the non-tradable factors.

Let $\boldsymbol{d}^\Omega_{O,t} \equiv \left[\beta^O_{D+1,t}\boldsymbol{g}'_{D+1,t} \quad \dots \quad \beta^O_{L,t}\boldsymbol{g}'_{L,t}\right]$, and let $\widehat{\boldsymbol{d}}^\Omega_{O,t}$ be its sample analogue, i.e., with $\widehat{\beta}^O_{k,t}$ replacing $\beta^O_{k,t}$ and $\widehat{\gamma}^O_t$ replacing γ^O_t. Then the following relationship holds between the projection of option returns on the tradable factors and the risk premia predictors,

$$R^{O,\Omega}_{t+\Delta t} = \left(\boldsymbol{d}^\Omega_{O,t}\right)' \boldsymbol{\lambda}_\Omega + \eta^\Omega_{O,t+\Delta t} \,.$$

With the true factor loadings unknown, we estimate $\boldsymbol{\lambda}_\Omega$ based on the feasible version of the above equation,

$$\widehat{R}^{O,\Omega}_{t+\Delta t} \equiv \left(\boldsymbol{\phi}^{gf_\Omega}_{O,t+\Delta t}\right)' \widehat{\boldsymbol{B}}_\Omega = \left(\boldsymbol{d}^\Omega_{O,t}\right)' \boldsymbol{\lambda}_\Omega + \widetilde{\eta}^\Omega_{O,t+\Delta t} \,. \tag{B.4}$$

Inference in the case of non-tradable factors follows the results of Section B.2.1 with $\boldsymbol{d}^\Omega_{O,t}$ replacing $\boldsymbol{d}^M_{O,t}$, $\widehat{\boldsymbol{B}}_\Omega$ replacing $\widehat{\boldsymbol{b}}_M$, and $\boldsymbol{\phi}^{gf_\Omega}$ replacing $\boldsymbol{\phi}^{gf_M}$ and it relies on the following assumption:

Assumptions B.3 *(v)* $E\left[\eta^\Omega_{O,t+\Delta t}|\boldsymbol{\phi}^{gf_\Omega}_{O,t+\Delta t}\right] = 0$

(vi) $\eta^\Omega_{O,t+\Delta t}$ *is spatially strong mixing, as in Driscoll and Kraay (1998).*

Internet Appendix C Thin Plate Regression Spline (TPRS) Basis

Our approach to the modeling of option exposures builds on Thin Plate Regression Spline basis (TPRS basis). In this Appendix we introduce the reader to the optimal construction of the parsimonious TPRS basis from its high-dimensional Thin Plate Smoothing Spline basis origin.

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Basis expansion theory is based on the idea that any sufficiently well-behaved and smooth function can be expressed as a linear combination of basis functions. Any basis expansion is a combination of basis functions and loading parameters. In our case, the functions of interest to be estimated are option exposures and the measurement equation used to estimate these exposures (i.e., the basis parameters) corresponds to the dynamics of deleveraged option returns (equations (1)-(2) in the main text), which are directly related to option exposures. We focus on a Spline-based basis because in this framework it is easy to control the smoothness – or differentiability – of the resulting estimates.

For illustration purposes, let us denote by $\tilde{\beta}_l(\mathbf{z}_{O,l,t})$ the true exposure of option O to factor l given the vector of signals $\mathbf{z}_{O,l,t}$. Once a basis of a given dimension κ is chosen, the value of $\tilde{\beta}_l(\cdot)$ at each of the $N \equiv \sum_{t=1}^{T} N_t$ (where T is the number of days in the sample) observations in $Z \equiv \{\mathbf{z}_{O,l,t}\}_{\forall O,t}$ can be approximated as a linear combination of the κ basis functions. For $\mathbf{z} \in Z$, we have

$$\tilde{\beta}_l(\mathbf{z}) \approx \beta_l(\mathbf{z}) \equiv \sum_{j=1}^{\kappa} \phi_j(\mathbf{z})\, b_l^j. \tag{C.1}$$

In our context, an appropriate basis expansion should achieve two key objectives. As we discuss in Section 2.2 of the main text, option pricing theory suggests that using a multidimensional vector $\mathbf{z}_{O,l,t}$ to estimate exposures is important. As a result, a good basis expansion for option exposures needs to efficiently handle a multidimensional vector of signals. Second, the basis expansion chosen should avoid overfitting, be computationally efficient, and limit ad-hoc specifications including ad-hoc choices of knots, i.e., points $\boldsymbol{z}_s \in Z_l$ which implicitly set the basis dimension κ. To meet these objectives, we choose the Thin Plate Regression Spline Basis (TPRS basis) which was originally developed by Wood (2003). The TPRS basis expansion extends Thin Plate Smoothing Splines (TPSS) by allowing for a low dimensional basis representation of the function of interest (i.e., low κ). TPSS in turn is a multivariate extension of Cubic Smoothing Splines (CSS). We first introduce the foundations of the TPSS-basis. We then extend our discussion to the TPRS basis.

Both CSS and TPSS builds on the notion of *roughness* of the estimated function. In the univariate case when the dimension of $\boldsymbol{z}$ is 1, the roughness of $\widetilde{\beta}_l$ can be measured by

$$\int_{z \in D} \left[\frac{d^m \widetilde{\beta}_l(z)}{(dz)^m} \right]^2 dz, \tag{C.2}$$

where D is the domain of $\widetilde{\beta}_l$. Intuitively, equation (C.2) measures the integrated variation of

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the m^{th} derivative of $\widetilde{\beta}_l$. The notion of *roughness* as defined by equation (C.2) is thus inversely related to the level of smoothness of β_l. From a fitting perspective, excessive *roughness* of the estimated function is indicative of the degree of overfitting. The CSS basis expansion is obtained by solving a functional optimization problem by minimizing the sum of squared basis residuals $\sum_{\mathbf{z}\in Z_l}\left(\tilde{\beta}_l(z)-\beta_l(z)\right)^2$ over all possible well-behaved functions $\beta_l(\cdot)$ subject to a penalty of the form (C.2) when z is a scalar and m is set to 2 in (C.2). TPSS extends the CSS algorithm by allowing for multivariate signals and by providing a general solution of the mean square problem taking into consideration all derivatives of β_l in (C.2) of the m^{th}-order which is also function of cross-derivatives in the multivariate $\mathbf{z}$ case.[39] The optimal solution of this problem gives the following TPSS-basis approximation of the function $\widetilde{\beta}_l : \mathbb{R}^d \to \mathbb{R}$

$$\tilde{\beta}_l(\mathbf{z}) = \sum_{s=1}^{M} \omega_{s,l} \cdot \nu_s(\mathbf{z}) + \sum_{u=1}^{N_l} \delta_{u,l} \cdot f_{m,d}(\mathbf{z}, \mathbf{z}_u), \tag{C.3}$$

where $\nu_s(\cdot) : \mathbb{R}^d \to \mathbb{R}$ and $f_{m,d}(\cdot, \mathbf{z}_u) : \mathbb{R}^d \to \mathbb{R}$ are basis functions which exact definitions are provided below, and $\omega_{s,l}$ and $\delta_{u,l}$ are the basis parameters to be estimated. Equation (C.3) can be rewritten in matrix form as

$$\widetilde{\beta}_l = \mathbf{V} \cdot \mathbf{w}_l + \mathbf{F} \cdot \delta_l + \varepsilon^{TPSS}, \tag{C.4}$$

where $\mathbf{V}$ is an $N_l \times M$ matrix with elements $V_{ij} = \nu_j(\mathbf{z}_i)$ for $i = 1, \ldots, N$ and $j = 1, \ldots, M$, and $\mathbf{F}$ is an $N_l \times N_l$ matrix with elements $F_{ij} = f_d(\mathbf{z}_i, \mathbf{z}_j)$ for $i, j = 1, \ldots, N_l$. In the previous equation, $\mathbf{w}_l$ and δ_l are two vectors of coefficients and $\widetilde{\beta}_l$ is the $N_l \times 1$ vector of true exposures.[40] Moreover, $M = \frac{(m+d-1)!}{d!(m-1)!}$ where $m > d/2$ is an integer that defines the order of the derivatives taken into account for the "*roughness*" penalization to obtain the TPSS solution.

In equations (C.3) and (C.4), $\nu_s(\cdot)$ are all the monomials of the elements of $\mathbf{z}$ and of order smaller than or equal to $m-1$.[41] Recall that our benchmark specification uses $\mathbf{z}_{O,l,t} = [k_{O,l,t}\ \tau_{O,l,t}\ v_t]$ and thus $d = 3$. In addition, we consider the case of $m = 2$ (i.e., quadratic

[39] In the two predictors case with $d = 2$ and with roughness measured using the second derivative ($m = 2$), we have $\int\int \left(\left[\frac{d^2\beta_l(\mathbf{z})}{(dz_1)^2}\right]^2 + \left[\frac{d^2\beta_l(\mathbf{z})}{dz_1 dz_2}\right]^2 + \left[\frac{d^2\beta_l(\mathbf{z})}{(dz_2)^2}\right]^2 \right) dz_1 dz_2$.

[40] To ensure the uniqueness of the coefficients, it is necessary that $\sum_{u=1}^{N} \delta_{u,l}\nu_s(\mathbf{z}_u) = 0$ for all s such that $\mathbf{V}'\delta_l = \mathbf{0}_{M\times 1}$. This restriction in turn guarantees that the penalized least square criterion $\varepsilon^{TPSS'}\varepsilon^{TPSS}$ is positive definite and admits a unique solution (i.e., the vectors $\mathbf{w}_l$ and δ_l are unique).

[41] Monomials are polynomials with just one term in each dimension of a given order. For example when $\mathbf{z}$ is of dimension 3, one monomial of $\mathbf{z} = [z_1\ z_2\ z_3]$ is $z_1^3 z_2^2 z_3^{74}$ for instance.

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"*roughness*" penalization) which together with d implies that $M = 4$. In this case, we have $\nu_1(\mathbf{z}_{O,l,t}) = 1$, $\nu_2(\mathbf{z}_{O,l,t}) = k_{O,l,t}$, $\nu_3(\mathbf{z}_{O,l,t}) = \tau_{O,l,t}$, and $\nu_4(\mathbf{z}_{O,l,t}) = v_t$.[42] The N_l functions $f_{m,d}(\cdot, \mathbf{z}_u)$ are then defined as

$$f_{m,d}(\mathbf{z}, \mathbf{z}_u) = \begin{cases} \theta_{e,d} \, ||\mathbf{z} - \mathbf{z}_u||^{2m-d} \ln(||\mathbf{z} - \mathbf{z}_u||) & \text{for } d \text{ even} \\ \theta_{o,d} \, ||\mathbf{z} - \mathbf{z}_u||^{2m-d} & \text{for } d \text{ odd} \end{cases}, \tag{C.5}$$

for $\mathbf{z} \in Z_l$, and where $||\cdot||$ denotes the Euclidian norm. In the previous definition, the constants $\theta_{e,d}$ and $\theta_{o,d}$ are given by

$$\begin{aligned} \theta_{e,d} &= \frac{(-1)^{m+1+d/2}}{2^{2m-1}\pi^{d/2}(m-1)!(m-d/2)!} \\ \theta_{o,d} &= \frac{\Gamma(d/2-m)}{2^{2m}\pi^{d/2}(m-1)!}. \end{aligned} \tag{C.6}$$

There are two important drawbacks of the TPSS-basis. First, when the dimension d of $\mathbf{z}$ is greater than 2 the evaluation of the basis functions becomes computationally expensive for large data panels. Second, penalized estimation becomes necessary which in turn makes inference challenging. Both problems are alleviated by the TPRS basis. The TPRS basis chooses a reduced-dimension basis among all the functions $f_{m,d}(\cdot, \mathbf{z}_u)$ while keeping $\nu_s(\cdot)$ unchanged. As a result, the TPRS basis is much less computationally demanding than TPSS-basis and its low-dimensional nature allows for OLS estimation of basis parameters without relying on penalization methods as the dimension reduction is already built into the algorithm.

An *ad-hoc* TPRS basis can be constructed by first choosing κ distinct points (knots) from Z_l, $\mathbf{z}_1$ through $\mathbf{z}_\kappa$ which then define the κ basis functions $f_{m,d}(\cdot, \mathbf{z}_u)$, $u = 1, \ldots, \kappa$. In a hypothetical situation where $\widetilde{\beta}_l$ is directly observed, a regression model for $\widetilde{\beta}_l$ can be formulated based on equation (C.3) for $\mathbf{z}_{O,l,t} \in Z_l$ as

$$\widetilde{\beta}_l(\mathbf{z}_{O,l,t}) = \sum_{s=1}^{M} w_{s,l} \cdot \nu_s(\mathbf{z}_{O,l,t}) + \sum_{u=1}^{\kappa} \delta_{\kappa,u,l} \cdot f_{m,d}(\mathbf{z}_{O,l,t}, \mathbf{z}_u) + \varepsilon_{O,l,t}^{TPRS}. \tag{C.7}$$

Defining the $N_l \times \kappa$ matrix $\widetilde{\mathbf{F}}_\kappa$ with element $\widetilde{F}_{ij} = f_d(\mathbf{z}_i, \mathbf{z}_u)$ with $i = 1, \ldots, N_l$ where a given $\mathbf{z}_i$ is a particular realization of $\mathbf{z}_{O,l,t}$ in Z_l and $j = 1, \ldots, \kappa$. One can rewrite equation (C.7) as

$$\widetilde{\beta}_l = \mathbf{V} \cdot \mathbf{w}_l + \widetilde{\mathbf{F}}_\kappa \cdot \delta_{\kappa,l} + \varepsilon^{TPRS}. \tag{C.8}$$

Because an inappropriate choice of the κ knots may significantly impacts model fit, Wood

[42] Note that the functions $\nu_s(\cdot)$ have a *roughness* equal to 0 by construction and thus are not penalized.

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(2003) develops a general algorithm that built on an optimal basis derived from the spectral approximation of $\mathbf{F}$ while imposing the uniqueness restriction $\sum_{u=1}^{N_l} \delta_{u,l}\nu_s(\mathbf{z}_u) = 0$, $\mathbf{V}'\delta_l = \mathbf{0}_{M\times 1}$.

For a given basis dimension κ, the goal of the TPRS-algorithm is to construct based on Z_l, $\kappa - M$ basis functions which, together with $\nu_s(\cdot)$, summarizes the information from the complete TPSS-basis represented by $\mathbf{F}$. The guiding principle is that the approximating low-dimensional basis $\widetilde{\mathbf{F}}_\kappa$ should allow for a minimal change in model fit from the full TPSS basis as described in Wood (2003). The construction starts with the eigendecomposition of the matrix $\mathbf{F} = \mathbf{U}\mathbf{D}\mathbf{U}'$ where $\mathbf{D}$ is the diagonal matrix of eigenvalues and $\mathbf{U}$ contains the eigenvectors. The eigenvalues in $\mathbf{D}$ are sorted so that $|D_{i,i}| \geq |D_{i+1,i+1}|$, and the eigenvectors in $\mathbf{U}$ have the corresponding order. The spectral approximation of $\mathbf{F}$ of rank κ satisfies

$$\widehat{\mathbf{F}}_\kappa = \mathbf{U}_\kappa \mathbf{D}_\kappa \mathbf{U}_\kappa', \tag{C.9}$$

where the columns of $\mathbf{U}_\kappa$ contain the first κ eigenvectors associated with the first κ eigenvalues and $\mathbf{D}_\kappa$ is the diagonal matrix of the first κ eigenvalues. For uniqueness of the TPRS basis representation, one still need to impose $\mathbf{V}'\delta_l = \mathbf{0}_{M\times 1}$. To this end, note that the κ columns of the $N_l \times \kappa$ matrix $\mathbf{U}_\kappa$ form an orthogonal basis. The dimension reduction of the TPRS basis constraints the TPSS-basis coefficient vector according to $\delta_l = \mathbf{U}_\kappa \delta_{\kappa,l}$. We thus have

$$\begin{aligned}\widetilde{\beta}_l &= \mathbf{V}\cdot\mathbf{w}_l + \widehat{\mathbf{F}}_\kappa \cdot \mathbf{U}_\kappa\delta_{\kappa,l} + \varepsilon^{TPRS} \\ &= \mathbf{V}\cdot\mathbf{w}_l + \mathbf{U}_\kappa\mathbf{D}_\kappa \cdot \delta_{\kappa,l} + \varepsilon^{TPRS},\end{aligned} \tag{C.10}$$

under the constraint $\mathbf{V}'\delta_l = \mathbf{V}'\mathbf{U}_\kappa\delta_{\kappa,l} = \mathbf{0}_{M\times 1}$. Up to this point, the TPRS basis replaced $\mathbf{F}$ with $\mathbf{U}_\kappa\mathbf{D}_\kappa$ of dimension $N_l \times \kappa$ while keeping the basis functions $\nu_s(\cdot)$ of dimension M unchanged. This implies that the total dimension of the basis equal to $\kappa + M$. To arrive at the final dimension κ, the TPRS basis exploits the constraint $\mathbf{V}'\mathbf{U}_\kappa\delta_{\kappa,l} = \mathbf{0}_{M\times 1}$ to restrict possible parameter vectors to a lower-dimensional space. To do this, TPRS basis finds an orthogonal column basis $\mathbf{Q}_{\kappa-M}$ which solves $\mathbf{V}'\mathbf{U}_\kappa\mathbf{Q}_{\kappa-M} = \mathbf{0}_{M\times(\kappa-M)}$. Then TPRS basis represents the coefficient vector $\delta_{\kappa,l}$ as $\delta_{\kappa,l} = \mathbf{Q}_{\kappa-M}\delta_{\kappa-M,l}$ where $\mathbf{V}'\mathbf{U}_\kappa\mathbf{Q}_{\kappa-M}\delta_{\kappa-M,l} = \mathbf{0}_{M\times 1}$ is satisfied for any $\delta_{\kappa-M,l}$ by construction. A $\mathbf{Q}_{\kappa-M}$ can then be easily found from the QR decomposition of $\mathbf{U}_\kappa'\mathbf{V}$. It is given by the final $\kappa - M$ columns of the orthogonal factor $\mathbf{Q}$.[43]

[43]The QR decomposition of a matrix $\boldsymbol{X}$ factorizes $\boldsymbol{X} = \boldsymbol{Q}\boldsymbol{R}$ where $\boldsymbol{Q}$ is an orthogonal matrix and $\boldsymbol{R}$ is an upper triangular matrix.

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Armed with $\mathbf{Q}_{\kappa-M}$, the TPRS basis representation can be written (without constraints) as

$$\begin{aligned}\widetilde{\beta}_l &= \mathbf{V}\cdot\mathbf{w}_l + \mathbf{U}_\kappa\mathbf{D}_\kappa\mathbf{U}_\kappa^{'}\cdot\mathbf{U}_\kappa\mathbf{Q}_{\kappa-M}\delta_{\kappa-M,l} + \varepsilon^{TPRS} \\ &= \mathbf{V}\cdot\mathbf{w}_l + \mathbf{U}_\kappa\mathbf{D}_\kappa\mathbf{Q}_{\kappa-M}\cdot\delta_{\kappa-M,l} + \varepsilon^{TPRS} \\ &= \mathbf{V}\cdot\mathbf{w}_l + \mathbf{F}_{\kappa-M}\cdot\delta_{\kappa-M,l} + \varepsilon^{TPRS},\end{aligned} \tag{C.11}$$

where we have used the fact that $\mathbf{U}_\kappa^{'}\cdot\mathbf{U}_\kappa$ is equal to the identity matrix to obtain the second equality, defined $\mathbf{F}_{\kappa-M}\equiv\mathbf{U}_\kappa\mathbf{D}_\kappa\mathbf{Q}_{\kappa-M}$ in the third, and where $\delta_{\kappa-M,l}$ is the matrix of TPRS basis parameters of dimension $N_l\times(\kappa-M)$. Each row i of the matrix is constructed such that, after multiplying it by the parameter vector $\delta_{\kappa-M}$, it approximates the i-th row of the full basis matrix $\mathbf{F}$ multiplied by the parameter vector $\delta_l=\mathbf{U}_\kappa\mathbf{Q}_{\kappa-M}\delta_{\kappa-M,l}$. Each element $\mathbf{F}_{\kappa-M,ij}$ of the matrix $\mathbf{F}_{\kappa-M}$ is a linear combination of the i-th row of the rank-κ spectral approximation of $\mathbf{F}$ where the coefficients of the linear combination are given by the j-th column of $\mathbf{U}_\kappa\mathbf{Q}_{\kappa-M}$.

We can re-write equation (C.11) as equation (14) in the paper. We have

$$\beta_l\left(\mathbf{z}_{O,l,t}\right) = \begin{bmatrix}\mathbf{V}\\ \mathbf{F}_{\kappa-M}\end{bmatrix}^{'}\cdot\begin{bmatrix}\mathbf{w}_l\\ \delta_{\kappa-M,l}\end{bmatrix} = \phi_{O,l,t}^{'}\mathbf{b}_l = \sum_{j=1}^{k}\phi_{O,l,t}^{j}b_l^j. \tag{C.12}$$

To implement the TPRS basis expansion, we use the `R` package `mgcv` version `1.8.26` which directly builds on Wood (2003) and Wood (2017).

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